ELECTROSTATICS 12

CONTENTS

ELECTROSTATICS:

The branch of physics which deals with study of charges which are in rest is called electrostatics. ***OR***

"Study of static charges is called electrostatics"

CHARGE:

If the attractive or repulsive property produced between two bodies due to removal or absorption of electrons then the body is called charge, denoted by "q" or "Q"

(i) A negatively charged object has excess of force electrons.

(ii) A positively charged object has deficiency of free electrons.

(iii) Similar charges repel each other.

(iv) Opposite charges attract each other.

(v) The magnitude of charge can be determine as follows.

$$\boxed{q = ne}$$

n = Number of electrons

e = Charge of electron = 1.6×10^{-19} C

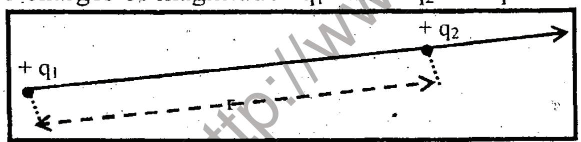
12.1 COULOMB'S LAW:

Statement:

"The electrostatic repulsive or attractive force between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them."

Mathematical Form:

Consider two charges of magnitude "q_1" and "q_2" are placed at distance "r" from eachother,



From statement

$$F \propto q_1 q_2 \text{ ----------------- (i)}$$

And

$$F \propto \frac{1}{r^2} \text{ ----------------- (ii)}$$

Comparing (i) and (ii)

$$F \propto q_1 q_2 \frac{1}{r^2}$$

Or

$$F = (\text{Constant}) \frac{q_1 q_2}{r^2}$$

Where this constant is called Coulomb's Law constant, denoted by "K" and its value depends upon medium between the charges and the system of units used.

$$\therefore \quad \boxed{F = K\frac{q_1 q_2}{r^2}} \text{ ---------------- (i)}$$

Since force is a vector quantity there fore in vector form (i) can be written as

$$\boxed{\vec{F}_{12} = K\frac{q_1 q_2}{r^2}\,\hat{r}_{12}}$$

Where $\vec{F}_{12}$ = Force exerted by "q_1" on "q_2"

$\hat{r}_{12}$ = Unit vector along the line joining the two charges from q_1 to q_2 value of "K" is

$$K = \frac{1}{4\pi \epsilon_o} = 9 \times 10^9 \text{Nm}^2/\text{c}$$

$$\boxed{\vec{F} = \frac{1}{4\pi \epsilon_o}.\frac{q_1 q_2}{r^2}.\hat{r}} \text{ ------------(ii)}$$

Equation (ii) represents the general form of electrostatic force between two charges in presence of air or free space between them.

ϵ_o = Permittivity of air or free space and in SI system its value is

$\epsilon_o = 8.85 \times 10^{-12}\ c^2/N\text{-}m^2$

COULOM'S FORCE BETWEEN CHARGES IN PRESENCE OF ANY DELECTRIC MEDIUM:

In presence of any dielectric medium other than air or free space, electrostatic force between the charges can be written as

$$\vec{F}' = \frac{1}{4\pi\epsilon}\cdot\frac{q_1q_2}{r^2}\hat{r} \text{ .... (ii)}$$

Where ϵ = Absolute permittivity of medium we also know that

$\epsilon_r = \frac{\epsilon}{\epsilon_o}$ = Relative permittivity of the medium

$\therefore \quad \epsilon = \epsilon_r\epsilon_o$

$$\therefore \quad (a) \Rightarrow \vec{F}' = \frac{1}{4\pi\epsilon_r\epsilon_o}\cdot\frac{q_1q_2}{r^2}\hat{r}$$

$$\vec{F}' = \frac{1}{\epsilon_r}\left(\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1q_2}{r^2}\hat{r}\right)$$

But

$$\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1q_2}{r^2}\hat{r} = \vec{F}$$

$$\therefore \quad \boxed{\vec{F}' = \frac{1}{\epsilon_r}\vec{F}} \text{ .... (iv)}$$

Or

$$\boxed{\epsilon_r = \frac{\vec{F}}{\vec{F}'}}$$

Equation "(iv)" shows that the presence of any medium other than air or vacume reduces the electrostatic force between the electric charges by factor "ϵ_r" which is called relative permittivity or dielectric constant.

Since value of ϵ_r always greater than one therefore, $\vec{F}' < \vec{F}$

12.2 ELECTRIC FIELD INTENSITY:

The strength of an electric field is known as its intensity.

When a unit positive charge brought in an electric field of any source charge at a point, the charge will experience a force at that point. Now electric field intensity can define as

"The force experienced by a unit positive charge placed at any point of the field is called electric field intensity, denoted by "E" Because intensity is a vector quantity so, it is also denoted by "$\vec{E}$".

Mathematical Form:

Consider a point charge q_o is placed at point "P" of an electric field of source charge "q" therefore, from definition.

$$E = \frac{F}{q_o} \quad \text{or} \quad F = Eq_o$$

In vector from

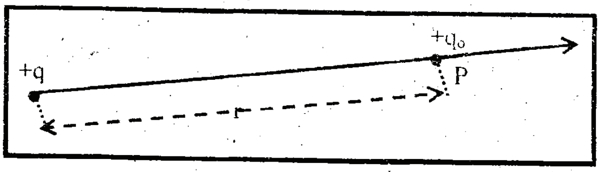
$$\vec{E} = \frac{\vec{F}}{q_o} \quad \text{or} \quad \vec{F} = \vec{E}q_o$$

Unit:

In S.I system unit of electric field intensity is Newton per coulomb (N/c).

12.3 ELECTRIC FIELD INTENSITY DUE TO A POINT CHARGE:

Consider a test charge q_o is placed at a point of an electric field of source charge "+q" in presence of air or free space between them. Also suppose that distance between +q and q_o is "r" as shown.



According to the Coulomb's law

$$F = \frac{1}{4\pi \epsilon_o} \cdot \frac{q_1 q_2}{r^2}$$

Where $q_1 = q$ and $q_2 = q_o$

Therefore, $F = \frac{1}{4\pi \epsilon_o} \cdot \frac{q q_o}{r^2}$

Or $\frac{F}{q_o} = \frac{1}{4\pi \epsilon_o} \cdot \frac{q}{r^2}$

From definition of intensity

$$\frac{F}{q_o} = E$$

$$\therefore \boxed{E = \frac{1}{4\pi \epsilon_o} \cdot \frac{q}{r^2}} \quad \text{.............(i)}$$

In vector form

$$\boxed{\vec{E} = \frac{1}{4\pi \epsilon_o} \cdot \frac{q}{r^2} \cdot \hat{r}} \quad \text{............(ii)}$$

It is the electric field intensity due to point charge in presence of air or vacume.

IN PRESENCE OF ANY DIELECYTRIC MEDIUM:

If medium is other than air, then the intensity is given by

$$\vec{E}' = \frac{1}{4\pi \epsilon} \cdot \frac{q}{r^2} \cdot \hat{r}$$

But $\epsilon = \epsilon_r \epsilon_o$

$$\therefore \quad \vec{E}' = \frac{1}{\epsilon_r}\left(\frac{1}{4\pi \epsilon_o} \cdot \frac{q}{r^2} \cdot \hat{r}\right)$$

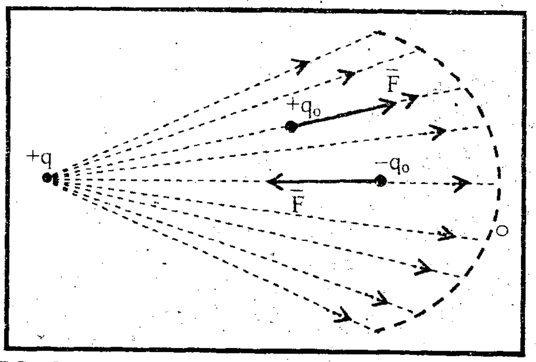
From (ii) $\vec{E}' = \frac{1}{\epsilon_r} \vec{E}$

This show that the intensity decreases by on amount ϵ_r

Or $\epsilon_r = \frac{\vec{E}}{\vec{E}'}$

12.4 ELECTRIC FIELD OF CHARGE:

"The particular region / space or area around a charged particle (source charge q) in which it exerts a force on other charged particles is called the electric field of the source charge."

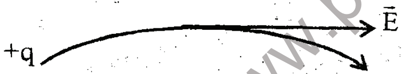


ELECTRIC LINES OF FORCE:

Electric field of any charge represented by a number of lines, called electric lines of force it is define as

"The paths on which any test charge moves towards or away from the source change are called electric lines of force."

Electric lines of force always initiate from +ve charge and terminate on –ve charge. Direction of electric field intensity "$\vec{E}$" and $\vec{F}$ is direction of tangent drawn at any point of line.



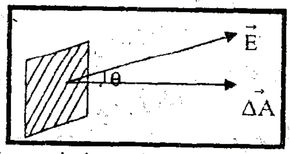
12.5 ELECTRIC FLUX $\Delta\phi$:

"The total number of electric lines of forces passing through a surface is called the electric flux".

The flux passing through a surface depends upon the electric intensity at the point and the vector area of the electric field, so electric flux can also be defined as ***"the dot product of electric field intensity and vector area of the surface."***

$$\Delta\phi = \vec{E} \cdot \Delta\vec{A}$$

$$\Delta\phi = E\,\Delta A\cos\theta$$



Being a dot product electric flux is a scalar quantity. Its unit is volt meter.

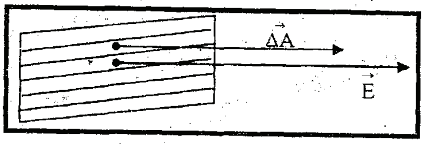
Condition of Maxima:

The flux through a given surface depends upon the angle at which the surface is placed with respect to the electric field.

Flux will be maximum when vector area $\overrightarrow{\Delta A}$ is parallel to the electric field $\vec{E}$,

i.e. $\theta = 0^0$. $\phi_{max.} = E\,\Delta A\cos 0^\circ$

$\phi_{max.} = E\,\Delta A$

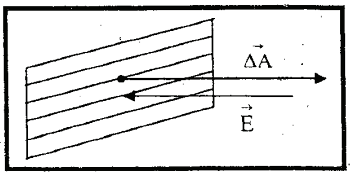


Positive flux is regarded as outgoing flux from the surface.

If the vector area is antiparallel to the electric field, i.e. $\theta = 180^0$. Then

$$\phi = E\Delta A \cos 180^0$$

$$\phi = -E\Delta A$$



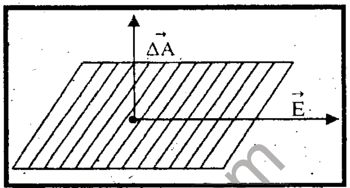
Negative flux is regarded as the incoming flux towards the surface.

Condition of Minima:

Flux will be minimum when vector area $\vec{\Delta A}$ is perpendicular on the electric field $\vec{E}$, i.e. $\theta = 90^0$

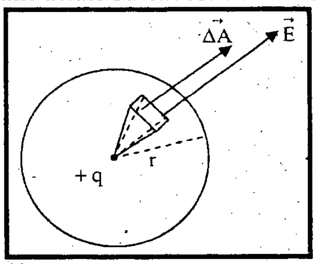
$$\phi_{min.} = E\Delta A \cos 90°$$

$$\phi_{min.} = 0$$



12.6 ELECTRIC FLUX THROUGH THE SURFACE OF SPHERE:

Suppose an isolated point positive charge "+q" is placed at the centre of Gaussian sphere of radius "r". As electric intensity varies inversely with the square of the radial distance, therefore, electric field intensity at all the points on the surface of sphere is different, therefore, dividing whole surface into a large number of small but equal flat surfaces of areas $\Delta A_1, \Delta A_2, \Delta A_3,$ - - - - - - - - - ΔA_N such that electric intensity "E" is the geometric mean over any flat surface and it remains constant at all surfaces. Maximum flux at all surfaces is given by:



$$\phi_1 = E\Delta A_1$$

$$\phi_2 = E\Delta A_2$$

$$\phi_3 = E\Delta A_3$$

$$\phi_N = E\Delta A_N$$

Total flux passing through the surface is:

$$\phi = \phi_1 + \phi_2 + \phi_3 + \text{---------} + \phi_N$$

$$\phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \text{---------} + E\Delta A_N$$

$$\phi = E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \text{---------} + \Delta A_N)$$

$$\phi = E\Sigma\Delta A \qquad \text{------------------ (1)}$$

Electric field intensity due to an isolated point charge is given by:

$$E = \frac{1}{4\pi r^2}\,\frac{q}{\varepsilon_0}$$

$$\therefore \textbf{eq.(1)} \Rightarrow \phi = \frac{1}{4\pi r^2}\,\frac{q}{\varepsilon_0} \times \Sigma\Delta A$$

Where $\Sigma\Delta A$ is the area of the Gaussian surface, which is $4\pi r^2$, therefore,

$$\phi = \frac{1}{\cancel{4\pi r^2}}\,\frac{q}{\varepsilon_0} \times \cancel{4\pi r^2}$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

This result shows that electric flux due to an isolated point charge depends

(i) Directly upon magnitude of charge enclosed in the surface.

(ii) Inversely upon permitivity of free space, and is independent of shape of surface in which charge is enclosed.

12.7 GAUSS'S LAW:

Introduction:

Gauss's Law provides the solutions of those problems in which more than one charge is enclosed in the surface.

Statement:

"The total outward flux over any closed hypothetical surface is equal to the charge enclosed divided by ϵ_o irrespective of the way in which charge is distributed."

Mathematically, total flux $\phi = \frac{1}{\varepsilon_0}$ x charge enclosed

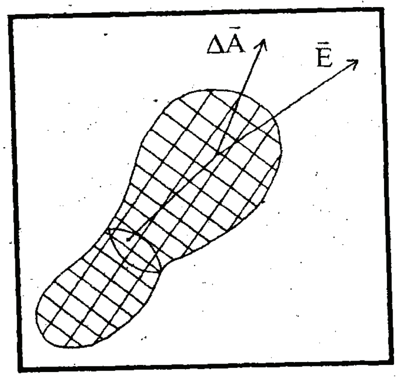
Proof:

Consider a closed surface of any shape and size, containing various charges $q_1, q_2, q_3, \text{--------}, q_N$ at different positions.

If a sphere is drawn with q_1 at the center and which lies within the surface, then the flux through it will be:

$$\phi_1 = \frac{q_1}{\varepsilon_0}$$

This flux finally passes through the surface. Similarly the flux due to charge q_2, q_3 etc. will be:



$$\phi_2 = \frac{q_2}{\varepsilon_0}$$

$$\phi_3 = \frac{q_3}{\varepsilon_0}$$

$$\phi_N = \frac{q_N}{\varepsilon_0}$$

The total flux passing through the surface is equal to the sum of all fluxes passing through the surface:

$$\phi = \phi_1 + \phi_2 + \phi_3 + \text{---------} + \phi_N$$

$$\phi = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \text{---------} + \frac{q_N}{\varepsilon_0}$$

$$\phi = \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \text{---------} + q_N)$$

OR $$\phi = \frac{1}{\varepsilon_0} \text{ x total charge}$$

Thus it is found that the flux passing through a closed surface depends upon the amount of charge enclosed directly and inversely upon the permitivity of medium and is independent of the size and shape of the surface and also the distribution of charges in the surface.

12.8 APPLICATIONS OF GAUSS'S LAW:

Case#1 Field of Uniform Spherical Charged Surface at a Distance "r" From Its Centre:

OR

Electric Field Intensity at an External Point of Uniform Spherical Charged Surface:

Let us suppose charge "q" is uniformly distributed in a hypothetical spherical surface of radius "a" and it is required to determine the electric intensity at point "P" which is at a radial distance "r" ($r > a$) from the center "O" of the sphere. Since electric field intensity does not remain same everywhere on the surface as it varies inversely with the square of the radial distance, therefore, we divide the whole surface into a large number of equal and small flat surfaces of area, ΔA_1, ΔA_2, ΔA_3, - - - - - - - - - - - - , ΔA_N

Such that Electric intensity $\vec{E}$ is the geometric mean over any flat surface and it remains constant at all surfaces. Maximum electric flux passing through such surface is given by.

$$\phi_1 = E\Delta A_1$$
$$\phi_2 = E\Delta A_2$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\phi_N = E\Delta A_N$$

Total flux is the sum of all flux passing through the surface.

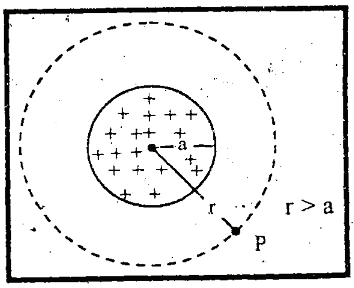
$$\phi = \phi_1 + \phi_2 + \ldots\ldots + \phi_N$$
$$\phi = E\Delta A_1 + E\Delta A_2 + \ldots\ldots + E\Delta A_N$$
$$\phi = E(\Delta A_1 + \Delta A_2 + \ldots\ldots + \Delta A_N)$$
$$\phi = \Sigma E\Delta A$$
$$\phi = E\Sigma\Delta A$$

Where $\Sigma\Delta A$ is the area of Gaussian surface, which is $4\pi r^2$.

$$\therefore \quad \phi = E \times 4\pi r^2 \qquad \text{------------------ (i)}$$

According to Gauss's Law

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{------------------ (ii)}$$



By equating eq.(i) and (ii) we get.

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

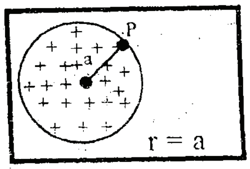
$$\Rightarrow \quad E = \frac{1}{4\pi r^2}\frac{q}{\varepsilon_0}$$

Case#2 Field at the surface of uniform spherical charged surface:

When it is required to determine the electric field intensity at the surface of uniform spherical charged surface then the radial distance will be.

$$r = a$$

$$\therefore \quad E = \frac{1}{4\pi a^2}\frac{q}{\varepsilon_0}$$



If σ is the surface charge density, then

$$\sigma = \frac{\text{Charge}}{\text{Area}}$$

$$\sigma = \frac{q}{4\pi a^2}$$

$$q = \sigma \times 4\pi a^2$$

$$\therefore \quad E = \frac{1}{\cancel{4\pi a^2}} \times \frac{\sigma \times \cancel{4\pi a^2}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Case#3 Field inside the uniform spherical charged surface:

If electric intensity is to be determined inside the spherical surface, then point "P" will be situated at radial distance r (r < a) from the centre "O" of the uniform spherical charged surface then the Gaussian surface containing point "P" will not contain any charge:

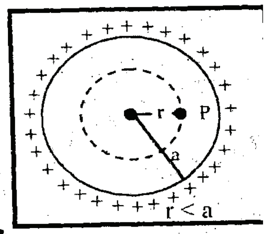
i.e. $q = 0$

Therefore, surface charge density at the centre of spherical surface would also be zero,

i.e. $\sigma = 0$

as $E = \frac{\sigma}{\epsilon_0}$

$\therefore \quad E = 0$

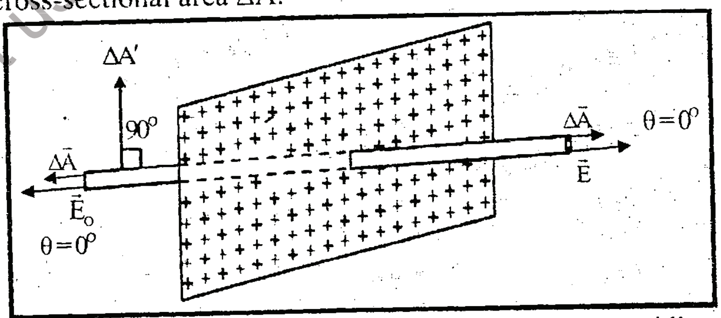


(2) Electric intensity near a large thin sheet of charges:

OR

Electric intensity due to an infinite sheet of charges:

Let us consider a sheet of area ΔA on which positive charge is uniformly distributed. Let σ be the surface charge density of the sheet. To determine the electric intensity E at a point P very close to the sheet, consider a Gaussian surface in the form of a cylinder of cross-sectional area ΔA.



As flat surfaces of the cylinder are of the same area and are equidistant from the sheet, so the fluxe ϕ_1 and ϕ_2 passing through them will also be the same:

i.e. $\phi_1 = \phi_2 = E\Delta A \cos\theta$

Since these faces of the cylinder are parallel to the electric lines of forces.

i.e. $\theta = 0^0$

therefore, $\phi_1 = \phi_2 = E\Delta A \cos 0^0$

$$\phi_1 = \phi_2 = E\Delta A$$

The total flux will be the sum of these two flux.

$$\phi = \phi_1 + \phi_2$$

$$\therefore \quad \phi = E\Delta A + E\Delta A$$

$$\phi = 2E\Delta A \quad \text{------------ (i)}$$

According to the Gauss's Law:

OR $$\phi = \frac{q}{\varepsilon_0}$$

If σ is the surface charged density, then

$$\sigma = \frac{q}{\Delta A}$$

OR $$q = \sigma\Delta A$$

$$\therefore \quad \phi = \frac{1}{\varepsilon_0} \times \sigma\Delta A \quad \text{------------ (ii)}$$

By equating eq.(i) and (ii), we get.

$$2E\,\Delta A = \frac{1}{\varepsilon_0} \times \sigma\Delta A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

In vector form $$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{r}}$$

Where $\hat{r}$ is the unit vector parallel to the face of the cylinder. Similarly electric field intensity near a large thin sheet of negative charge is given by:

$$\boxed{\vec{E} = -\frac{\sigma}{2\varepsilon_0}\hat{r}}$$

3) Electric intensity between two oppositely charged plates:

Consider two metal plates separated by a small distance. Let one of the plate is positively charged so that its surface charge density is $+\sigma$ and the other plate is charged negatively with surface charge density $-\sigma$, therefore, the electric field intensity between the plates will be acting from the positive sheet to the negative sheet.

If "P" is the point at the centre between the plates, where these plates behave like plane sheets of infinite extent, then the electric field intensity is:

$$\vec{E}_+ = \frac{\sigma}{2\varepsilon_0}\hat{r}$$

$$\vec{E}_- = -\frac{\sigma}{2\varepsilon_0}\hat{r}$$

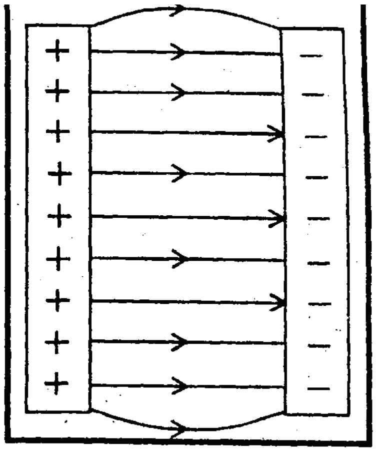
As point 'p' lies between the two charged plates, therefore net intensity will be their difference.

$$E = E_+ - E_-$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{r} - \left(-\frac{\sigma}{2\varepsilon_o}\hat{r}\right)$$

$$\vec{E} = \frac{2\sigma}{2\varepsilon_o}\hat{r}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{r}$$



12.9 ELECTRIC POTENTIAL OR RELATION BETWEEN ELECTRIC POTENTIAL AND ELECTRIC INTENSITY:

Electric potential is defined as:

"The work done on unit positive charge in displacing it against the direction of electric field".

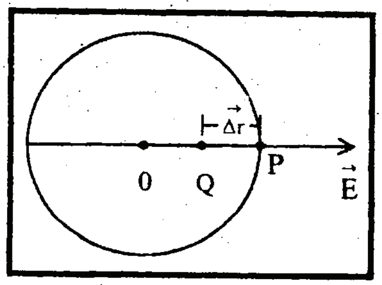
Mathematically....

$$\text{Electric Potential} = \frac{\text{Work}}{\text{Charge}} \quad \text{———— (i)}$$

OR $$V = \frac{W}{q_0}$$

Explanation:

Let a very small test charge q_0 is displaced from point "P" to point "Q" along any arbitrary path in the field as shown in the figure. The displacement of charge q_0 is Δr. The force on this charge q_0 in the field is given by:



$$F = q_0\vec{E}$$

From $$W = \vec{F}.\vec{d}$$

We have $$W = q_0\vec{E}\,.\,\overrightarrow{\Delta r}$$

OR $$\frac{W}{q_0} = \vec{E}\,.\,\overrightarrow{\Delta r}$$

OR $$V = \vec{E}\,.\,\overrightarrow{\Delta r}$$

Thus mathematically electric potential is defined as, ***"the dot product of electric field intensity and radial displacement vector of charge in the field"***

OR $$V = E\Delta r\cos\theta$$

Since displacement of charge is against the electric field, therefore,

$$\theta = 180^0$$

and $$V = E\Delta r\cos 180^0$$

$$\cos 180^0 = -1$$

$$V = -E\Delta r$$

OR $$E = -\frac{V}{\Delta r}$$

This result shows that electric intensity is the negative potential gradient.

S. I. Unit:

S. I. Unit of electric potential is volt.

From eq.(1) $\quad \text{Volt} = \dfrac{\text{Joule}}{\text{Coulomb}}$

Definition of one Volt:

"If one coulomb charge is displaced against the field by doing one Joule work on it then the electric potential is said to be of one volt". **(2013)**

Prove that $\dfrac{1\text{Volt}}{\text{meter}} = \dfrac{1\text{Newton}}{\text{Coulomb}}$, name the physical quantity which has these units.

$$\text{L.H.S} = \frac{1\text{Volt}}{\text{meter}}$$

$$= \frac{\text{Joule/Coulomb}}{\text{meter}} \qquad \because \quad \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$$

$$= \frac{\text{Joule}}{\text{Coulomb} \times \text{meter}}$$

$$= \frac{\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}} \qquad \because \quad \text{Joule} = \text{Newton} \times \text{meter}$$

$$\frac{1\text{Volt}}{\text{meter}} = \frac{\text{Newton}}{\text{Coulomb}} \quad \text{Proved}$$

OR

$$\frac{1\text{Newton}}{\text{Coulomb}} = \frac{1\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}}$$

$$= \frac{\text{Joule}}{\text{Coulomb} \times \text{meter}} \qquad \because \quad \text{Newton} \times \text{meter} = \text{Joule}$$

$$= \frac{\text{Joule}}{\text{Coulomb}} \times \frac{1}{\text{meter}}$$

$$= \text{Volt} \times \frac{1}{\text{meter}} \qquad \because \quad \frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$$

$$\frac{1\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{meter}}$$ Proved the physical quantity is electric intensity.

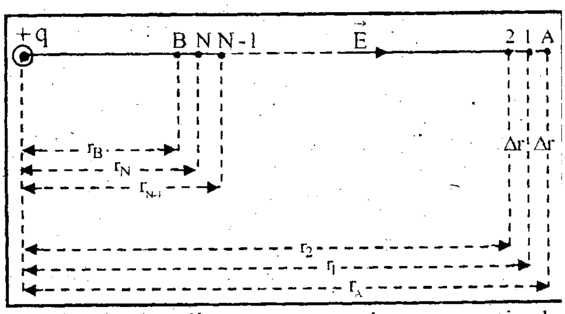
12.10 ELECTRIC POTENTIAL NEAR AN ISOLATED POINT CHARGE:

OR

ABSOLUTE ELECTRIC POTENTIAL

Definition:

The absolute potential at a point is the work done is taking a unit positive charge from a point at infinity having zero potential to the point against the electric field intensity.



Explanation:

Consider two points A and B in a straight line at distances r_A and r_B respectively from an isolated point charge +q as shown in the figure. In order to determine the electric potential at B, a test charge q_0 is moved from A to B. For this purpose some work has to be performed on q_0. Since electric intensity does not remain constant through A to B, so we divide the whole distance into a large number of small and equal distances r_1, r_2, ------ r_N, such that electric field which is geometric mean over any surface assumed to be constant at all surfaces.

First we determine the work done in moving charge q_0 from point A to point 1

$$\Delta W = Fd\cos\theta$$

Here $F = q_0E$

$d = \Delta r$

and $\theta = 180^0$

$$\therefore \quad \Delta W_{A\to 1} = q_0E\Delta r\cos 180^0$$

$$\Delta W_{A\to 1} = -q_0E\Delta r$$

OR

$$\frac{\Delta W_{A\to 1}}{q_0} = -E\Delta r$$

but

$$\frac{\Delta W_{A\to 1}}{q_0} = \Delta V_{A\to 1}$$

$$\therefore \quad \Delta V_{A\to 1} = -E\Delta r$$

Electric field intensity due to an isolated point charge is given by:

$$E = \frac{Kq}{r^2}$$

$$\therefore \quad \Delta V_{A\to 1} = -Kq\frac{(r_A - r_1)}{r^2}$$

Where 'r' is the geometric mean distance of q_0 when moved from point A to point 1.

$$r = \sqrt{r_A r_1}$$

$$r^2 = r_A r_1$$

$$\therefore \quad \Delta V_{A\to 1} = -Kq\left(\frac{r_A - r_1}{r_A r_1}\right)$$

$$\Delta V_{A\to 1} = -Kq\left(\frac{r_A}{r_A r_1} - \frac{r_1}{r_A r_1}\right)$$

$$\Delta V_{A\to 1} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right)$$

Similarly

$$\Delta V_{1\to 2} = -Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Delta V_{N\to B} = -Kq\left(\frac{1}{r_B} - \frac{1}{r_N}\right)$$

Total electric potential from A to B is the algebraic sum of all electric potentials.

$$\Delta V_{A\to B} = \Delta V_{A\to 1} + \Delta V_{1\to 2} + \text{-------} + \Delta V_{N\to B}$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right) - Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \text{------------} + Kq\left(\frac{1}{r_B} - \frac{1}{r_N}\right)$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A} + \frac{1}{r_2} - \frac{1}{r_1} + \text{------------} + \frac{1}{r_B} - \frac{1}{r_N}\right)$$

$$\Delta V_{A\to B} = -Kq\left(-\frac{1}{r_A} + \frac{1}{r_B}\right)$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Or potential difference $V_B - V_A = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

The absolute potential at point B is obtained by considering the point A to be situated at infinity.

$$r_A = \infty$$

$$\Rightarrow \quad \frac{1}{r_A} = \frac{1}{\infty}$$

$$\frac{1}{r_A} = 0$$

$$\therefore \quad V_B = -\frac{Kq}{r_B}:$$

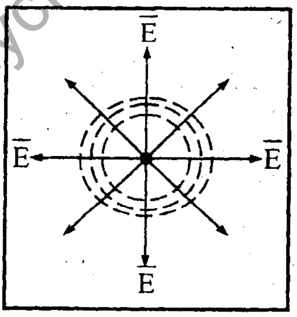
Absolute potential due to a point charge '+q' at a distance 'r' from it is given by:

$$V = -\frac{Kq}{r}$$

OR

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

12.11 EQUIPOTENTIAL SURFACE:



In an electric field, there are the points at which same electrostatic potential exist. A surface passing through such points is known as ***equipotential surface***. On such a surface, the potential energy of a charged particle remains same at all points, so a charge can move on such a surface without doing any work against electric field. It implies that an equipotential surface must be perpendicular on the field at all points.

Two equipotential surfaces can never intersect each other, because in that case there will be two potentials at the point of intersection, which is not possible.

12.12 ELECTRON VOLT:

Electron volt is the unit of energy of a charged particle. It can be defined as:

"An electron volt is the amount of energy acquired by an electron when it falls through a potential difference of one volt."

1 electron volt = charge on electron x 1 volt.

$1\ eV = 1.6 \times 10^{-19} C \times 1V$

$1\ eV = 1.6 \times 10^{-19} VC$

As $\text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$

OR $J = VC$

$1\ eV = 1.6 \times 10^{-19} J$

12.13 CAPACITOR:

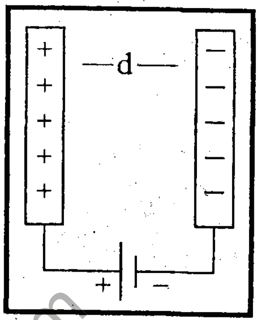
A device used for storing electric charge is called capacitor. A simplest capacitor is the parallel plate capacitor which consist of two parallel metallic plates that have equal and opposite charges separated by a small distance as shown in the figure. The two plates of a capacitor are connected to a battery where the plates acquire equal amount of opposite charge. A typical capacitor is capable of storing a large amount of charge in a small space. The capability of a capacitor to store charge is called its ***capacity*** or ***capacitance***. The space in a capacitor is filled by air or some insulating material called as dielectric.

If Q is the charge stored in the plates when a potential difference V is applied across the plates, then

$$Q \propto V$$
$$Q = CV$$

Where C is the capacitance of capacitor

S.I. Unit of Capacitance:



As capacitance of capacitor is

$$C = \frac{Q}{V}$$

If $Q = 1$ Coulomb

And $V = 1$ Volt

$$\therefore \quad C = \frac{1\,\text{Coulomb}}{1\,\text{Volt}}$$

OR $C = 1$ Farad

Definition of One Farad:

"The capacitance of a capacitor is one farad if a charge of one coulomb given to the plates produces potential difference of one volt between them."

Other units of capacitance are:

Micro farad	$1\mu F = 10^{-6}$ F
Nano farad	$1nF = 10^{-9}$ F
Pico farad	$1PF = 10^{-12}$ F

12.14 COMBINATION OF CAPACITORS:

There are many situations in electrical circuits where more than one capacitor are used. The capacitors can either be combined in series or in parallel.

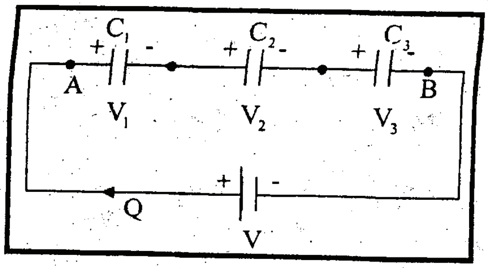
a) Capacitors in Series:

Definition:

When the plates of capacitor are connected end to end, then the combination is called series combination.

Explanation:

Let us suppose capacitors of capacitances C_1, C_2 and C_3 are connected in series between two points 'A' and 'B' as shown in figure. As all capacitors are connected in a single path, therefore when they are charged, each capacitor acquires the same amount of charge irrespective of their capacitances. Also the potential difference V applied across the points A and B is equal to the sum of potential difference across each capacitor.



$$V = V_1 + V_2 + V_3 \qquad \text{------------- (1)}$$

From $V = \frac{Q}{C}$

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

$$\therefore \text{ eq.(i)} \Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{\not{Q}}{C} = \not{Q}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad \textbf{Ans.}$$

Where C is the net or equivalent capacitance and the reciprocal of which is equal to the sum of reciprocal of all capacitors connected in series. This result implies that in series combination net capacitance of circuit decreases.

b) Capacitors in Parallel:

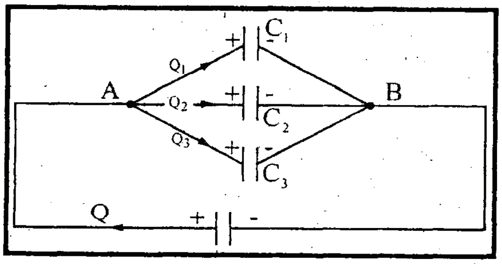
Definition:

When one plate of each capacitor is common point and second plate to second common point, then the combination is called parallel combination.

Explanation:

Let us suppose capacitors of capacitances C_1, C_2 and C_3 are connected in parallel between two points A and B as shown in figure.

As each capacitor is connected between the same two points, so when a potential difference V is applied between points A and B, then all capacitors would have the same potential difference across them. On giving charge Q, to point A, capacitors C_1, C_2 and C_3 will acquire charges Q_1, Q_2 and Q_3 respectively, depending upon their capacitances, hence



$$Q = Q_1 + Q_2 + Q_3 \quad \text{---------------} \ (1)$$

From $Q = CV$

We have $Q_1 = C_1V$, $\quad Q_2 = C_2V$, and $\quad Q_3 = C_3V$

$$\therefore \text{ eq.(1)} \Rightarrow CV = C_1V + C_2V + C_3V$$

$$C\not{V} = \not{V}(C_1 + C_2 + C_3)$$

$$\boxed{C = C_1 + C_2 + C_3} \quad \textbf{Ans.}$$

Where C is the net or equivalent capacitance which is equal to the sum of capacitances of all capacitors connected in parallel. This result implies that in parallel combination the net capacitance of circuit increases.

12.15 DEPENDENCE OF CAPACITANCE UPON DIELECTRIC:

1) When air is the medium between the plates of capacitor:

Let us suppose a capacitor with two parallel plates, each of area 'A' which are separated by a distance 'd' as shown in figure. Let air be the insulating material or dielectric between the plates. Capacitance of such a capacitor is given by:

$$C = \frac{Q}{V} \quad \text{---------------- (i)}$$

Where V is the voltage applied across the plates of capacitor and it can be replaced by

$$V = Ed$$

Also if σ is the charge density, then

$$\sigma = \frac{Q}{A}$$

OR $Q = \sigma A$

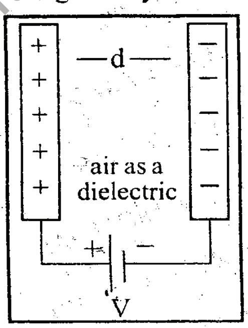
$$\therefore \text{eq.(i)} \Rightarrow C = \frac{\sigma A}{Ed} \quad \text{---------------(ii)}$$

Electric intensity between two oppositely charged plates is given by

$$E = \frac{\sigma}{\epsilon_0}$$

$$\therefore \text{eq.(3)} \Rightarrow C = \frac{\not{\sigma} A}{\frac{\not{\sigma} d}{\epsilon_0}}$$

OR $C = \frac{\epsilon_0 A}{d}$



Where ϵ_0 is the permitivity of free space. Above equation shows that capacitance of capacitor depends

(i) Directly upon permitivity of free space and area of plates:

(ii) Inversely upon distance between the plates.

2) When dielectric slab is the medium between the plates of capacitor:

If an insulating material or dielectric of relative permittivity ϵ_r is placed between the plates of capacitor, then the capacitance of capacitor is given by:

$$C' = \frac{\epsilon A}{d}$$

But $\epsilon = \epsilon_0 \epsilon_r$

Therefore $C' = \frac{\epsilon_0 \epsilon_r A}{d}$

OR $C' = \epsilon_r \frac{\epsilon_0 A}{d}$

OR $C' = \epsilon_r C$

This result shows that capacitance of capacitor increases in the presence of dielectric other than air.

3) When air and a dielectric slab are the mediums between the plates of capacitors:

OR

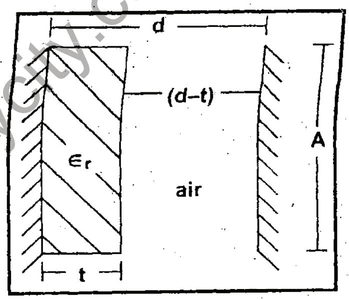
Compound Capacitor:

OR

When A Dielectric Slab of Thickness t<d is Slipped Between the Plates of Capacitor

"If more than one mediums are placed between the plates of a parallel plate capacitors, then such a capacitor is called compound capacitor".

Let us suppose a parallel plate capacitor of plates of area "A" is separated by a distance "d" from each other. Let a dielectric slab of same area "A" dielectric constant ϵ_r and of thickness "t" (t<d) is slipped between the plates of capacitor, such that a compound capacitor of two series capacitors is formed. Capacitance of capacitor of thickness "t" is given by:



$$C_1 = \frac{\epsilon_o \epsilon_r A}{t}$$

Capacitance of capacitor of thickness (d-t) is given by

$$C_2 = \frac{\epsilon_0 \epsilon_r' A}{(d-t)}$$

Where $\epsilon_r{}'$ is the permitivity of air

If "C" is the equivalent capacitance of a compound capacitor formed by two series capacitors, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{\dfrac{\epsilon_0 \epsilon_r A}{t}} + \frac{1}{\dfrac{\epsilon_0 \epsilon_r A}{(d-t)}}$$

$$\frac{1}{C} = \frac{t}{\epsilon_0 \epsilon_r A} + \frac{(d-t)}{\epsilon_0 \epsilon_r' A}$$

$$\frac{1}{C} = \frac{1}{\epsilon_0 A}\left(\frac{t}{\epsilon_r} + \frac{(d-t)}{\epsilon_r{}'}\right)$$

$$C = \epsilon_o A \times \frac{1}{\left(\dfrac{t}{\epsilon_r} + \dfrac{(d-t)}{\epsilon_r{}'}\right)}$$

For air $\epsilon_r' = 1$

$$\boxed{C = \frac{\epsilon_0 A}{(d-t) + \dfrac{t}{\epsilon_r}}}$$ **Ans.**