HEAT 11

CONTENTS

HEAT

11.1- Caloric Theory:

In order to explain the nature of heat, in the beginning of nineteenth century a famous scientist Calorie presented the theory called as Caloric Theory. According to this theory ***"Heat is present in every material body in the form of a hypothetical weight less fluid called Caloric"***. Further if there is no transfer of heat takes place from the material body then the amount of caloric remains constant in the body. Difference of temperature between the bodies was assumed to be due to the presence of different amounts of caloric.

The reason of acceptance of caloric theory was its successful explanation of heat transfer, such as conduction and convection.

Drawbacks of Caloric Theory:

One of the main problem with the caloric theory was its unability to explain the heat generated by the friction, which was first noticed by Count Rumford. Later on when it was proved that the sun-rays reaches the earth after traveling through space and not through the material medium, caloric theory could not explain this way of heat transfer which is called as Radiation. Hence caloric theory was rejected.

11.2- Internal Energy:

Internal energy is the energy contained in a body. It is the sum of all types of energies possessed by a body. For a mono atomic or on ideal gas Microscopically it is defined as ***the sum of Translatory Kinetic energies of molecules.***

11.3- Heat Energy:

Heat is that internal energy which transfers from one object to another object due to the temperature difference between them. It is the energy in transit. Once the energy is transferred to a body then it becomes internal energy of the body.

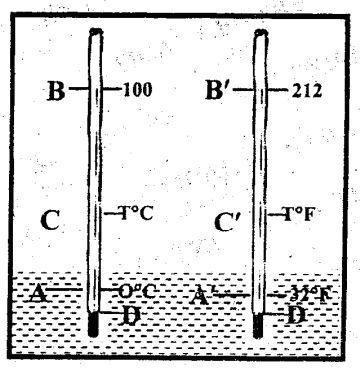
The S.I. unit of heat is Joule.

11.4- Temperature:

The measure of the degree of hotness or coldness of a body is called Temperature. Temperature is the property which determines the direction of flow of heat when two bodies are brought in thermal contact with each other. Microscopically temperature is defined as, ***"the average of Translatory Kinetic Energy of molecules."***

11.5- Relation between Celsius and Fahrenheit Scales:

Let us consider that both Celsius thermometer and Fahrenheit thermometer are dipped in the same water bath placed at the room temperature, as shown in the figure. The final and initial points in Celsius thermometer are labelled as A and B, and that are in the Fahrenheit thermometer as A' and B' respectively, where as the mercury level in both are labelled as C and C'. As the mercury level in both thermometers will rise to the same height, therefore,the ratio of mercury level raised in the Celsius thermometer to the total height of mercury level is equal to the ratio of mercury level raised in the Fahrenheit thermometer to the total height of mercury level, mathematically.



$$\frac{AC}{AB} = \frac{A'C'}{A'B'}$$

OR

$$\frac{T^\circ_C - 0}{100 - 0} = \frac{T^\circ_F - 32}{212 - 32}$$

$$\frac{T^\circ_C}{100} = \frac{T^\circ_F - 32}{180}$$

$$T^\circ_C = \frac{100}{180}(T^\circ_F - 32)$$

$$\boxed{T^\circ_C = \frac{5}{9}(T^\circ_F - 32)}$$

OR

$$\boxed{T^\circ_F = \frac{9}{5}T^\circ_C + 32}$$

11.6- Thermometric Properties:

Property of substance which change uniformly with the change of temperature is called thermometric property. For example electrical resistance of wire changes with the change of temperature, the volume of a gas at constant pressure changes with the change of temperature.

11.7- Thermal Expansion:

The countable increase in the dimension of any substance by means of Thermal energy is called Thermal Expansion.

OR

"The increase in size of a substance when heated is called Thermal Expansion."

There are following three types of thermal expansion:

(1) Linear Expansion **(2) Cubical or Volumetric Expansion**

(3) Superficial Expansion

11.8- Linear Expansion:

Definition:

"The increase in length of a substance on heating is called Linear Expansion."

Explanation:

Suppose the length of a uniform thin metallic rod at some initial temperature T_1 is L_1. Let the rod is heated through a small temperature ΔT, such that the increase in length of rod is ΔL and the final temperature and final length of rod are T_2 and L_2 respectively, mathematically

Change or increase in length of rod, $\Delta L = L_2 - L_1$

Change or increase in temperature of rod, $\Delta T = T_2 - T_1$

Experimentally it has been found that the change in length of rod is directly proportional to the original length and the change in temperature of the rod, mathematically

$$\Delta L \propto L_1 \text{ —— (a)} \quad \text{and} \quad \Delta L \propto \Delta T \text{ —— (b)}$$

From comparing (a) and (b)

$$\Delta L \propto L_1 \Delta T$$

$$\boxed{\Delta L = \alpha L_1 \Delta T} \quad \text{----------(1)}$$

$$L_2 - L_1 = \alpha L \Delta T$$

$$L_2 = L_1 + \alpha L_1 \Delta T$$

$$\boxed{L_2 = L_1\{1 + \alpha \Delta T\}}$$

Where "α" is the constant of Proportionality known as **"The Co-efficient of Linear Expansion"**, which determines the nature of material of the rod.

Co-efficient of Linear Expansion:

From equation (1) $\alpha = \dfrac{\Delta L}{L_1 \Delta t}$

It is defined as "The change in length per unit length per Kelvin rise is temperature"

The value of α is different for different materials.

Unit: The S.I unit of co-efficient of linear expansion is $^\circ C^{-1}$ or K^{-1}.

11.9- Volumetric Expansion:

Definition:

"The increase in volume of a substance on heating, is called volume or cubical expension."

Explanation:

Suppose the volume of a cube at some initial temperature T_1 is V_1. Let the cube is heated through a small temperature ΔT, such that the increase in volume of cube is ΔV and the final temperature and final volume of solid are T_2 and V_2 respectively, mathematically.

Change or increase in volume of cube, $\Delta V = V_2 - V_1$

Change or increase in temperature of cube, $\Delta T = T_2 - T_1$.

Experimentally it has been found that the change in volume of cube is directly proportional to the original volume and the change in temperature of the cube, mathematically

$$\Delta V \propto V_1 \text{ ——(a)} \quad \text{and} \quad \Delta V \propto \Delta T \text{ ——(b)}$$

From comparing (a) and (b)

$$\Delta V \propto V_1 \Delta T$$

$$\boxed{\Delta V = \beta V_1 \Delta T} \quad \text{----------(1)}$$

$$V_2 - V_1 = \beta V_1 \Delta T$$

$$V_2 = V_1 + \beta V_1 \Delta T$$

$$\boxed{V_2 = V_1 \{1 + \beta \Delta T\}}$$

Where "β" is the constant of proportionality known as **"the Co-efficient of Cubical Expansion"**

Co-efficient of Volume Expansion:

From equation (1)

$$\boxed{\beta = \frac{\Delta V}{V_1 \Delta T}}$$

It is defined as "The change in volume per unit volume per Kelvin rise is temperature".

The value of β is different for different materials.

Unit: The S.I unit of co-efficient of cubical expansion is $°C^{-1}$ or K^{-1}.

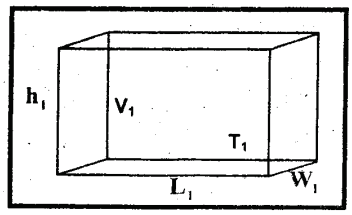
11.10- Q. Show that the Co-efficient of Cubical Expansion of a Solid Substance is the Thrice of Its Co-efficient of Linear Expansion:

Or

Prove that $\beta = 3\alpha$:

Let us consider a cube whose length is L_1, width is W_1 and height is h_1 at some initial temperature T_1, such that the initial volume of cube is

$$V_1 = L_1 W_1 h_1 \text{ ----------(1)}$$



Let the cube is uniformly heated and let the temperature rise is ΔT, such that the final length, width and height of cube are L_2, W_2 and h_2 respectively, which are given by,

$$L_2 = L_1 \{1 + \alpha \Delta T\}$$

$$W_2 = W_1 \{1 + \alpha \Delta T\}$$

$$h_2 = h_1 \{1 + \alpha \Delta T\}$$

Final volume of the cube is,

$$V_2 = L_2 W_2 h_2$$

OR $$V_2 = L_1 \{ 1 + \alpha\Delta T \} W_1 \{ 1 + \alpha\Delta T \} h_1 \{ 1 + \alpha\Delta T \}$$

$$V_2 = L_1 W_1 h_1 \{ 1 + \alpha\Delta T \}^3$$

$$V_2 = V_1 \{ 1 + \alpha\Delta T \}^3 \qquad \because V_1 = L_1 W_1 h_1$$

Expanding the bracket term by,

$$\{ a + b \}^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$V_2 = V_1 \{ 1 + 3\alpha\Delta T + 3\alpha^2\Delta T^2 + \alpha^3\Delta T^3 \}$$

as "α" is very very small, therefore neglecting its higher powers.

$$V_2 = V_1 \{ 1 + 3\alpha\Delta T \} \text{ ---------- } (2)$$

But $$V_2 = V_1 \{ 1 + \beta\Delta T \} \text{ ---------- } (3)$$

Therefore by comparing equation (2) & (3) we get.

$$\cancel{V_1} \{ 1 + \beta\Delta T \} = \cancel{V_1} \{ 1 + 3\alpha\Delta T \}$$

$$\cancel{1} + \beta\Delta T = \cancel{1} + 3\alpha\Delta T$$

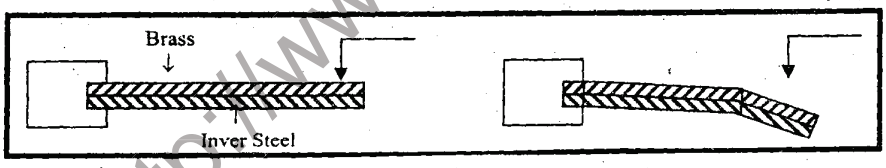
$$\beta\cancel{\Delta T} = 3\alpha\cancel{\Delta T}$$

$$\boxed{\beta = 3\alpha}$$ **PROVED.....**

Or $$\boxed{\alpha = \frac{1}{3}\beta}$$

11.11- Bimetallic Thermostat:

Thermostats are commonly used for maintaining desired temperatures. A bimetallic thermostat consists of two dissimilar metal strips whose co-efficients of linear expansion are different. The Strips are joined very well. The metallic strip works as an electric contact breaker in electrical heating circuit. When the desired temperature of the body is reached, the Strip bends and the circuit breaks. Thus the current stops flowing through the heating coil, when the temperature falls, the Strip contracts and the connection is restored.

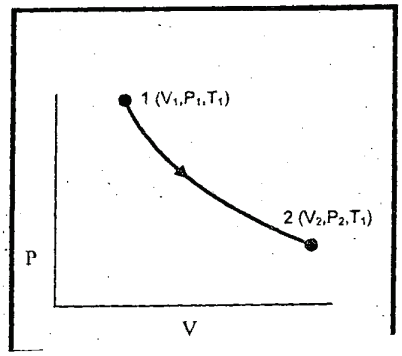


11.12- Bimetallic Thermometer:

A bimetallic thermometer consists of two dissimilar metal Strips whose Co-efficient of linear expansions are different. The Strips are joined very well. The Strip is often is in the form of a coil. Its one end is fixed and the other end is attached to a pointer.

This kind of thermometer is usually used as ordinary air thermometer, oven thermometer and in automobiles for automatic choke.

11.13- Boyle's Law:



The relation between the Volume and Pressure of a gas was studied by Robert Boyle and formulated it in the form of a law known as **Boyle's law**, which states that ***"For the given mass of a gas, the volume of a gas is inversely proportional to the pressure of the gas provided the temperature of gas remains constant"*** mathematically

$$V \alpha \frac{1}{P}$$

OR $PV = \text{Constant}$

For different states of the gas it can be written as

$$P_1V_1 = P_2V_2 = \text{Constant}$$

The Pressure – volume graph of Boyles law is a hyperbola.

The inverse relation of volume and pressure of a gas remains valid until the mass of gas remains constant. If how ever the mass of gas varies then the product PV will be directly proportional to the mass of the gas.

$$PV \propto m$$

OR $\frac{PV}{m} = \text{Constant}$

For different states of gas

OR $\frac{P_1V_1}{m_1} = \frac{P_2V_2}{m_2} = \text{Constant}$

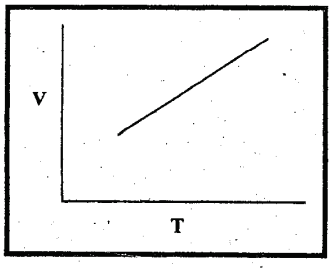
Real gases obey Boyle's law at low pressure

11.14- Charle's Law:

The relation between the volume of a gas and its absolute temperature was first studied by Charle's called the **Charle's Law**, which states that ***"for the given mass of a gas, the volume of a gas is directly proportional to the absolute temperature of the gas provided the pressure of the gas remains constant"***, mathematically

$$V \propto T$$

OR $\frac{V}{T} = \text{Constant}$



For different states of gas it can be wirtten as

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

The Volume – Temperature graph of a Charle's Law is a straight line.

The direct relation of volume and temperature of a gas remains valid until the mass of gas remain constant. If however the mass of gas varies then the ratio will be directly proportional to the mass of the gas.

$$\frac{V}{T} \propto m$$

OR $\frac{V}{mT} = \text{Constant}$

For different states of gas

$$\frac{V_1}{m_1 T_1} = \frac{V_2}{m_2 T_2} = \text{Constant}$$

Real gases Obey Charle's law at high temperature

11.15- General Gas Law:

The effect of simultaneous change of Temperature, Pressure and Volume for a given mass of a gas is called as general gas law.

From Boyles law $\frac{PV}{m} = \text{Constant} =$ ---------- **(1)**

And from Charle's law $\frac{V}{mT} = \text{Constant} =$ ---------- **(2)**

By merging both the equations, we get

$$\frac{PV}{mT} = R_m \qquad \text{---------- (3)}$$

Where R_m is the constant called gas constant, which has no significance

$$\text{As number of moles} = \frac{\text{Mass in Grammes}}{\text{Molecular Mass}}$$

$$n = \frac{m}{M}$$

OR $m = nM$

$$\therefore \quad \frac{PV}{nMT} = R_m$$

$$\frac{PV}{nT} = MR_m$$

Where MR_m is the product of molecular mass of gas and gas constant and this product is a universal constant denoted by "R".

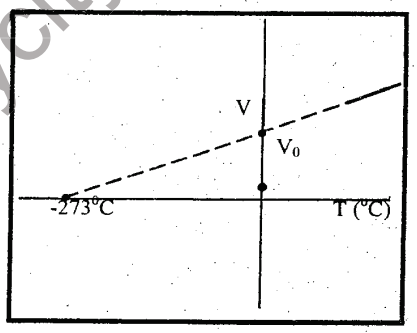
$$\frac{PV}{nT} = R$$

OR $PV = nRT$

Where "R" is called universal gas constant, in S.I. units its value is 8.314 J/mole-K.

11.16- Absolute Zero:

According to the Charle's Law the Volume of a gas at constant pressure is directly proportional to its absolute temperature, and the graph of volume and temperature is a straight line. From the graph it is found that at 0°C the gas still possesses a volume "V_o". When the straight line of a graph is extra plotted to lower temperature as shown in the figure, it intersects the temperature axis at –273°C. This implies that if a gas could be cooled off to –273°C, it would have no volume. Lord Kelvin suggested this point as zero of Kelvin Scale which is called **Absolute Zero**. Thus absolute zero is the theoretical limit of least possible temperature.



11.17- Kinetic Molecular Theory of Gases:

Kinetic molecular theory of gases explains the physical behaviour of gases in terms of the energy and interaction of gas molecules. Assumptions of this theory were put forward by Maxwell, Boltzman, Celsius, Kelvin, etc. which are as follows:

(1) All gases are composed of molecules, which posses similar physical behaviour. A finite volume of a gas contains a large number of molecules. (about 3×10^{25} molecules of air in $1m^3$ at S.T.P).

(2) Sizes of molecules are considered as negligible as compared to the distances between them.

(3) Molecules of a gas always remain in random motion, which follows the laws of mechanics.

(4) There is no interaction between the molecules of a gas except when they collide with each other, which are perfectly elastic.

(5) The collision of molecules with the walls of container are regarded as the pressure of the gas.

(6) The average translatory Kinetic energy of molecules is directly proportional to the absolute temperature of the gas.

11.18-Interpretation of Pressure of a Gas on the Basis of Kinetic Molecular Theory of Gases:

OR

Q. Prove that $P = \frac{1}{3}\rho\overline{V^2}$

Let us consider "n" moles of a perfect gas are contained in a cubical vessel of equal dimensions each of length L. As gas molecules move randomly therefore for the sake of simplicity we first consider only one molecule of mass "m" moving along x-direction only. The molecule from left face of the cube moves with velocity V_x and collides with the right face of the cube and rebounds back with the velocity $-V_x$.

AS Force = Rate of change of momentum

$$F = \frac{mV_x - (-mV_x)}{t}$$

$$F = \frac{2mV_x}{t}$$

OR $$F = 2mV_x \times \frac{1}{t} \text{ ---------- (2)}$$

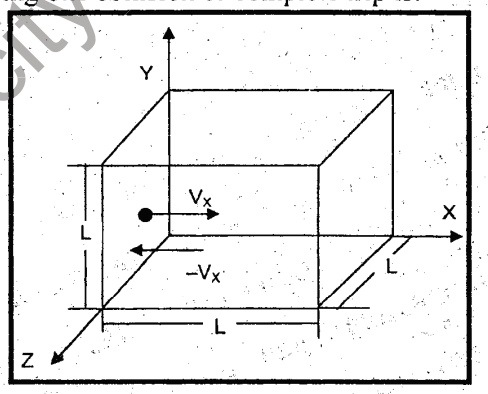
The distance traveled by the molecule during one collision or complete trip is:

$$S = 2L$$

From $S = Vt$

We have $2L = V_x \times t$

OR $\frac{1}{t} = \frac{V_x}{2L}$



Equation. (2) => $F_1 = 2mV_x \times \frac{V_x}{2L}$

$$F_1 = \frac{mV_x^2}{L}$$

The above equation is for the force applied by a single molecule on the face of the cube, the force applied by "N" molecules is

$$F = \frac{mV_{1x}^2}{L} + \frac{mV_{2x}^2}{L} + \frac{mV_{3x}^2}{L} + \text{-----} + \frac{mV_{Nx}^2}{L}$$

$$F = \frac{m}{L}(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \text{-----} + V_{Nx}^2)$$

As Pressure is the force per unit area

$$P = \frac{F}{A} \qquad (\text{where } A = L^2)$$

$$P = \frac{\frac{m}{L}(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \text{-----} + V_{Nx}^2)}{L^2}$$

$$P = \frac{m}{L^3}(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \text{----} + V_{Nx}^2) \text{ --- (3)}$$

If n_v is the number of molecules per volume, then

$$n_v = \frac{N}{L^3}$$

OR $L^3 = \frac{N}{n_v}$

Equation (3) => $P = \frac{m}{N/n_v}(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + ----+ V_{Nx}^2)$

OR $P = n_v m \frac{(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + -------+ V_{Nx}^2)}{N}$

Where

$$\frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + ----------+ V_{Nx}^2}{N} = \overline{V_x^2}$$

and $n_v m = \rho$, the density of gas

$\therefore$ $P = \rho \overline{V_x^2}$ ---------- **(4)**

The total velocity of molecules could be expressed as

$$\overline{V^2} = \overline{V_X^2} + \overline{V_Y^2} + \overline{V_Z^2}$$

But where

$$\overline{V_Y^2} = \overline{V_Z^2} = \overline{V_X^2}$$

Therefore

$$\overline{V^2} = \overline{V_x^2} + \overline{V_x^2} + \overline{V_x^2}$$

OR $\overline{V^2} = 3\overline{V_x^2}$

$$\overline{V_X^2} = \frac{\overline{V^2}}{3} \text{ ---------- } (5)$$

Now equation (4) ⇒

$$\boxed{P = \frac{1}{3}\rho\overline{v^2}}$$ **Proved**

11.19-Q.Prove that the Average Translatory Kinetic Energy of Gas Molecules is Directly Proportional to the Absolute Temperature of Gas:

OR

Prove that T ∝ K.E

Proof:

According to the kinetic molecular theory, the pressure of a gas is given by:

Equation (4) => $P = \frac{1}{3}\rho\overline{v^2}$

Where $\rho = n_v m$, the density of gas

And $n_v = \frac{N}{V}$, the number molecules per unit volume of a gas,

therefore, $\rho = \frac{Nm}{V}$

$\therefore$ $P = \frac{1}{3}\frac{Nm\overline{v^2}}{V}$

OR $PV = \frac{1}{3}Nm\overline{v^2}$

Multiplying and dividing R.H.S. by 2, we get.

$$PV = \frac{2}{3} N\left(\frac{1}{2}m\overline{v^2}\right) \quad \text{----------} (1)$$

According to the general gas law

$$PV = nRT$$

As avagadro's number is the number of molecules present in a mole of a gas therefore,

$$N_A = N/n$$

OR $n = N/N_A$

$$\therefore \quad PV = \frac{NRT}{N_A}$$

Where $\frac{R}{N_A} = K$; the Boltzman's constant

$$\therefore \quad PV = NKT \quad \text{----------} (2)$$

By equating equation (1) and (2), we get

$$\frac{2}{3} \cancel{N}\left(\frac{1}{2}m\overline{v^2}\right) = \cancel{N} K T$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2} KT$$

$$\boxed{(K.E)_{ave} = \frac{3}{2} KT}$$

Since the factor

$$\frac{3}{2}K = \text{Constant}$$

$$\therefore \quad K.E = \text{Constant} \times T$$

OR $\boxed{(K.E)_{ave} \propto T}$ **Proved**

11.20- Boyle's Law on the Basis of Kinetic Molecular Theory of Gases:

According to the kinetic molecular theory of gases, the Pressure of a gas is given by:

$$P = \frac{1}{3} \rho \overline{v^2}$$

but $\rho = n_V m$, the density of gas and

$n_V = \frac{N}{V}$, the number of molecules of a gas per unit volumes

$$\therefore \quad \rho = \frac{Nm}{V}$$

Thus $P = \frac{1}{3} \frac{Nm}{V} \overline{v^2}$

OR $PV = \frac{1}{3} Nm\overline{v^2}$

Multiplying and dividing by 2

$$PV = \frac{2}{3} N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$PV = \frac{2}{3} N (K.E)_{ave}$$

But $(K.E)_{ave} = \frac{3}{2} KT$

$\therefore \quad PV = \frac{2}{3} N \times \frac{3}{2} KT$

$PV = NKT$

Where temperature in Boyle's law remains constant so the factor

$NKT = \text{Constant}$

Hence

$$\boxed{PV = \text{Constant}}$$

which proves the Boyle's Law

11.21- Charle's Law on the Basis of Kinetic Molecular Theory of Gases:

According to the kinetic molecular theory of gases, the Pressure of a gas is given by:

$$P = \frac{1}{3} \rho \overline{v^2}$$

but $\rho = n_V m$, the density of gas and

$n_V = \frac{N}{V}$,the number of molecules of a gas per unit volumes

$\therefore \quad \rho = \frac{Nm}{V}$

Thus $P = \frac{1}{3} \frac{Nm}{V} \overline{v^2}$

OR $PV = \frac{1}{3} Nm \overline{v^2}$

Multiplying and dividing by 2

$$PV = \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2}\right)$$

$$PV = \frac{2}{3} N (K.E)_{ave}$$

But $(K.E)_{ave} = \frac{3}{2} KT$

$\therefore \quad PV = \frac{2}{3} N \times \frac{3}{2} KT$

$PV = NKT$

OR $\frac{V}{T} = \left(\frac{NK}{P}\right)$

Where pressure of gas in Charle's Law remains constant, so the factor

$\frac{NK}{P} = \text{Constant}$

$\therefore \quad \boxed{\frac{V}{T} = \text{Constant}}$

Which prove the Charle's Law.

11.22- Heat Capacity:

All substances naturally posses ability of absorbing some heat. This natural ability of substances is called **Heat Capacity**. Heat capacity of a substance depends upon many factors, such as nature, mass, temperature etc. Upon absorption of heat either temperature of substance changes or state of substance changes. The quantity of heat, which changes the temperature of substance, is called **Specific Heat** and the quantity of heat, which changes the state of substance, is called **Latent Heat**. When the temperature of substance changes its state remains same and when state of substance changes its temperature remains same.

11.23- Latent Heat:

Latent heat of a substance is defined as ***the amount of heat required to change the state of unit mass of a substance***. Mathematically,

$$H = \frac{\Delta Q}{m}$$

OR $\boxed{\Delta Q = mH}$

S.I. unit of Latent Heat is J/Kg.

Latent heat is of two types,

1) Latent Heat of Fusion:

Latent heat of fusion is ***the amount of heat required to melt the unit mass of a solid***.

$$H_f = \frac{\Delta Q}{m}$$

OR $\boxed{\Delta Q = mH_f}$

2) Latent Heat of Vapourization:

Latent heat of Vapourization is ***the amount of heat required to vopourize the unit mass of a liquid***.

$$H_v = \frac{\Delta Q}{m}$$

OR $\boxed{\Delta Q = mH_v}$

11.24- Specific Heat Capacity:

All substances absorb heat when heated through some range of temperature. This ability of objects of absorbing heat depends upon their masses and as well as upon their nature also, called as **Specific Heat Capacity.** The specific heat capacity of a substance is defined as ***"The amount of heat required to rise the temperature of a unit mass of a substance through one Kelvin or one degree Celsius"***.

If a substance of mass "m" is heated through a small range of temperature ΔT, such that ΔQ is the amount of heat absorbed, then specific heat capacity of a substance is given by:

$$C = \frac{\Delta Q}{m\Delta T}$$

S.I. Unit of Specific Heat Capacity is $J.Kg^{-1}K^{-1}$.

11.25- Molar Specific Heat Capacity:

As we know that Specific Heat capacity of a substance depends upon the mass of the substance in addition to the nature of the substance, which is given by:

$$C = \frac{\Delta Q}{m\Delta T} \quad \text{----------} \quad (1)$$

But for gasses their heat capacity is independent of the mass of the gas, therefore, if "n" is the no of moles of a gas and "M" is the molecular mass of the gas, then

$$\text{No. of moles of Gas} = \frac{\text{mass of gas}}{\text{Molecular mass of gas}}$$

$$n = \frac{m}{M}$$

OR $$m = nM$$

$$\therefore \text{Eq.(1)} \Rightarrow C = \frac{\Delta Q}{nM\Delta T}$$

$$MC = \frac{\Delta Q}{n\Delta T} \text{ ---------- (2)}$$

Where MC is the product of molecular mass of the gas and the specific heat capacity of the gas, which is called ***the molar specific heat capacity*** of a gas, which is given by:

$$MC = C'$$

$$\text{Eq.(2)} \Rightarrow C' = \frac{\Delta Q}{n\Delta T}$$

Thus molar specific heat capacity of a gas can be defined as ***"the amount of heat required to rise the temperature of one mole of a gas through one Kelvin or one degree Celsius"***.

The S.I. unit of molar specific heat capacity is $J.mol^{-1}.K^{-1}$.

Molar specific heat capacity is of two types which are.

(i) Molar specific heat capacity at constant pressure (C_p)

It is define as *"the amount of heat required to rise the temperature of one mole of a gas through one kelvin at constant pressure."*

$$C_p = \frac{\Delta Q_p}{n\Delta T}$$

(ii) Molar specific heat capacity at constant volume (C_v).

It is define as *"the amount of heat required to rise the temperature of one mole of a gas through one kelvin at constant volume."*

$$C_v = \frac{\Delta Q_v}{n\Delta T}$$

11.26- Q.Why a gas possess two types of molar specific heat capacities?

Ans. A gas possess two types of molar specific heat capacities, because when a gas is heated, its rise in temperature depends upon the fact that whether the gas is allowed to expand or not. When a gas is allowed to expand, than its volume changes but pressure remains constant. Such type of heat capacity is called **molar specific heat capacity at constant pressure.**

When a gas is not allowed to expand, then its pressure changes but volume remains constant. Such a type of capacity is called **molar specific heat capacity at constant volume.**

Q. Why Cp > Cv ?

Ans. At constant pressure, the amount of heat given to the gas changes its internal energy and does work also. But at constant volume the amount of heat given to the gas changes its internal energy only and no work is done. That is why molar heat capacity at constant pressure is greater then molar heat capacity at constant volume

11.27- Q. Prove that the Difference of Molar Specific Heat Capacity at Constant Pressure and at Constant Volume is Equal to the Universal Gas Constant:

OR

Prove that $C_p - C_v = R$:

Let us consider "n" moles of an ideal or perfect gas enclosed in a cylinder fitted with a frictionless movable piston. The cylinder is placed on a heat reservoir, the temperature of which can be raised or lowered as needed. From kinetic molecular theory, molecules of ideal gas always remain in random motion and collide elastically with the walls of cylinder and with its piston, thereby exerting pressure on the piston which is balanced by the weight of the piston and load placed on the piston.

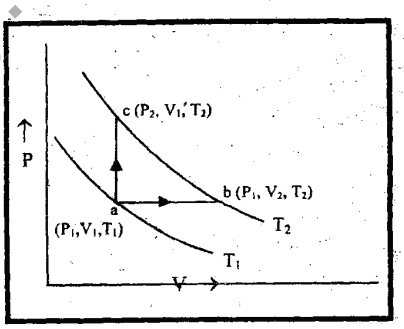
Consider two isotherms (or curves) of an ideal gas at temperatures T_1 and T_2. Suppose the gas is at state "a" with temperature T_1 and the temperature of the gas is to be raised to T_2. For this purpose there are two paths which are a→b and a→c.

The path a → b is a constant pressure path. The volume of gas changes to V_2 from V_1,

$$\Delta V = V_2 - V_1$$

Therefore, some work is said to be done by the gas

$$\Delta W = P\Delta V$$



And consequently the rise in temperature of gas is

$$\Delta T = T_2 - T_1$$

Thus at constant pressure, the amount of heat given to the gas changes its internal energy and also does work against the external Pressure, mathematically

$$\Delta Q_P = \Delta U + \Delta W$$

OR $$nC_P\Delta T = \Delta U + P\Delta V \text{ ---------- } \mathbf{(1)}$$

Applying the general gas equation PV = nRT to the Constant Pressure Path a → b:

Initially $PV_1 = nRT_1$ ---------- **(i)**

Finally $PV_2 = nRT_2$ ---------- **(ii)**

Subtracting Equation **(i)** from equation **(ii)**

$$PV_2 - PV_1 = nRT_2 - nRT_1$$

$$P\Delta V = nR\Delta T$$

Eq.(1) ⇒ $nC_P\Delta T = \Delta U + nR\Delta T$ ---------- **(2)**

The path a → c is a constant volume path, that is the volume of gas does not change:

$$\Delta V = 0$$

And hence no work is said to be done by the gas

$$\Delta W = 0$$

The rise in temperature of the gas is "ΔT"

The change in internal energy of the gas is "ΔU"

Thus at constant volume, the amount of heat energy given to the gas does nothing but only changes its internal energy, mathematically

$$\Delta Q_V = \Delta U + \Delta W$$

$\Delta Q_V = \Delta U + O$

OR $\Delta Q_V = \Delta U$ $[\because \Delta Q_v = nC_v\Delta T]$

OR $nC_V\Delta T = \Delta U$ ---------- (3)

By substituting the value of "ΔU" from equation (3) in equation (2), we get

$nC_P\Delta T = nC_V\Delta T + nR\Delta T$

$\cancel{n}C_P\cancel{\Delta T} = \cancel{n}\cancel{\Delta T}(C_V + R)$

$C_P = C_V + R$

OR $\boxed{C_P - C_V = R}$ Proved

11.28- Q. For a Monoatomic Gas Prove that,

$$C_v = R\frac{3}{2} \text{ and } C_p = R\frac{5}{2}$$

Proof:

$K.E. = \frac{3}{2}KT$ ---------- (1)

But $K = \frac{R}{N_A}$

And $N_A = \frac{N}{n}$

$\therefore K = \frac{R}{N/n}$

$K = \frac{Rn}{N}$

Putting $N = 1$

$K = Rn$

Eq.(1)⇒ $K.E. = \frac{3}{2}RnT$

OR $\Delta K.E. = \frac{3}{2}Rn\Delta T$

OR $\frac{\Delta K.E.}{n\Delta T} = \frac{3}{2}R$ ---------- (2)

Where $\frac{\Delta K.E.}{n\Delta T}$ is the amount of heat required to rise the temperature of 'n' mole of a gas at constant volume, which is the molar specific heat capacity at constant volume, i.e.

$\frac{\Delta K.E.}{n\Delta T} = C_v$ ---------- (3)

By comparing equation (2) and (3), we get

$\boxed{C_v = \frac{3}{2}R}$

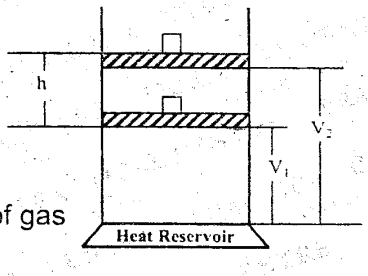
As $C_p - C_v = R$

$C_p - \frac{3}{2}R = R$

$\boxed{C_p = \frac{5}{2}R}$

11.29- Work Done in Thermodynamics:

Consider an ideal or perfect gas enclosed in a cylinder fitted with frictionless and movable piston of cross-sectional area "A". A constant pressure on the gas is applied by placing a load on the piston. Let the initial volume of the gas is V_1, and the cylinder is placed on a heat reservoir and ΔQ amount of heat flows into the system and the gas expands by pushing the piston upward through a displacement hY and the Volume of the gas becomes V_2.



as $P = \frac{F}{A}$

OR $F = PA$

Work done by the gas is ΔW = Force x Displacement

$$\Delta W = PA\Delta h$$

Where $Ah = \Delta V$, the change in volume of gas

$$\therefore \quad \boxed{\Delta W = P\Delta V} \text{ ----------(1)}$$

That is in thermodynamics work is said to be done when volume of the system change at constant pressure.

Applying gas law for initial and final conditions of system

$$PV_2 = n\,R\,T_2 \text{ ----------(i)}$$

$$PV_1 = n\,R\,T_1 \text{ ---------- (ii)}$$

Subtracting e.q. **(ii)** from e.q. **(i)**

$$PV_2 = n\,R\,T_2$$

$$-PV_1 = -n\,R\,T_1$$

$$P(V_2 - V_1) = n\,R\,(T_2 - T_1)$$

$$P\Delta V = n\,R\,\Delta T \text{ ---------- (2)}$$

By comparing e.q. **(1)** and **(2)**, we get

$$\boxed{\Delta W = n\,R\,\Delta T}$$

11.30- Thermodynamics:

The branch of physics which deals with the transformation of heat energy into mechanical energy is called thermodynamics.

11.31- First Law of Thermodynamics:

The first law of thermodynamics is merely the statement of the law of conservation of energy when it is stated with reference to heat energy and mechanical energy. It can be stated in the following ways.

"When heat energy is transformed into mechanical energy or when other mechanical energy is converted into heat, the total amount of energy remains constant".

OR

"For an isolated system the sum of all forms of energy (i.e. heat energy and mechanical energy) remains constant."

EXPLANATION:

Consider a system consisting of an ideal or perfect gas which is fitted with a frictionless piston. Let ΔQ amount of heat energy is supplied to the system and the some quantity of which converts into the work, ΔW, then the remaining energy ($\Delta Q - \Delta W$) is retained by the system, due to which internal energy of the system changes from U_1 to U_2. It is found experimentally that during the repetition of this process the change in internal energy ($U_2 - U_1 = \Delta U$) does not depend upon the path adopted by the system, but it only depends upon the initial and final states of the system. Thus we can write as:

$$\Delta Q - \Delta W = U_2 - U_1$$

OR $\Delta Q - \Delta W = \Delta U$ ---------- (1)

$\Delta Q = \Delta U + \Delta W$

11.32- Sign Conventions:

(i) ΔQ is taken as positive when heat enters the system and negative when heat leaves the system.

(ii) ΔW is taken positive when work is done by the system and negative when work is done on the system.

(iii) ΔU is taken positive when internal energy of the system increases and negative when it decreases.

11.33- Isolated System:

An isolated system is the one which has no surroundings, that is there is no flow of heat in or out from the system and hence system is not capable to do work, therefore:

$$\Delta Q = 0$$

And $\Delta W = 0$

From the first law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

We have $0 = \Delta U + 0$

OR $\Delta U = 0$

OR $U_2 = U_1$

Above result proves the law of conservation of energy i.e. the internal energy of an isolated system cannot be changed by any process taking within the system.

11.34- Cyclic Process:

A cyclic process is the one which starts and ends up at the same state, i.e. system finally attains its initial state:

$$U_2 = U_1$$

$$U_2 - U_1 = 0$$

OR $\Delta U = 0$

From the first law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

OR $\Delta Q = 0 + \Delta W$

OR $\Delta Q = \Delta W$

It means that the work obtained from a cyclic system can be at maximum equal to the energy supplied to it and no machine in any number of cycles can perform more work than the energy gained by the machine. A perpetual motion machine of the first kind was the concept of an imaginary machine which could do more work than the energy gained by it. The above result of the first law proves that it is impossible to construct such a machine.

11.35- Applications of the First Law of Thermodynamics:

(1) ISOBARIC PROCESS:

Definition:

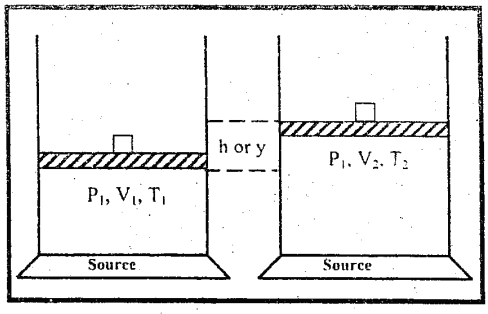
An isobaric process is the one in which pressure of the system remains constant, so that the Charle's Law is applicable.

Explanation:

Consider an ideal gas system enclosed in a cylinder provided with a frictionless piston. Let the system is placed on a heat reservoir and let ΔQ be the amount of heat supplied to the system, due to which kinetic energy of gas molecules increases which increases the internal energy of the system from U_1 to U_2

$$\Delta U = U_2 - U_1$$

also the piston of cylinder moves upward which means change in volume of system form V_1 to V_2 but pressure of system remains constant.



$$\Delta V = V_2 - V_1$$

Hence some work is said to be done by the system against constant pressure

$$\Delta W = P\Delta V$$

$$\Delta W = P(V_2 - V_1)$$

According to the first Law of Thermodynamics

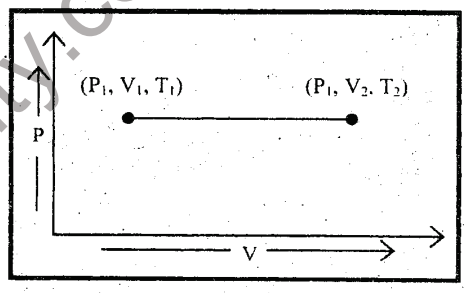
$$\Delta Q = \Delta U + \Delta W$$

OR $$\Delta Q = \Delta U + P(V_2 - V_1)$$

Above result shows that in an isobaric process, all heat energy given to the system is utilized in two ways. One in increasing the internal energy of the system and second in doing some work done against external pressure.

Graphical Representation:

On PV–diagram, the graph of an isobaric process is a straight horizontal line called as an ***Isobar***.



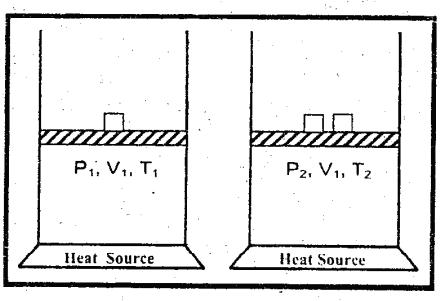
(2) ISOCHORIC PROCESS:

Definition:

An isochoric process is the one in which volume of the system remains constant, so that the pressure law is applicable.

Explanation:

Consider a system of an ideal gas in a cylinder provided with a piston, which is fixed. Let the system is placed on a heat reservoir and ΔQ be the amount of heat supplied to the system which increases the kinetic energy of gas molecules and hence internal energy of the system changes from U_1 to U_2



$$\Delta U = U_2 - U_1$$

As piston can not move, therefore, there will be no change in volume of system

$$\Delta V = 0$$

Hence there will be no work done by the system

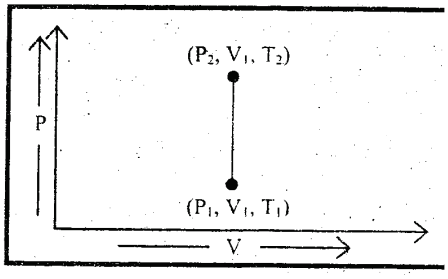
$$\Delta W = 0$$

According to the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

OR $$\Delta Q = \Delta U + O$$

$$\Delta Q = \Delta U$$



The above result shows that in an isochoric process the heat energy given to the system does nothing, but only changes the internal energy of the system.

Graphical Representation:

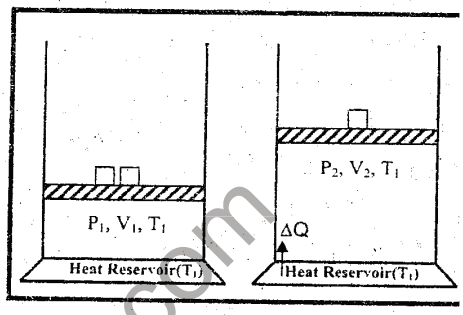
On P–V diagram, the graph of an Isoc horic Process is a vertical straight line called as *Isochor*.

(3) ISOTHERMAL PROCESS:

Definition:

An isothermal process is the one in which temperature of the system remains constant, so that the Boyle's law is applicable.

Explanation:



Consider a system of a gas in a cylinder, which is provided with a frictionless movable piston. Walls of the cylinder and piston are ideally heat insulating and its base is ideally heat conducting. The cylinder is placed on a heat reservoir at a temperature T_1. The gas is allowed to expand by decreasing the load on the piston and the temperature of system is maintained by supplying some heat energy to the system from heat reservoir. Such an expansion is called **Isothermal Expansion.**

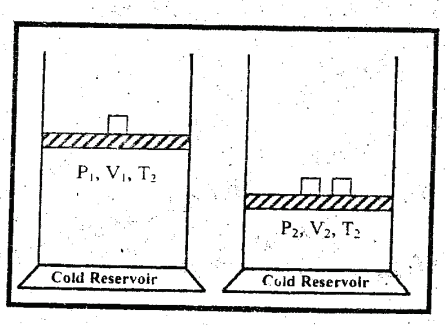
As $U_2 = U_1$

=> $\Delta U = 0$

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta Q = O + \Delta W$$
$$\Delta Q = \Delta W$$

Above result shows that in an isothermal process the heat energy given to the system is converted all into doing some work.

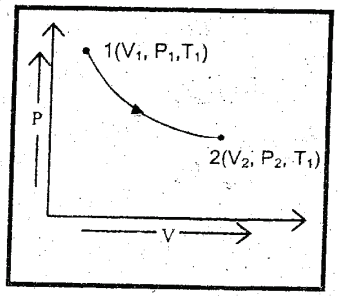


If the cylinder is placed on a cold reservoir at a temperature T_2, then the gas is allowed to compressed by increasing the load on the piston. The temperature of the system is maintained by allowing the heat to leave out form the system to the surroundings. Such a contraction is called **Isothermal contraction.** Mathematically

$$-\Delta Q = -\Delta W$$

OR $$\Delta Q = \Delta W$$

Graphical Representation:



On P-V diagram the graph of an isothermal process is a smooth curve called as **isotherm**.

(4) ADIABATIC PROCESS:

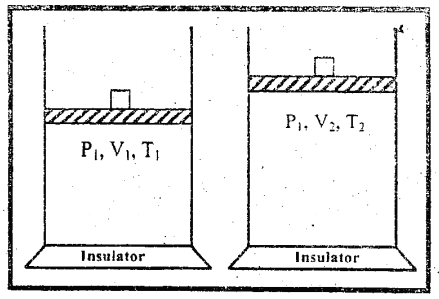
Definition:

An adiabatic process is the one in which system has no surroundings that is no heat can flow in or out from the system.

Explanation:

Consider a system of a gas in a cylinder provided with a movable frictionless piston. Let the system was initially at temperature T_1. Now let the system is placed on

insulator and the gas expands and cools off adiabatically and its temperature falls to T_2. Thus some internal energy of the system converts into work done.



Since, $\Delta Q = 0$

From the first law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$O = \Delta U + \Delta W$$

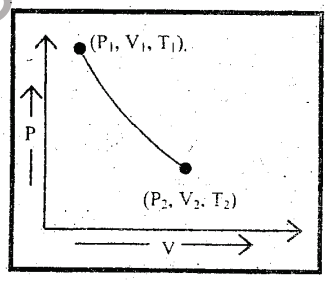
OR $$-\Delta U = \Delta W$$

Above equation shows that in an adiabatic process, work is done at the cost of internal energy of the system. This process is called adiabatic expansion.

If the system was initially at temperature T_2 and then it is placed on Insulator then the gas compresses and its temperature rises. Thus the gas at the cost of its internal energy does some work. This process is called Adiabatic Compression.

Graphical Representation:

On P-V diagram, the graph of an adiabatic process as a smooth cure called as an **adiabatic curve**.



11.36- Second Law of Thermodynamics:

The limitations regarding the transformation of heat energy into mechanical energy are stated by the second law of thermodynamics. It can be stated in a number of ways. The two basic statements were stated by Lord Kelvin and Clausius.

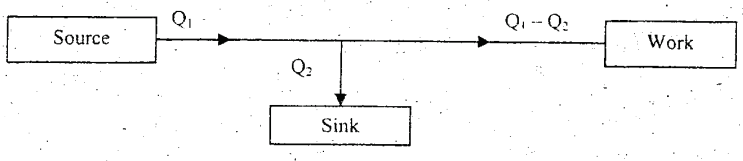
11.37- Kelvin's Statement:

STATEMENT:

"It is impossible to drive a continuous supply of work by cooling a body to a temperature lower than of the coldest of its surroundings".

EXPLANATION:

A machine which works on Kelvin's statement is called as **Heat engine**, the working of which can be understood by the following flow chart.



i.e. a continuous supply of heat can never be obtained from a single supply of heat. The conversion of heat into work is possible only when the working substance works between two different temperatures, It means that, heat engine cannot convert all the heat energy into mechanical energy without giving a part of it to the cold body or the sink.

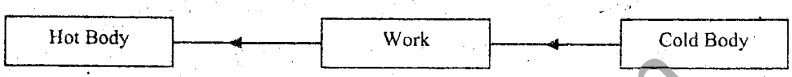
11.38- Clausius Statement:

STATEMENT:

"It is imposable to cause heat to flow from a cold body to a hot body without doing any work".

EXPLANATION:

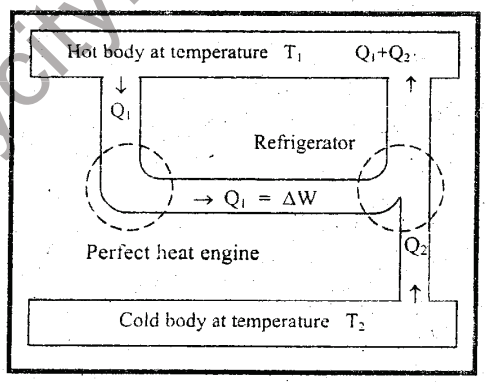
A **refrigerator** is the machine which conveys heat from a body at lower temperature to a body at higher temperature. This can be achieved only when it is connected by a suitable working substance or external agency which can provide it enough energy so that heat can flow in reveres direction.



11.39- Equivalence of Kelvin and Clausius Statement:

The two statements of the second law of thermodynamics seem to be different but the fact is contrary. If any one of the two statements is violated, another one also violates.

Suppose that Kelvin statement is false, that is it is possible to construct an engine which takes heat from the source and convert it completely into work and rejects no heat to the sink. It means that source is capable to do work by itself. Thus it will extract heat from the cold body, which provides the basis of such a refrigerator which transfers heat from cold body to the hot body without the expenditure of energy which is contrary to the claussius statement.



11.40- Carnot Engine:

Carnot engine is an ideal heat engine which was assumed by Sadi Carnot. It is free from all imperfections of real heat engine, thus its efficiency would be an ideal efficiency and could never be obtained by any real heat engine.

Carnot engine consists of a cylinder with ideally heat insulating frictionless piston. Further whose walls are ideally heat insulating and base is ideally heat conducting. An ideal gas is enclosed in the cylinder. For the operation of carnot engine, it is also supposed that source and sink are of infinite capacities.

11.41- Carnot Cycle:

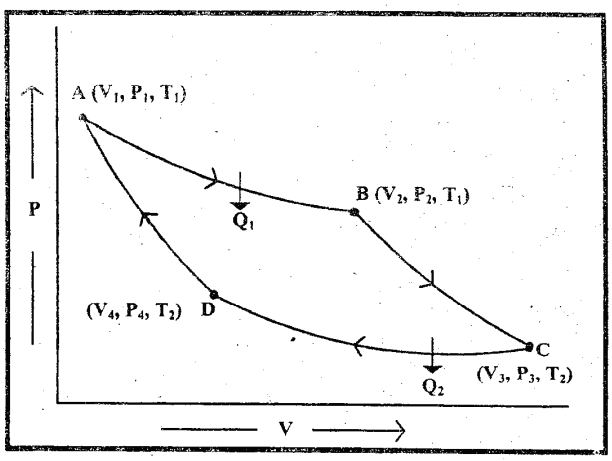
The operating cycle of a carnot engine is called carnot cycle. It consists of four following processes.

PROCESS: 1

In this process the cylinder is placed on a hot body having temperature T_1. The gas is allowed to expand by decreasing the load on the piston slowly and at the same time some heat Q_1, from the heat reservoir flows into the cylinder, so that its temperature remains constant and its volume change to V_2 from V_1. This expansion is called as Isothermal expansion.

PROCESS: 2

In this process the cylinder is placed on an insulator and the gas continues to expand a little. Since no heat can enter or leave the system, therefore this expansion is called adiabatic expansion. In this process the temperature of gas decreases from T_1 to T_2 and its volume change to V_3 from V_2.



PROCESS: 3

In this process the cylinder is placed on a cold body at a temperature T_2 and the gas is compressed very slowly by increasing the load on the piston. The temperature of the gas is maintained at T_2 by the transfer of heat Q_2 from the gas to the cold reservoir and its volume change to V_4 from V_3. This compression is called Isothermal Compression.

PROCESS: 4

In this process the cylinder is once again placed on the insulator and the gas is compressed until it attains its initial state. In this process no heat can enter or leave the system, therefore this compression is called as adiabatic compression. The temperature of the gas increasing from T_2 to T_1 and its volume comes back to V_1 from V_4.

If a graph is plotted between volume and pressure of the carnot cycle a loop is obtained. Curves "AB" and "CD" are the isothermal whereas curves "BC" and "DA" are the adiabatic.

11.42- Efficiency of Engine:

The ratio between output and input is called efficiency mathematically it can be written as:

$$\text{Efficiency} = \frac{\text{output}}{\text{Input}}$$

OR

$$\text{Efficiency} = \frac{\text{Workdone}}{\text{Heat absorbed}}$$

Derivation of Efficiency of Carnot Engine:

As carnot engine operates in a cyclic process therefore, during the cycle, Q_1 heat is added to the system and Q_2 heat is rejected,

$$\Delta Q = Q_1 - Q_2$$

After the completion of the cycle, the system returns to its initial state, i.e.

$$U_f = U_i$$

$$U_f - U_i = 0$$

$$\Delta U = 0$$

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

OR $\Delta Q = \Delta W$

OR $\Delta W = Q_1 - Q_2$

The efficiency of a heat engine is given by

$$E = \frac{\text{Output}}{\text{Input}} \times 100$$

OR

$$E = \frac{\Delta W}{Q_1} \times 100$$

$$E = \left(\frac{Q_1 - Q_2}{Q_1}\right) \times 100$$

$$E = \left(\frac{\cancel{Q_1}}{\cancel{Q_1}} - \frac{Q_2}{Q_1}\right) \times 100$$

$$E = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

Since the heat transferred to or from the carnot engine is directly proportional to the absolute temperature of the cold and hot bodies, therefore

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$E = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

Thus efficiency of Carnot engine depends only upon the temperature of hot and cold bodies.

11.43- Theoretical Limit of Efficiency of Carnot Engine:

In order to obtain 100 % efficiency of carnot engine, factor $\frac{Q_2}{Q_1}$ in equation $E = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$ should be zero

$$\frac{Q_2}{Q_1} = 0$$

=> $Q_2 = 0$

i.e. no heat is transferred to the sink, which is not possible

Also in equation $E = \left(1 - \frac{T_2}{T_1}\right) \times 100$, for hundred percent efficiency factor $\frac{T_2}{T_1}$ should be zero.

$$\frac{T_2}{T_1} = 0$$

=> $T_2 = 0$ Kelvin

i.e. sink is kept at absolute zero which is no attainable by any mean. Hence efficiency of ideal heat engine could not be hundred percent or even carnot engine is not perfect heat engine.

11.44- Entropy:

Entropy of a system is the measure of the molecular disorder. It is also defined as ***the measure of the unavailability of useful energy in the system***. Mathematically change in entropy is defined as ***the heat transferred to the system at the constant absolute temperature***,

$$\Delta S = \frac{\Delta Q}{T}$$

S.I. unit of entropy is jk^{-1}.

The change in entropy is taken as positive when heat is added to the system and negative when heat is removed from the system. In any process, molecular disorder of the system either increases or remains constant, which is expressed in the from of a law called **"The law of increase of entropy"** which states that ***"All natural processes always take place in such a direction so as to cause increase in the entropy of the system and surrounding"***.

The law of increase of entropy is a direct result of the second law of thermodynamic, so the second law can be stated as ***"when an isolated system undergoes a change, the disorder of the system, either increases or remains constant"***.