17.14 FORMULAE:

(1) Mass Variation

$m = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$; $m \rightarrow$ Relativistic mass, $m_o \rightarrow$ Rest mass

(2) Time Dilation

$t = \dfrac{t_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$; $t \rightarrow$ Relativistic time, $t_o \rightarrow$ Proper time

(3) Length Contraction

$L = L_o\sqrt{1 - \dfrac{v^2}{c^2}}$; $L \rightarrow$ Relativistic length, $L_o \rightarrow$ Proper length

(4) Mass Energy Relation

$\mathbf{E = mc^2}$

(5) Kinetic energy of particle in non-inertial frame

(i) $\mathbf{K.E = E - E_o}$; **$E \rightarrow$ Total or Relativistic energy**

$\mathbf{E_o = m_o c^2}$; **$E_o \rightarrow$ Rest mass energy**

(ii) $K.E = \dfrac{p^2}{2m_o}$; **$p \rightarrow$ Momentum of Particle**

(6) Stefen's law

$\mathbf{E = \sigma T^4}$; **$\sigma \rightarrow$ Stepen's constant**

$\sigma = 5.67 \times 10^{-8}\ W\ m^2\ K^{-4}$

(7) Raleigh Jean's law

$E = \dfrac{Constant}{\lambda^4}$

(8) Wein's Displacement law

$\mathbf{\lambda_{max} T = constant}$; **$\lambda_{max} \rightarrow$ wave length of maximum energy radiated**

Constant = 0.0029 m.k

(9) Plank's law or Energy of Photon or Quanta or Radiations

(i) $\mathbf{E = h\upsilon}$; **$h \rightarrow$ Plank's constant**

$h = 6.63 \times 10^{-34}\ J.S^{-1}$

(ii) $E = \dfrac{hc}{\lambda}$

(10) Relation between light speed, wave length and frequency

$C = \upsilon\lambda$

(11) Work function

$\phi = h\upsilon_o$; $\upsilon_o \rightarrow$ **threshold frequency**

(12) Kinetic energy of Photo electrons from

(i) Stopping Potential

$\frac{1}{2}m_o v^2 = eV_o$; **$V_o \rightarrow$ Stopping Potential**

(ii) $\frac{1}{2}m_o v^2 = E - \phi$

(13) Cut – off wave length

$$\lambda_c = \frac{hc}{eV_o}$$

(14) Wave length of minimum energy

$$\lambda_{min} = \frac{hc}{\varphi}$$

(15) Change or shift in wave length of scattered X – rays

$\lambda_2 - \lambda_1 = \frac{h}{m_o c}(1 - \cos\theta)$; **$\theta \rightarrow$ scattering angle of X – rays**

(16) De Broglie's wave length or wave length of particle from velocity

$\lambda = \frac{h}{p}$ ***OR*** $\lambda = \frac{h}{m_o v}$

(17) Wave length of particle in accelerating frame from accelerating potential

$$\lambda = \frac{h}{\sqrt{2m_o eV}}$$

(18) Pair production

$$E = 2(m_o c^2) + (K \cdot E)_{e^-} + (K \cdot E)_{e^+}$$

(19) Uncertainty Principle

(i) Momentum – Position Uncertainty

$\Delta p \Delta x = \hbar$; $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} J.S^{-1}$

(ii) Energy time uncertainty

$\Delta E \, \Delta t = \hbar$

17.15 SOLVED NUMERICALS OF BOOK:

PROBLEMS

17.1: ***In the inertial frame of Pendulum the time period is measured to be 3 seconds. What will be the period of the Pendulum for on observer moving at a speed of 0.95C with respect to the Pendulum?***

DATA:

$t_o = 3$ Sec

$V = 0.95C$

$t = ?$

SOLUTION:

$$t = \frac{t_o}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$t = \frac{3}{\sqrt{1 - \left(\frac{0.95C}{C}\right)^2}}$$

$$t = \frac{3}{\sqrt{1 - 0.9025}}$$

$$t = \frac{3}{0.312}$$

$$\boxed{t = 9.6 \text{ sec}}$$

17.2: ***What will be the length of a bar in the stationary frame if it is length along x – direction is 1m and the motion is with a velocity of 0.75C with respect to the observer at rest?***

DATA:

$L_o = 1$ m

$V = 0.75C$

$L = ?$

SOLUTION:

$$L = L_o \sqrt{1 - \frac{V^2}{C^2}}$$

$$L = 1\sqrt{1 - \left(\frac{0.75C}{C}\right)^2}$$

$$L = 1\sqrt{1 - 0.5625}$$

$$\boxed{L = 0.66 \text{ m}}$$

17.3 ***Given $m_oC^2 = 0.511$ MeV. Find the total energy and the kinetic energy of an electron moving with the speed $v = 0.85C$***

DATA:

$E_o = 0.511$ MeV

$V = 0.85C$

$E = ?$

$K{\cdot}E = ?$

SOLUTION:

as $$m = \frac{m_o}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$mC^2 = \frac{m_oC^2}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$E = \frac{E_o}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$E = \frac{0.511}{\sqrt{1 - \left(\frac{0.85C}{C}\right)^2}}$$

$$E = \frac{0.511}{\sqrt{1 - 0.7225}}$$

$$E = \frac{0.511}{0.5268}$$

$$\boxed{E = 0.970 \text{ MeV}}$$

$$K{\cdot}E = E - E_o$$

$$K{\cdot}E = 0.970 - 0.511$$

$$K{\cdot}E = 0.459 \text{ MeV}$$

17.4 The total energy of a proton of mass 1.67×10^{-27} kg is three times its rest energy. Find **(a)** Protons rest energy. **(b)** Speed of proton. **(c)** Kinetic energy of proton in eV.

DATA: $m_o = 1.67 \times 10^{-27}$ Kg

$E = 3E_o$

a) $E_o = ?$ b) $V = ?$ c) $K.E = ?$

SOLUTION:

a) $E_o = m_o C^2$

$E_o = (1.67 \times 10^{-27})(3 \times 10^8)^2$

$E_o = 1.503 \times 10^{-10}$ J

OR $E_o = \dfrac{1.503 \times 10^{-10}}{1.6 \times 10^{-19}}$

$E_o = 9.39 \times 10^8$ eV

OR $\boxed{E_o = 939 MeV}$

b) $E = 3E_o$

$m\cancel{C^2} = 3m_o\cancel{C^2}$

$m = 3m_o$

$$\frac{\cancel{m_o}}{\sqrt{1-\frac{V^2}{C^2}}} = 3\cancel{m_o}$$

$$\frac{1}{3} = \sqrt{1-\frac{V^2}{C^2}}$$

$$\frac{1}{9} = 1-\frac{V^2}{C^2}$$

$$\frac{V^2}{C^2} = \frac{8}{9}$$

$$V^2 = \frac{8}{9}C^2$$

$$V = \sqrt{\frac{8}{9}C^2}$$

$\boxed{V = 0.94C}$

c) $K.E = E - E_o$

$K.E = 3E_o - E_o$

$K.E = 2E_o$

$K.E = 2 \times 939$

$\boxed{K = 1878 \text{ MeV}}$

17.5 ***A Particle of rest mass m_o has a speed $V = 0.8C$. Find its relativistic momentum, its Kinetic energy and total energy.***

DATA:

rest mass $= m_o$

$V = 0.8C$

a) $p = ?$

b) $E = ?$

c) $K.E = ?$

SOLUTION:

a) $\mathbf{p = mv}$

$$p = \frac{m_o V}{\sqrt{1-\frac{V^2}{C^2}}}$$

$$p = \frac{m_o \times 0.8C}{\sqrt{1-\left(\frac{0.8\cancel{C}}{\cancel{C^2}}\right)^2}}$$

$$p = \frac{0.8\, m_o C}{\sqrt{1-0.64}}$$

$$p = \frac{0.8\, m_o C}{\sqrt{0.36}}$$

$$p = \frac{0.8\, m_o C}{0.6}$$

$$p = \frac{4}{3} m_o c$$

b) $\mathbf{E = mc^2}$

$$E = \frac{m_o C^2}{\sqrt{1-\frac{V^2}{C^2}}}$$

$$E = \frac{m_o C^2}{\sqrt{1-\left(\frac{0.8\cancel{C}}{\cancel{C}}\right)^2}}$$

$$E = \frac{m_o C^2}{\sqrt{1-0.64}}$$

$$E = \frac{m_o C^2}{\sqrt{0.36}}$$

$$E = \frac{m_o C^2}{0.6}$$

$$\boxed{E = \frac{5}{3} m_o C^2}$$

c) $\mathbf{K.E = E - E_o}$

$$K.E = \frac{5}{3} m_o C^2 - m_o C^2$$

$$K.E = \frac{5 m_o C^2 - 3 m_o C^2}{3}$$

$$\boxed{K.E = \frac{2}{3} m_o C^2}$$

17.6: ***What will be the velocity and momentum of a particle of rest mass m_o whose kinetic energy is equal to its rest mass energy?***

DATA:

rest mass = m_o

$V = ?$

$P = ?$

$K{\cdot}E = E_o$

SOLUTION:

Given that $K{\cdot}E = E_o$

$E - E_o = E_o$

$E = 2\,E_o$

$mC^2 = 2\,m_oC^2$

$m = 2\,m_o$

$$\frac{m_o}{\sqrt{1-\frac{V^2}{C^2}}} = 2\,m_o$$

$$\frac{1}{2} = \sqrt{1-\frac{V^2}{C^2}}$$

$$\frac{1}{4} = 1 - \frac{V^2}{C^2}$$

$$\frac{V^2}{C^2} = 1 - \frac{1}{4}$$

$$\frac{V^2}{C^2} = \frac{3}{4}$$

$$V^2 = \frac{3}{4}C^2$$

$$V = \frac{\sqrt{3}}{2}C$$

$$\boxed{V = 0.87C}$$

P = mv

$$p = 2\,m_o \times \frac{\sqrt{3}}{2}C$$

$$\boxed{p = \sqrt{3}\,m_o C}$$

17.7 ***The sun radiates energy at a rate of 3.8×10^{26}w. At what rate the mass of the sun diminishes?***

DATA:

$P = 3.8 \times 10^{26}$W

Time = 1 year = 3.15×10^{7}sec

rate of mass diminished per year = ?

SOLUTION:

$$P = \frac{E}{t}$$

$E = P \times t$

$mC^2 = P \times t$

$$m = \frac{p \times t}{C^2}$$

$$m = \frac{3.8 \times 10^{26} \times 3.15 \times 10^{7}}{(3 \times 10^{8})^2}$$

$$\boxed{m = 1.33 \times 10^{17}\ \text{Kg/year}}$$

17.8: ***What will be the work function of a substance for a threshold frequency of 43.9×10^{13} Hz?***

DATA:

$\upsilon_o = 43.9 \times 10^{13}$ Hz

$\Phi = ?$

$h = 6.63 \times 10^{-34}$J s^{-1}

SOLUTION:

$\Phi = h\,\upsilon_o$

$\Phi = 6.63 \times 10^{-34} \times 43.9 \times 10^{13}$

$\Phi = 2.93 \times 10^{-19}$ J

$$\Phi = \frac{2.93 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\boxed{\Phi = 1.83\ \text{eV}}$$

17.9: ***What will be the value of $\lambda min = \frac{hc}{eV_o}$; if $h = 6.63 \times 10^{-34}$ Js^{-1}, $e = 1.6 \times 10^{-19}$ C and $V_o = 10^4 V$, $C = 3 \times 10^8$ ms^{-1}***

DATA:

$\lambda min = ?$

$h = 6.63 \times 10^{-34} Js^{-1}$

$e = 1.6 \times 10^{-19}$ C

$V_o = 10^4$ V

SOLUTION:

$$\lambda min = \frac{hc}{eV_o}$$

$$\lambda min = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10^4}$$

$$\boxed{\lambda = 1.24 \times 10^{-10} \text{ m}}$$

17.10: ***In a Compton scattering process, the fractional change in wavelength of x – ray photon is 1% at an angle $\theta = 120^o$, find the wavelength of x – ray used in the experiment.***

DATA:

$$\frac{\Delta\lambda}{\lambda_1} = \frac{1}{100}$$

$\theta = 120^o$

$\lambda_2 = ?$

SOLUTION:

$$\frac{\Delta\lambda}{\lambda_1} = \frac{1}{100}$$

$$\Delta\lambda = \frac{\lambda_1}{100}$$

$$\Delta\lambda = \frac{h}{m_o C}(1 - \cos\theta)$$

$$\frac{\lambda_1}{100} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times (3 \times 10^8)}(1 - \cos 120^o)$$

$$\frac{\lambda_1}{100} = 2.43 \times 10^{-12} \times 1.5$$

$$\frac{\lambda_1}{100} = 3.64 \times 10^{-12}$$

$$\lambda_1 = 3.64 \times 10^{-12} \times 100$$

$$\boxed{\lambda_1 = 3.64 \times 10^{-10} \text{ m}}$$

17.11: ***Find the wave length of 2.0g light ball moving with a velocity (a) 1.0mm per century (b)1.0mS^{-1}. (Given $h = 6.63 \times 10^{-34}$ Js^{-1} and 1 year = 3.15×10^7 S)***

DATA:

$\lambda = ?$

$m_o = 2g = 2 \times 10^{-3}$ Kg

a) V = 1 mm per century

b) V = 1 m/s

SOLUTION:

$$\lambda = \frac{h}{m_o V} \quad \text{(1)}$$

a) 1mm $= 1 \times 10^{-3}$ m

1 century $= 3.15 \times 10^9$ S

$$V = \frac{1 \times 10^{-3}}{3.15 \times 10^9} = 3.17 \times 10^{-13} \text{m/s}$$

$$\text{eq (1)} \Rightarrow \lambda = \frac{6.63 \times 10^{-34}}{2 \times 10^{-3} \times 3.17 \times 10^{-13}}$$

$$\boxed{\lambda = 1.05 \times 10^{-18} \text{ m}}$$

b) $$\text{eq (1)} \Rightarrow \lambda = \frac{6.63 \times 10^{-34}}{2 \times 10^{-3} \times 1}$$

$$\boxed{\lambda = 3.315 \times 10^{-31} \text{m}}$$

17.12: ***An electron exist within a region of 10^{-10} m. Find its momentum uncertainty and the approximate kinetic energy.***

DATA:

$\Delta x = 10^{-10}$ m

a) $\Delta p = ?$

b) K.E = ?

SOLUTION:

a) $\Delta p \, \Delta x = \hbar$

$$\Delta p = \frac{\hbar}{\Delta x}$$

$$\Delta p = \frac{1.05 \times 10^{-34}}{10^{-10}}$$

$$\boxed{\Delta p = 1.05 \times 10^{-24} \text{ Kgm/s}}$$

$$\Delta p = \frac{\hbar}{\Delta x}$$

$$\Delta p = \frac{1.05 \times 10^{-34}}{10^{-10}}$$

$\Delta p = 1.05 \times 10^{-24}$ Kgm/s

b) $$K.E = \frac{\Delta p^2}{2m_o}$$

$$K.E = \frac{(1.05 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31} \, 1 \times 1.6 \times 10^{-19}}$$

$$\boxed{K.E = 3.8 \text{ eV}}$$

17.13: ***Sodium surface is shined with light of wavelength 3×10^{-7} m. If the work function of Na = 2.46 eV, find the K.E of the Photoelectrons and also the cut off wave length.***

DATA:

$\lambda = 3 \times 10^{-7}$ m

$\Phi = 2.46$ eV

K.E = ?

$\lambda_c = ?$

SOLUTION:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$E = 4.14$ eV

K.E = E - Φ

K.E = 4.14 – 2.46

K.E = 1.68 eV

$\Phi = 2.46 \times 1.6 \times 10^{-19}$

$\Phi = 3.936 \times 10^{-19}$ J

$$\lambda_c = \frac{hc}{\Phi}$$

$$\lambda_c = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3.936 \times 10^{-19}}$$

$$\boxed{\lambda_c = 5.05 \times 10^{-7}\ \text{m}}$$

17.14: ***X – Rays of wave length. λ_o are scattered from a carbon black at an angle of 45° with respect to the incident beam. Find the shift in wave length.***

DATA:

given $\lambda_1 = \lambda_o$

then $\lambda_2 = \lambda$

$\theta = 45^o$

$\lambda - \lambda_o = ?$

SOLUTION:

$$\lambda - \lambda_o = \frac{h}{m_o C}(1 - \cos\theta)$$

$$\lambda - \lambda_o = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 45^o)$$

$$\boxed{\lambda - \lambda_o = 7.11 \times 10^{-13}\text{m}}$$

17.15: ***If the electron beam in a TV Picture tube is, accelerated by 10,000V what will be the De'Broglie's wave length?***

DATA:

V = 10,000V

$\lambda = ?$

SOLUTION:

$$\lambda = \frac{h}{\sqrt{2 m_o e V}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 10^{-31} \times 1.6 \times 10^{-19} \times 10.000}}$$

$$\boxed{\lambda = 1.23 \times 10^{-11}\ \text{m}}$$

17.16: ***What minimum energy proton can be used to observe an object of size 2.5×10^{-10} m.***

DATA:

$\Delta E = ?$

$\Delta x = 2.5 \times 10^{-10}$m

SOLUTION:

$\Delta p \Delta x = h$

$$\Delta p = \frac{h}{\Delta x}$$

$$\Delta p = \frac{6.63 \times 10^{-34}}{2.5 \times 10^{-10}}$$

$\Delta p = 2.652 \times 10^{-24}$Kgm/s

$\Delta E = mc^2$

$\Delta E = (mc)c$

$\Delta E = \Delta pc$

$$\Delta p = \frac{2.652 \times 10^{-24} \times 3 \times 10^{8}}{1.6 \times 10^{-19}}$$

$$\boxed{\Delta E = 4972.5\ \text{eV}}$$

17.17: ***What will be the De'broglie's wavelength associated with a mass of 0.01 Kg moving with a Velocity 10 m/s.***

DATA:

$\lambda = ?$
$m_o = 0.01\ Kg$
$V = 10\ m/s.$

SOLUTION:

$$\lambda = \frac{h}{m_o V}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{0.01 \times 10}$$

$$\boxed{\lambda = 6.63 \times 10^{-33}\ m}$$

17.18: ***Certain excited state of hydrogen atom have a life time 2.5 x 10^{-19} S. What will be the minimum uncertainty in energy?***

DATA:

$\Delta t = 2.5 \times 10^{-19}\ S$
$\Delta E = ?$

SOLUTION:

$$\Delta E\ \Delta t = \hbar$$

$$\Delta E = \frac{\hbar}{\Delta \hbar}$$

$$\Delta E = \frac{1.05 \times 10^{-34}}{2.5 \times 10^{-19} \times 1.6 \times 10^{-19}}$$

$$\boxed{\Delta E = 2625\ eV}$$

17.19: ***X-Rays are scattered from a target material. The scattered radiation is viewed at an angle of 90^{o} with respect to the incident beam. Find the Compton shift in wave length.***

DATA:

$\theta = 90^{o}$
$\lambda_2 - \lambda_1 = ?$

SOLUTION:

$$\lambda_2 - \lambda_1 = \frac{h}{m_o C}(1 - \cos\theta)$$

$$\lambda_2 - \lambda_1 = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 90^{o})$$

$$\boxed{\lambda_2 - \lambda_1 = 2.43 \times 10^{-12}\ m}$$

17.20: ***Find the frequency of a photon when an electron of 20 KeV is brought to rest in a collision with a heavy nucleus.***

DATA:

$\upsilon = ?$
$E = 20 KeV$

SOLUTION:

$E = 20 \times 10^{3} \times 1.6 \times 10^{-10}$
$E = 3.2 \times 10^{-15} J$

$$E = h\upsilon$$

$$\upsilon = \frac{E}{h}$$

$$\upsilon = \frac{3.2 \times 10^{-15}}{6.63 \times 10^{-34}}$$

$$\boxed{\upsilon = 4.83 \times 10^{18}\ Hz}$$

17.16 SOLVED NUMERICALS OF PAPERS:

YEAR 2013.

Q. Pair annihilation occurred due to a head-on-collision of an electron and a positron having the same kinetic energy producing pair of Photons each having 2.5MeV. What were their kinetic energies before collision? (Given $M_oC^2 = 0.511$ MeV)

DATA:

$E = h\nu = 2.5$ MeV

$E_o = m_oc^2 = 0.511$ MeV

$(K.E)^{e-} = (K.E)^{e+} = K.E$

SOLUTION:

As we know that for Annihilation of matter

$2h\nu = 2m_oc^2 + (K.E)_{e-} + (K.E)_{e-}$

Or $2h\nu = 2m_oc^2 + K.E + K.E$

Or $2E = 2m_oc^2 + 2K.E$

$2E = 2(m_oc^2 + K.E)$

Or $E = m_oc^2 + K.E$

Or $K.E = E - m_oc^2$

$= 2.5 - 0.511$

$\boxed{K.E = 1.989 \text{ MeV}}$

$\therefore$ $(K.E)_{e-} = (K.E)^{e-} = (K.E)$

$\boxed{(K.E)^{e-} = (K.E)_{e+} = 1.982 \text{ MeV}}$

YEAR 2011.

Q.2(xii) Find the shortest wavelength of photon emitted in the Balmer series and determine its energy in eV. ($R_H = 1.097 \times 10^{-7}$ m^{-1}).

DATA:

Rest mass = m_o

$K = 2E_o$

(a) Velocity = V = ?

(b) Momentum = P = ?

SOLUTION:

As given that

$K = 2E_o$ ------------(1)

Using [K = K.E]

$E = E_o + 2E_o$

Or $E = 3E_o$

$mc^2 = 3m_oc^2$

$m = 3m_o$ -----------(2)

(a) For Velocity

Equation (2) also written as

$$\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = 3m_o \quad \left[m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \right]$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3$$

$$\frac{1}{3} = \sqrt{1-\frac{v^2}{c^2}}$$

Squaring on both sides

$$\frac{1}{9} = 1 - \frac{v^2}{c^2}$$

Or $\frac{v^2}{c^2} = 1 - \frac{1}{9}$

$$\frac{v^2}{c^2} = \frac{9-1}{9}$$

$$\frac{v^2}{c^2} = \frac{8}{9}$$

$$v^2 = \frac{8}{9}c^2$$

Or $\sqrt{v^2} = \sqrt{\frac{8}{9}c^2}$

$\boxed{v = \frac{\sqrt{8}}{3}c}$ --------(A)

Or $\boxed{v = \frac{2\sqrt{2}}{3}c}$

Or $v = \frac{2 \times 1.44 \times 3 \times 10^8}{3}$

$\boxed{v = 2.828 \times 10^8 \text{ m/sec}}$

(b) For momentum

$P = mv$

Put m and v from (2) and (A)

$P = 3m_o \times \frac{\sqrt{8}}{3}c$

$\boxed{P = \sqrt{8}\, m_oc}$

Or $\boxed{P = 2\sqrt{2}\, m_oc}$

YEAR 2010.

Q.2(xiv)If the electron beam in a television picture tube is accelerated by 10000volt, What will be the de-Broglies's hypothesis ($h = 6.63 \times 10^{-34}$ J sec) ($m = 9.1 \times 10^{-31}$kg)

SOLUTION: Problem No. 17.15 of Book

ANSWER $\lambda = 1.23 \times 10^{-11}$m

YEAR 2009.

Q.7 (d) A sodium surface is shined with the light of wavelength 3×10^{-7} m. If work function of sodium metal is 2.46ev. Find K.E of the photo electrons and also cut off wavelength.

SOLUTION:

SIMILAR TO QUESTION# 17.3

ANSWER (a) K.E = 1.68ev

(b) $\lambda_c = \lambda_o = 5.05 \times 10^{-7}$ m

YEAR 2008.

Q.7 (d) Calculate the relativistic speed at which mass of a particle becomes double its rest mass.

DATA:

$m = 2m_O$

Relativistic speed = v = ?

SOLUTION:

As we know that

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

OR $$2m_o = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

Squaring on both sides

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

OR $$1 - \frac{1}{4} = \frac{v^2}{c^2}$$

$$\frac{4-1}{4} = \frac{v^2}{c^2}$$

OR $$\frac{3}{4} = \frac{v^2}{c^2}$$

$$v^2 = \frac{3}{4}c^2$$

$$v^2 = \sqrt{\frac{3}{4}c^2}$$

OR $$v = \frac{\sqrt{3}}{2}c$$

OR $$v = \frac{1.732}{2} \times 3 \times 10^8 \text{m/sec}$$

$$V = 2.598 \times 10^8 \text{ m/sec}$$

YEAR 2007.

Q.8 (d) In Compton scattering process the fractional change in wavelength of x –rays photon is 1% at an angle 120, find the wavelength of x – rays used in this experiment.

DATA:

Fractional change $= \frac{\Delta\lambda}{\lambda_1} = 1\% = = \frac{1}{100} = 10^{-2}$

Angle $= \theta = 120°$

Wavelength of x – rays $= \lambda_1 = ?$

SOLUTION:

As $\Delta\lambda = \lambda c(1 - 105\ \theta)$ ———— (1)

Where

$$\lambda_c = \frac{h}{m_o c}$$

$$\lambda_c = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} kg \times 3 \times 10^{8}} = \frac{6.63 \times 10^{-34}}{27.33 \times 10^{-23}}$$

$$\lambda_c = 2.426 \times 10^{-12} m$$

$\therefore (1) \Rightarrow\ \Delta\lambda = 2.426 \times 10^{-12} (1 - \text{Cos } 120°)$

$\Delta\lambda = 2.426 \times 10^{-12} [1 - (-0.5)]$

$\Delta\lambda = 2.426 \times 10^{-12} \times 1.5$

$$\frac{\Delta\lambda}{\lambda_1} = \frac{3.6389 \times 10^{-12}}{\lambda_1}$$

$$10^{-2} = \frac{3.639 \times 10^{-12}}{\lambda_1}$$

$$\lambda_1 = \frac{3.639 \times 10^{-12}}{10^{-2}}$$

$\lambda_1 = 3.639 \times 10^{-10}$ m

$[\because 10^{-10} m = 1 A°]$

OR **$\lambda_1 = 3.64$ A°**

YEAR 2006.

Q.7(d) An electron exists within a region 10^{-10} m find its momentum uncertainify and approximate kinetic energy.

SOLUTION:

SIMILAR TO QUESTION# 17.12

ANSWER (a) $\Delta P = 1.05 \times 10^{-24}$ N – sec

(b) K·E = 3.8ev

YEAR 2005.

Q.8 (d) Estimate the relativistic mass and the wavelength associated with an electron moving at 0.9 C ($h = 6.63 \times 10^{-34}$ J – sec).

DATA:

Rest mass of electron = $m_O = 9.11 \times 10^{-31}$ kg
Velocity of electron = v = 0.9 C
Plank's constant = $h = 6.63 \times 10^{-34}$ J-sec
(a) Relativistic mass of electron = m = ?
(b) Wavelength = λ = ?

SOLUTION:

As we know that

$$m = \frac{m_o}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

OR

$$m = \frac{9.11 \times 10^{-31}}{\sqrt{1-\left(\frac{0.9\cancel{c}}{\cancel{c}}\right)^2}}$$

$$m = \frac{9.11 \times 10^{-31}}{\sqrt{1-0.81}}$$

$$m = \frac{9.11 \times 10^{-31}}{0.43589}$$

$$\boxed{m = 2.0899 \times 10^{-30}\ kg}$$

Using De-Broglie Hypothesis

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{2.0899 \times 10^{-30} \times 0.9 \times 3 \times 10^{8}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{5.6427 \times 10^{-22}}$$

$$\boxed{\lambda = 1.175 \times 10^{-12}\ m}$$

OR $\lambda = 1.175 \times 10^{-2} \times 10^{-10}$ m

$$\boxed{\lambda = 1.175 \times 10^{-2} \times A^\circ}$$

YEAR 2004.

Q.8 (d) Compare the energy of photon of wavelength 2×10^{-6} m with the energy of x – ray photon of wavelength 2×10^{-10} m ($h = 6.63 \times 10^{-34}$ J-sec)

DATA:

Wavelength of photon = $\lambda_p = 2 \times 10^{-6}$ m
Wavelength of x – ray = $\lambda_x = 2 \times 10^{-6}$ m
Plank's constant = $h = 6.63 \times 10^{-34}$ J-sec
Compare the energy = $E_p : E_x = \frac{E_p}{E_x} = ?$

SOLUTION:

As we know that
$E = h\gamma$

OR $E = \frac{hc}{\lambda}$ ——————(1)

For Photon

$E_p = \frac{hc}{\lambda_p}$ ——————(2)

For x – ray

$E_x = \frac{hc}{\lambda_x}$ ——————(3)

Now $E_p : E_x = \frac{E_p}{E_x}$

$$E_p : E_x = \frac{h_c}{\lambda_p} \Big/ \frac{h_c}{\lambda_x}$$

$$E_p : E_x = \frac{\cancel{hc}}{\lambda_p} \times \frac{\lambda_x}{\cancel{hc}}$$

$$E_p : E_x = \frac{\lambda_x}{\lambda_p}$$

$$E_p : E_x = \frac{2 \times 10^{-10}}{2 \times 10^{-6}}$$

$$\boxed{E_p : E_x = 10^{-4}}$$

OR

$$\boxed{\frac{E_p}{E_x} = 10^{-4}}$$

YEAR 2004.

Q.8 (d) 2nd Method

As $E = h\upsilon$

OR $E = \frac{hc}{\lambda}$ ——————— (1) $\left[\begin{array}{l} \upsilon = \frac{c}{\lambda} \\ c = 3 \times 10^8 m/sec \end{array}\right]$

FOR PHOTON

$$E_p = \frac{hc}{\lambda_p}$$

$$E_p = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-6}}$$

$E_p = 9.945 \times 10^{-20}$ J ——————— (2)

FOR X – RAY

(1)$\Rightarrow$ $E_x = \frac{hc}{\lambda_x}$

$$E_x = \frac{1.989 \times 10^{-25}}{2 \times 10^{-10}}$$

$E_x = 9.945 \times 10^{-16}$ J ——————— (3)

Now $E_p : E_x = \frac{E_p}{E_x}$

$$E_p : E_x = \frac{9.945 \times 10^{-20}}{9.945 \times 10^{-16}}$$

$$\boxed{E_p : E_x = 10^{-4}}$$

OR $\boxed{\frac{E_p}{E_x} = 10^{-4}}$

YEAR 2002.P.M.

Sodium surface is shined with the light of wave length 3×10^{-7}m. If the work function of Na = 2.46 eV, find the Kinetic energy of the photo electrons. [$C = 3 \times 10^8$m/s, $h = 6.63 \times 10^{-34}$J-S].

DATA:

$\lambda = 3 \times 10^{-7}$m

$\phi = 2.46$ eV

K= ?

SOLUTION:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$E = 4.14$ eV

$K = E - \phi$

$K = 4.14 - 2.46$

$$\boxed{K = 1.68 \text{ eV}}$$

YEAR 2002.P.M.

Find the speed at which the mass of a particle will be doubled. [$c = 3 \times 10^8$ m/s]

SOLUTION:

$$m = 2m_0$$

$$\frac{m_o}{\sqrt{1-\frac{V^2}{C^2}}} = 2m_0$$

$$\frac{1}{2} = \sqrt{1-\frac{V^2}{C^2}}$$

$$\frac{1}{4} = 1-\frac{V^2}{C^2}$$

$$v = \frac{\sqrt{3}}{2}c$$

$$v = 0.8666 \times 3 \times 10^8$$

$$\boxed{v = 2.6 \times 10^8 \text{ m/s}}$$

YEAR 2002.P.E.

Given m_0c^2=0.511 MeV. Find the total energy E and the Kinetic energy K of an electron moving with a speed v = 0.85c.

(Rest mass of electron = 9.1×10^{-31}Kg and velocity of light is 3×10^8 m/s).

DATA:

$m_0c^2 = E_0 = 0.511$MeV

$v = 0.85c$

$m_0 = 9.1 \times 10^{-31}$Kg

$c = 3 \times 10^8$ m/s

E=?, K=?

SOLUTION:

$$m = \frac{m_o}{\sqrt{1-\frac{V^2}{C^2}}}$$

$$mc^2 = \frac{m_oc^2}{\sqrt{1-\frac{V^2}{C^2}}}$$

$$E = \frac{E_o}{\sqrt{1-\frac{V^2}{C^2}}}$$

$$E = \frac{0.511\text{MeV}}{\sqrt{1-\left(\frac{0.85\cancel{c}}{\cancel{c}}\right)^2}}$$

$$\boxed{E = 0.97 \text{ MeV}}$$

$$K = E - E_0$$

$$K = 0.97 - 0.511$$

$$\boxed{K = 0.459 \text{ MeV}}$$

17.17 NUMERICALS FOR SELF PRACTICE:

YEAR 2012.

Q.2(iv) Given $m_oc^2 = 0.511MeV$. Find the total energy "E" and the kinetic energy "K" of an electron moving with a speed 0.85c

($m_o = 9.1 \times 10^{-31}$Kg, $c = 3\times10^8$ m/Sec).

Ans. **Q#17.3 of Book**

Q.2(xv) A sodium surface is exposed to a light of wavelength 3×10^{-7}m. If the work function of sodium metal is 2.46eV find the K.E of the photoelectrons and cut-off wavelength.

Ans. **Q#17 of Book**

YEAR 1999.

The range of visible light is 4000°A to 7000°A, will photo-electrons be emitted by a copper surface of work function 4.4eV, when illuminated by visible light?

Ans. $E < \phi$

YEAR 1994.

The work function of a metal is 2eV. The light of wavelength 3000°A is made to fall on it. Find the Kinetic energy of the fastest emitted photoelectrons.

Ans. K.E = 2.14 eV

YEAR 1992.

The work function of a photo emissive surface is 4.0 eV. What will be the velocity of the fastest photo electrons emitted from it by an incident light of frequency 3×10^{15} Hz. ($h=6.63\times10^{-34}$ J. sec).

Ans. $v = 1.72 \times 10^6$ m/s

YEAR 1989.

The work function of a certain metal is 3.0 eV, when this metal is illuminated by the infra-red light of 1.2×10^{15} Hertz, find the maximum K.E. of the emitted photo electrons. ($h=6.63\times10^{-34}$ J. sec).

Ans. K.E = 1.97eV

YEAR 1987.

Find the cut-off wave length for a given metal whose work function is 4.4 eV. ($h=6.63\times10^{-34}$ J. sec).

Ans. $\lambda_C = 282.2$ nm