12.17 SOLVED NUMERICALS OF BOOK:

Problem: 12.1:

Two unequal point charges repel each other with a force of 0.2 Newtons when they are 10cm apart. Find the force which each exerts on the other when they are **(i)** 1cm apart and **(ii)** 5cm apart.

Data:

$r = 10cm = 0.1m$
$F = 0.2N$
(i) $r = 1cm = 0.01m$
$F = ?$
(ii) $r = 5cm = 0.05m$
$F = ?$

Solution:

$K q_1 q_2 = \text{constant}$

$$F = \frac{K q_1 q_2}{r^2} \text{ --------(i)}$$

$$0.2 = \frac{K q_1 q_2}{(0.1)^2}$$

$$\boxed{K q_1 q_2 = 0.002 \ Nm^2}$$

(i) eq.(i)$\Rightarrow$ $F = \frac{0.002}{(0.01)^2}$

$$\boxed{F = 20N}$$

(ii) eq.(i)$\Rightarrow$ $F = \frac{0.002}{(0.05)^2}$

$$\boxed{F = 0.8N}$$

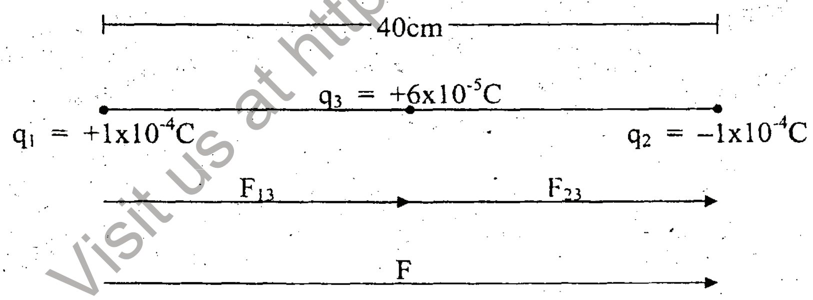
Self Test (1):

(i) What is electric force exerted on each of two identical conductors each with charge of 1μC and separated by 1.0cm? **[Ans. F = 0.9N]**

(ii) Two positive charges of magnitude 20μC and 100μC are placed at a distance of 150cm from each other. Calculate the electric force of repulsion between them. **[Ans. F = 8N]**

Problem: 12.2:

Two point charges $+1\times10^{-4}C$ and $-1\times10^{-4}C$ are placed at a distance of 40cm from each other. A charge of $+6\times10^{-5}C$ is placed midway between them. What is the magnitude and direction of force on it?



Required:

(i) Magnitude of force on q_3 = ?
(ii) Direction of force on q_3 = ?

Solution:

(i) $F_{13} = \frac{K q_1 q_3}{r^2}$

$$F_{13} = \frac{9\times10^9 \times 1\times10^{-4} \times 6\times10^{-5}}{(0.2)^2}$$

$F_{13} = 1350N$

Similarly $F_{23} = 1350N$

As F_{13} and F_{23} are acting along the same line, therefore, their resultant force will be their sum,

$F = F_{13} + F_{23}$

$F = 1350 + 1350$

$\boxed{F = 2700 \ N}$ **Ans.**

(ii) Direction of force will be towards the negative charge.

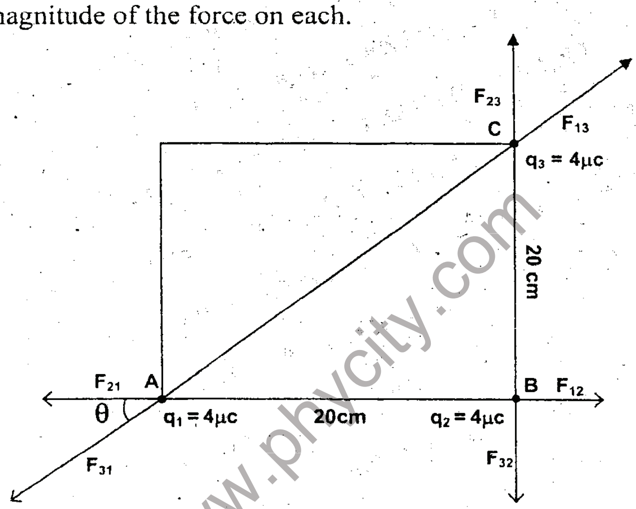
Self Test (2):

Two point charges of $+2 \times 10^{-4}$ coul and -2×10^{-4} coul are placed at a distance of 40cm from eachother. A charge of $+5 \times 10^{-5}$ coul is placed midway between them. What is the magnitude and direction of the force on it? **(2012)**
($K = 9 \times 10^9\ Nm^2/c^2$).

Ans. **4.5×10^3 N towards –ve charge**

Problem: 12.3:

Three point charges each of 4μc. are placed at the three corners of a square of side 20cm. Find the magnitude of the force on each.



Required:

(i) Magnitude of force on $q_1 = ?$
(ii) Magnitude of force on $q_2 = ?$
(iii) Magnitude of force on $q_3 = ?$

Solution:

$$F_{12} = \frac{K q_1 q_2}{r^2}$$

$$F_{12} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{(0.2)^2}$$

$$F_{12} = 3.6N$$

Similarly $F_{21} = 3.6N$

$$F_{23} = 3.6N$$

$$F_{32} = 3.6N$$

Determination of length of diognal AC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$AC = \sqrt{(20)^2 + (20)^2}$$

$$AC = \sqrt{800}$$

$$AC = 28.28cm$$

OR $r = 0.28m$

$$F_{13} = \frac{K\,q_1\,q_3}{r^2}$$

$$F_{13} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{(0.28)^2}$$

$$F_{13} = 1.8N$$

Similarly $F_{31} = 1.8N$

(i) As F_{21} and F_{31} are not perpendicular on each other, therefore, magnitude of their force will be determined by the Law of parallelogram.

$$F' = \sqrt{F_{21}^2 + F_{31}^2 + 2F_{21}F_{31}\cos\theta}$$

$$F' = \sqrt{(3.6)^2 + (1.8)^2 + 2(3.6)(1.8)\cos 45^\circ}$$

$$\boxed{F' = 5.036N}$$

OR $\boxed{F' = 5.04N}$

(ii) As F_{12} and F_{32} are perpendicular on each other, therefore magnitude of their resultant force will be determined by the pythaghorus theorem.

$$F''^2 = F_{12}^2 + F_{32}^2$$

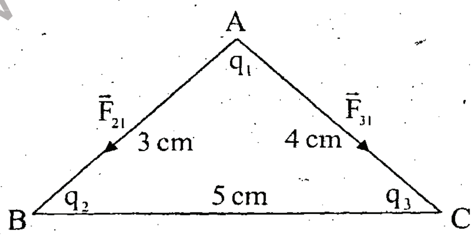
$$F''^2 = \sqrt{F_{12}^2 + F_{32}^2}$$

$$F'' = \sqrt{(3.6)^2 + (3.6)^2}$$

$\boxed{F'' = 5.09\ N}$ **Ans.**

(iii) Magnitude of force on q_3 is the same as that on q_1, because they are placed diognally across.

Problem: 12.4:



Three charges,

$q_1 = +7 \times 10^{-6}$ C

$q_2 = -4 \times 10^{-6}$ C and

$q_3 = -5 \times 10^{-6}$ C are placed

at the vertices of a triangle as shown in the diagram.

The sides of the triangle measure 3, 4 and 5cm. Determine the magnitude and direction of the force on the charge q_1.

Data:

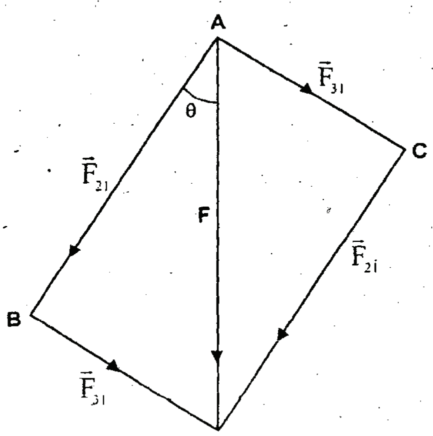
$q_1 = +7 \times 10^{-6}$ C

$q_2 = -4 \times 10^{-6}$ C

$q_3 = -5 \times 10^{-6}$ C

(i) Magnitude of force on q_1 = ?

(ii) Direction of force on q_1, θ = ?



Solution:

(i) $$F_{21} = \frac{K\,q_2\,q_1}{r^2}$$

$$F_{21} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 7 \times 10^{-6}}{(0.03)^2}$$

$$F_{21} = 280\ N$$

$$F_{31} = \frac{K q_3 q_1}{r^2}$$

$$F_{31} = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 7 \times 10^{-6}}{(0.04)^2}$$

$$F_{31} = 196.875 \text{ N}$$

As F_{21} and F_{31} are perpendicular on each other, therefore, magnitude of their resultant force will be determined by the Pythagoras theorem.

$$F^2 = F_{21}{}^2 + F_{31}{}^2$$

$$F = \sqrt{F_{21}^2 + F_{31}^2}$$

$$F = \sqrt{(280)^2 + (196.875)^2}$$

$$\boxed{F = 342.3 \text{ N}} \quad \textbf{Ans.}$$

(ii) $\theta = \tan^{-1}\left(\frac{F_{31}}{F_{21}}\right)$

$$\theta = \tan^{-1}\left(\frac{196.875}{280}\right)$$

$$\boxed{\theta = 35.1 \text{ along AB}}$$

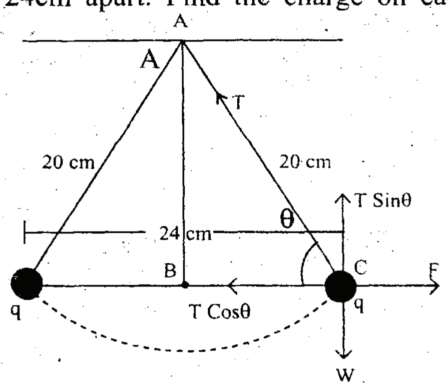
$$\theta_1 = 90^\circ - \theta$$

$$\theta_1 = 90^\circ - 35.1^\circ$$

$$\boxed{\theta_1 = 54.9^\circ \text{ along AC}} \quad \textbf{Ans.}$$

Problem # 12.5:

Two small spheres, each having a mass of 0.1gm are suspended from the same point by silk threads each of 20cm long. The spheres are given equal charges and they are found to repel each other, coming at rest 24cm apart. Find the charge on each sphere.



Data:

$m_1 = m_2 = m = 0.1\text{g}$

$m = 1 \times 10^{-4} \text{ Kg}$

$L = 20\text{cm} = 0.2\text{m}$

$r = 24\text{cm} = 0.24\text{m}$

Magnitude of charge on each sphere = ?

Solution:

Let us suppose that the magnitude of charge on each sphere be 'q' placed at a distance 'r' from each other. From coulomb's law,

$$F = \frac{K q^2}{r^2}$$

$$q^2 = \frac{F r^2}{K}$$

$$q = \sqrt{\frac{F r^2}{K}} \quad \text{---------------- (1)}$$

Determination of force:

Applying the first condition of equilibrium,

$$\Sigma F_x = 0$$
$$F - T\cos\theta = 0$$
$$F = T\cos\theta \quad \text{----------------- (2)}$$
$$\Sigma F_y = 0$$
$$T\sin\theta - W = 0$$
$$W = T\sin\theta \quad \text{----------------- (3)}$$

Dividing eq.(3) by eq.(2),

$$\frac{W}{F} = \frac{T\sin\theta}{T\cos\theta}$$
$$\frac{W}{F} = \tan\theta$$
$$F = \frac{W}{\tan\theta}$$
$$OR \quad F = \frac{mg}{\tan\theta} \quad \text{----------------- (4)}$$

Determination of θ:

In right angle triangle ΔABC

$$\cos\theta = \frac{BC}{AC}$$
$$\cos\theta = \frac{12}{20}$$
$$\cos\theta = 0.6$$
$$\theta = \cos^{-1}(0.6)$$
$$\theta = 53.1^\circ$$

eq.(4) =>
$$F = \frac{1 \times 10^{-4} \times 9.8}{\tan 53.1^\circ}$$
$$F = 7.36 \times 10^{-4}\ N$$

eq.(1) =>
$$q = \sqrt{\frac{7.36 \times 10^{-4} \times (0.24)^2}{9 \times 10^9}}$$
$$\boxed{q = 6.86 \times 10^{-8} C}$$

Problem # 12.6:

Two charges of $+2\times10^{-7}$C and -5×10^{-7}C are placed at a distance of 50cm from each other. Find a point on the line joining the charges at which the electric field is zero.

Solution:

Let us suppose 'P' be the point situated at a distance x from q_1 where net electric field is zero.

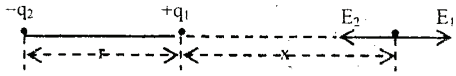
$$E_1 = \frac{K q_1}{x^2}$$
$$E_2 = \frac{K q_2}{(x + r)^2}$$

As Point P lies at the either side of charges, therefore, net electric field at point P will be the sum of fields of the two charges.

$$E_1 + (-E_2) = E$$

$$E_1 - E_2 = E$$

$$\frac{Kq_1}{x^2} - \frac{Kq_2}{(x+r)^2} = 0$$



$$\frac{Kq_1}{x^2} = \frac{Kq_2}{(x+r)^2}$$

$$\frac{(x+r)^2}{x^2} = \frac{5}{2}$$

$$\left(\frac{x+r}{x}\right)^2 = \frac{5}{2}$$

$$\sqrt{\left(\frac{x+r}{x}\right)^2} = \sqrt{2.5}$$

$$\frac{x+r}{x} = 1.58$$

$$x + r = 1.58x$$

$$1.58x - x = r$$

$$0.58x = 0.5$$

$$x = \frac{0.5}{0.58}$$

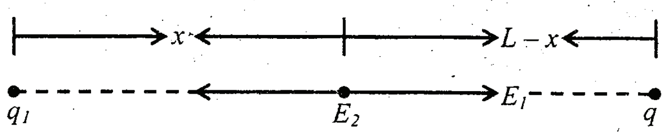
$$\boxed{x = 0.86\ \text{m}}$$

Or $\boxed{x = 86\ \text{cm}}$

As distance cannot be a negative number therefore, the point where net electric field is zero lies at 86cm from positive charge and 136cm from negative charge.

Self Test (3):

The accompanying figure shown a charge $q_1 = 1.0\mu C$, 100m from a charge $q_2 = 2.0\mu C$. At what point on line passing through the two charge is the electric field zero?



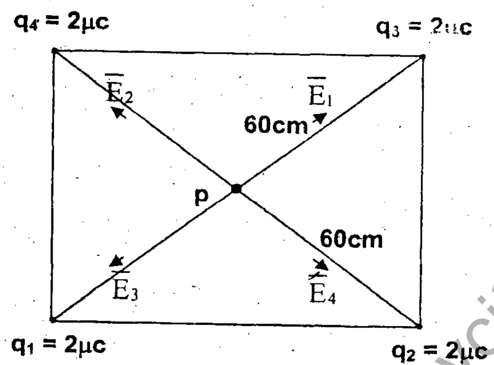
Problem# 12.7:

What are the electric field and potential at the center of a square whose diognals are 60cm each when.

(a) Charges each of 2μc are placed at the four corners.

(b) Charges of +2μc are placed on the adjacent corners and -4μc on other adjacent corners.

(a)



Required:

At the center of square 'P'

(i) $E = ?$

(ii) $V = ?$

Solution:

(i) As point 'P' lies between q_1 and q_3 and also between q and q therefore net intensity at 'P' will the difference of their intensities.

$$E = E_1 + E_2 - E_3 - E_4$$

As $$E = \frac{Kq}{r^2}$$

$$E = \frac{Kq_1}{r^2} + \frac{Kq_2}{r^2} - \frac{Kq_3}{r^2} - \frac{Kq_4}{r^2}$$

$$E = \frac{K}{r^2}(q_1 + q_2 - q_3 - q_4)$$

$$E = \frac{K}{r^2}(\not{2} + \not{2} - \not{2} - \not{2})$$

$$\boxed{E = 0}$$ **Ans.**

$$V = V_1 + V_2 + V_3 + V_4$$

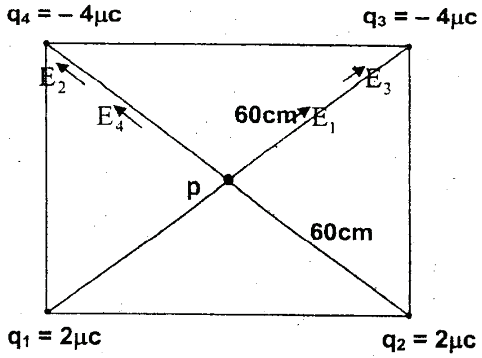
As $$V = \frac{Kq}{r}$$

$$V = \frac{K}{r}(q_1 + q_2 + q_3 + q_4)$$

$$V = \frac{9 \times 10^9}{0.3}(2 \times 10^{-6} + 2 \times 10^{-6} + 2 \times 10^{-6} + 2 \times 10^{-6})$$

$$\boxed{V = 2.4 \times 10^5 \text{ Volt}}$$ **Ans.**

(b)



Required:

At the center of square 'P'

(i) $E = ?$

(ii) $V = ?$

Solution:

(i) As point 'P' lies between q_1 and q_3, therefore net intensity at 'P' will be the difference of their intensities.

$$E^{I} = E_1 - (-E_3)$$

As $$E^{I} = \frac{Kq}{r^2}$$

$$E^{I} = \frac{Kq_1}{r^2} + \frac{kq_3}{r^2}$$

$$E^{I} = \frac{9 \times 10^9}{(0.3)^2}(2 \times 10^{-6} + 4 \times 10^{-6})$$

$$E^{I} = 1 \times 10^{11}(6 \times 10^{-6})$$

$$E^{I} = 6 \times 10^5 \text{ Volt/m}$$

Similarly net intensity of q_2 and q_4 at the center of square will also be

$$E^{II} = 6 \times 10^5 \text{ Volt/m}$$

As E^{I} and E^{II} are along the diagonals of square, which are Perpendicular on each other, therefore their net intensity will be obtained by the pythaghorus theorem.

$$E = E^{I^2} + E^{II^2}$$

$$E = \sqrt{E^{I^2} + E^{II^2}}$$

$$E = \sqrt{(6 \times 10^5)^2 + (6 \times 10^5)^2}$$

$$\boxed{E = 8.5 \times 10^5 \text{ Volt/m}}$$ **Ans.**

(ii)

$$V = V_1 + V_2 - V_3 - V_4$$

$$V = \frac{Kq_1}{r} + \frac{Kq_2}{r} - \frac{Kq_3}{r} - \frac{Kq_4}{r}$$

$$V = \frac{K}{r}(q_1 + q_2 - q_3 - q_4)$$

$$V = \frac{9 \times 10^9}{0.3}(2 \times 10^{-6} + 2 \times 10^{-6} - 4 \times 10^{-6} - 4 \times 10^{-6})$$

$$V = 3 \times 10^{10}(-4 \times 10^{-6})$$

$$\boxed{V = -1.2 \times 10^5 \text{ Volt}}$$ **Ans.**

Problem # 12.8:

A particle carrying a charge of 10^{-5} C starts from rest in a uniform electric field of intensity $50 vm^{-1}$. Find the force on the particle and the kinetic energy it acquires when it has moved 1m. **(2003 P.M)**

Data:

$q_o = 10^{-5}C$
$d = 1$ m
$E = 50$ volt/m
$F = ?$
$K.E = ?$

Solution:

$$F = q_o E$$

$$F = 10^{-5} \times 50$$

$$\boxed{F = 5 \times 10^{-4} N}$$ **Ans.**

$K.E =$ Work done
$K.E = Fd$
$K.E = 5 \times 10^{-4} \times 1$

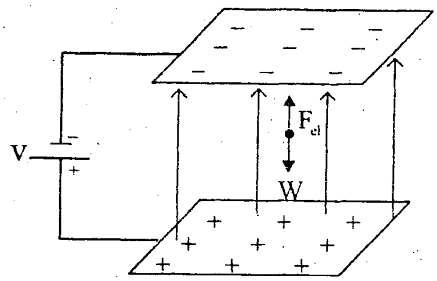
$$\boxed{K.E = 5 \times 10^{-4} J}$$ **Ans.**

Problem # 12.9:

A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19}C is to be held motionless between two horizontal parallel plates 10cm apart. Find the Voltage required to be applied between the plates. **(2002 P.E)**

Data:

$m = 1.67 \times 10^{-27}$Kg
$+e = 1.6 \times 10^{-19}$C
$d = 10$cm $= 0.1$m
$V = ?$



Solution:

Proton will held motionless between the horizontal plates when

$F_{el} =$ Weight of Proton
$q_0E = mg$
For Proton $q_0 = e$
$\therefore eE = mg$
But $E = \frac{V}{d}$

$$\frac{eV}{d} = mg$$

$$V = \frac{mgd}{e}$$

$$V = \frac{1.67 \times 10^{-27} \times 9.8 \times 0.1}{1.6 \times 10^{-19}}$$

$$\boxed{V = 1.02 \times 10^{-8} \text{ Volt}}$$ **Ans.**

Self Test (4):

(i) A Proton of mass 1.67×10^{-27} kg and charge 1.0×10^{-19} C is to be held motionless between two horizontal parallel plater 60m a port, find the voltage required to be applied between the plates. **(2009)**

Ans. **[$V = 6.137 \times 10^{-9}$ volt]**

(ii) Calculate the potential difference between two plates when they are separated by a distance of 0.005m and able to hold on electron motionless between them. ($m_e = 9.1 \times 10^{-31}$ kg , $e = 1.6 \times 10^{-19}$ c) **(1991)**

Ans. **[2.8×10^{-13} volt]**

(iii) A proton of mass 1.67×10^{-27}kg and change 1.6×10^{-19} coul is to be held motionless between two parallel horizontal plates. Find the distance between the plates when the potential difference of 6×10^{-9} volt is applied across the plates. **(2011)**

Ans. **[d = 5.866cm]**

Problem # 12.10:

A small sphere of weight 5×10^{-3}N is suspended by a silk thread 50mm long which is attached to a point on a large charged insulating plane. When a charge of 6×10^{-8}C is placed on the ball the thread makes an angle of 30° with the vertical. What is the charge density on the plane? **(2002 P.M)**

Data:

$W = 5 \times 10^{-3}$ N

$q_o = 6 \times 10^{-8}$ C

$\theta = 90° - 30° = 60°$

$\epsilon_o = 8.85 \times 10^{-12}$ C²/N-m²

$\sigma = ?$

Solution:

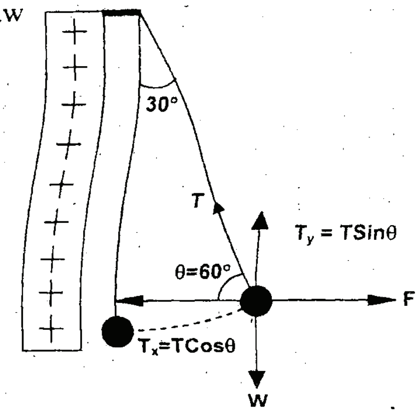
According to the 2nd condition of gauss's law

$$E = \frac{\sigma}{2\epsilon_o} \quad \text{---------------(1)}$$

$$But\ E = \frac{F}{q_o}$$

$$\frac{F}{q_o} = \frac{\sigma}{2\epsilon_o}$$

$$OR\ \sigma = \frac{2F\epsilon_o}{q_o} \quad \text{---------------(2)}$$



Determination of Force:

According to the 1st condition of equilibrium

$$\sum F_x = 0$$
$$F - T\cos\theta = 0$$
$$F = T\cos\theta \quad \text{----------------(3)}$$
$$\sum F_y = 0$$
$$T\sin\theta - W = 0$$
$$W = T\sin\theta \quad \text{----------------(4)}$$

By Dividing eq.(4) by (3) we get

$$\frac{W}{F} = \frac{T\sin\theta}{T\cos\theta}$$
$$\frac{W}{F} = \tan\theta$$
$$F = \frac{W}{\tan\theta}$$
$$F = \frac{5\times10^{-3}}{\tan 60^\circ}$$
$$F = 0.002889\text{ N}$$

eq.(2) ⇒ $$\sigma = \frac{2 \times 0.002889 \times 8.85 \times 10^{-12}}{6 \times 10^{-8}}$$

$$\boxed{\sigma = 8.5 \times 10^{-7}\ c/m^2}$$ **Ans.**

Problem # 12.11:

How many electrons should be placed on each of the two similar spheres each of 10g so that electrostatic repulsion be balanced by gravitational force? **(2010, 2006)**

Data: Charge on electron, $-e = 1.6 \times 10^{-19}$ C

Mass of each sphere, $m = 10$g

$m = 0.01$Kg

$F_{el} = F_G$

Number of electrons placed on each sphere = ?

Solution:

Let us suppose 'n' be the number of electrons placed on each sphere and 'q' be the total charge on each sphere placed at a distance 'r' from each other. Given that.

$$F_{el} = F_G$$
$$\frac{Kq^2}{r^2} = \frac{Gm^2}{r^2}$$
$$q = \sqrt{\frac{Gm^2}{K}}$$
$$q = \sqrt{\frac{6.67\times10^{-11}\times(0.01)^2}{9\times10^9}}$$
$$q = 8.6\times10^{-13}\text{ C}$$
$$q = ne$$
$$n = \frac{q}{e}$$
$$n = \frac{8.6\times10^{-13}}{1.6\times10^{-19}}$$
$$\boxed{n = 5.38\times10^{6}\text{ electrons}}$$ **Ans.**

Problem # 12.12:

There is a potential difference of 150 Volts between two conductors of a power line. A charge of 600C is carried from one conductor to the other. What work is required? If the time necessary to transport the charge is 1.25sec, how much power is used?

Data:

$V = 150$ Volts
$q_0 = 600C$
$W = ?$
$t = 1.25$ sec
$P = ?$

Solution:

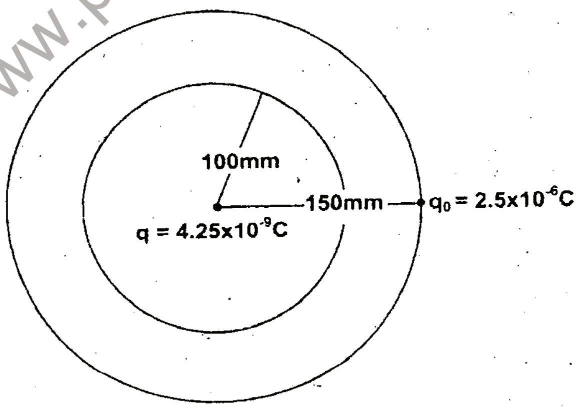
$W = q_0 V$
$W = 600 \times 150$
$\boxed{W = 90,000J}$ **Ans.**
$P = \dfrac{W}{t}$
$P = \dfrac{90,000}{1.25}$
$\boxed{P = 72,000 \text{ watt}}$ **Ans.**

Problem # 12.13:

A metal Sphere of 100mm 's has a charge of $4.25 \times 10^{-9}C$. What is the Potential when,
(i) at its surface
(ii) at its centre
What is the Potential energy of a charge $10^{-6}C$ at a point 150mm from the centre of sphere?

Data:

$q = 4.25 \times 10^{-9}C$
$q_0 = 2.5 \times 10^{-6}C$
Radius of inner surface = 100mm
OR $r_1 = 0.1m$
Radius of outer surface = 150mm
OR $r_2 = 0.15m$



(i) at the inner surface, V = ?
(ii) at the centre, V = ?
(iii) at the outer surface, U = ?

Solution:

(i) $V = \dfrac{Kq}{r}$ ------------- (1)

$V = \dfrac{9 \times 10^9 \times 4.25 \times 10^{-9}}{0.1}$

$\boxed{V = 382.5 \text{ Volt}}$ **Ans.**

(ii) As distance is the same from centre of the sphere to the inner surface as that from inner surface to the centre of sphere, therefore

$V = 382.5$ Volt

$U = \dfrac{K q q_0}{r}$

$U = \dfrac{9 \times 10^9 \times 4.25 \times 10^{-9} \times 2.5 \times 10^{-6}}{0.15}$

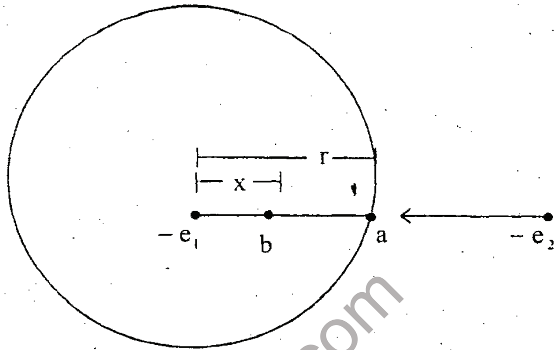
$\boxed{U = 6.37 \times 10^{-4} J}$ **Ans.**

Problem # 12.14:

An electron having an initial speed of 10^3m/s is directed from a distance of 1mm at another electron whose position is fixed. How close to the stationary electron will other approach.

Data:

$$-e = 1.6 \times 10^{-19}C$$
$$r = 1mm = 1 \times 10^{-3}m$$
$$v = 10^3 m/s$$
$$m = 9.1 \times 10^{-31}Kg$$
$$x = ?$$



Solution:

According to the law of conservation of energy loss in K.E of electron $(-e_2)$ = gain in potential energy of electron $(-e_2)$.

$$\frac{1}{2}mv^2 = U_b - U_a \text{ --------------(1)}$$

as $U = \frac{K q_1 q_2}{r}$

for electron $q_1 = q_2 = e$

$\therefore U = \frac{K e^2}{r}$

$$\text{eq.(1)} \Rightarrow \frac{1}{2}mv^2 = \frac{Ke^2}{x} - \frac{Ke^2}{r}$$

$$\frac{1}{2}mv^2 = Ke^2\left(\frac{1}{x} - \frac{1}{r}\right)$$

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times (1 \times 10^3)^2 = 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \left(\frac{1}{x} - \frac{1}{1 \times 10^{-3}}\right)$$

$$4.55 \times 10^{-25} = 2.304 \times 10^{-28}\left(\frac{1}{x} - 10^3\right)$$

$$\frac{4.55 \times 10^{-25}}{2.304 \times 10^{-28}} = \frac{1}{x} - 1000$$

$$1974.83 = \frac{1}{x} - 1000$$

$$2974.83 = \frac{1}{x}$$

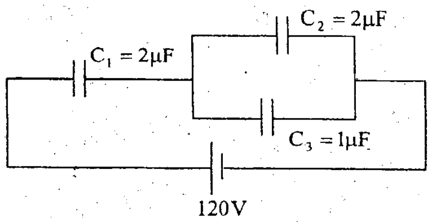
$$x = \frac{1}{2974.3}$$

$$\boxed{x = 0.00034 \text{ m}} \text{ Ans.}$$

Problem # 12.15:

Find the equivalent capacitance and charge on each of the capacitor shown in the diagram.

Required:



(i) $C = ?$ (ii) $q_1 = ?$

$q_2 = ?$

$q_3 = ?$

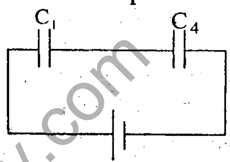
Solution:

(i) Let C_4 be the equivalent capacitance of C_2 and C_3 connected in parallel.

$C_4 = C_2 + C_3$

$C_4 = 2 + 1$

$C_4 = 3\mu F$



Let C be the equivalent capacitance of C_1 and C_4 connected in series.

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_4}$

$\frac{1}{C} = \frac{1}{2} + \frac{1}{3}$

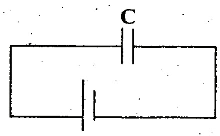
$\frac{1}{C} = \frac{5}{6}$

$C = 1.2\mu F$ Ans.

(ii) $q = CV$

$q = 1.2 \times 120$

$q = 144\mu C$



As C is the equivalent capacitance of C_1 and C_4 connected in series

$\therefore q_1 = q$

$q_1 = 144\mu C$ Ans.

$Q_4 = q$

$q_4 = 144\mu C$

$V_4 = \frac{q_4}{C_4}$

$V_4 = \frac{144}{3}$

$V_4 = 48V$

As C_4 is the equivalent capacitance of C_2 and C_3 connected in parallel.

$\therefore V_2 = V_4$

$V_2 = 48V$

$q_2 = C_2V_2$

$q_2 = 2 \times 48$

$q_2 = 96\mu C$ Ans.

and $V_3 = V_4$

$V_3 = 48V$

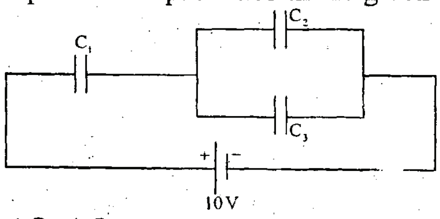
$q_3 = C_3V_3$

$q_3 = 1 \times 48$

$q_3 = 48\mu C$ Ans.

Self Test (5):

(i) Find the equivalent capacitance in the given circuit and charge on each capacitor. **(2002 P.M)**



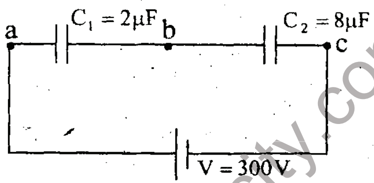
$C_1 = 6\mu F$
$C_2 = 2\mu F$
$C_3 = 4\mu F$

Problem # 12.16:

Two capacitors of $2\mu C$ and $8\mu C$ are joined in series and a potential difference of 300Volts is applied. Find the charge and potential difference for each capacitor.

Required:

(i) $q_1 = ?$
$q_2 = ?$
(ii) $V_1 = ?$
$V_2 = ?$



Solution:

(i) Let C be the equivalent capacitance of C_1 and C_2, connected in series.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{8}$$

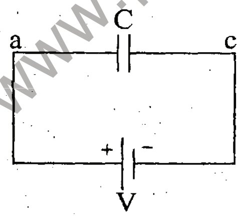
$$\frac{1}{C} = \frac{5}{8}$$

$$C = 1.6\mu F$$

$$q = CV$$

$$q = 1.6 \times 300$$

$$q = 480\mu C$$



As C is the equivalent capacitance of C_1 and C_2, connected in series.

$$\therefore \quad q_1 = q$$

$$\boxed{q_1 = 480\mu C}$$ **Ans.**

$$q_2 = q_1$$

$$\boxed{q_2 = 480\mu C}$$ **Ans.**

(ii) $$V_1 = \frac{q_1}{C_1}$$

$$V_1 = \frac{480}{2}$$

$$\boxed{V_1 = 240V}$$ **Ans.**

$$V_2 = \frac{q_2}{C_2}$$

$$V_2 = \frac{480}{8}$$

$$\boxed{V_2 = 60V}$$ **Ans.**

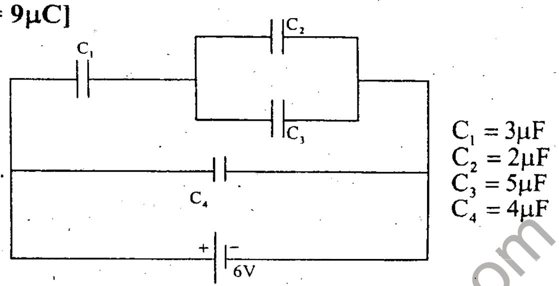
Self Test (6):

(i) Two capacitors of 2μF and 8μF capacitance are connected in series and a potential difference of 200 volts is applied. Find the charge and the potential difference for each capacitor. **(1999)**

Ans. **[V_1 = 40 volt]**

(ii) Calculate the equivalent capacitance and charge on 5μC capacitor as shown in the figure: **(1993)**

Ans. **[C_e = 6.1μF , q_3 = 9μC]**



(iii) Two capacitors of capacitance 3μμF and 6μμF are connected in series and the resulting combination is connected across 1000volts. Compute,

(i) The equivalent capacitance of the combination.

(ii) The total charge on the combination and the charge taken by each capacitor.

(iii) The potential difference across each capacitor. **(1986)**

Ans. **[V_2 = 333.3 volt]**

Problem # 12.17:

A capacitor of 100 PF is charged to a potential difference of 50volts. Its plates are then connected in parallel to another capacitor and it is found that the potential difference between its plates falls to 35 volts. What is the capacitance of the second capacitor?

Data:

C_1 = 100 PF
V_1 = 50 V

Joined in parallel

V = 35 V
C_2 = ?

Solution:

Charge of C_1 when charged separately.

$q_1 = C_1V_1$
$q_1 = 100 \times 50$
q_1 = 5000 PC

Charge of C_1 when joined in parallel

$q_1' = C_1V$
$q_1' = 100 \times 35$
q_1' = 3500 PC

Charge of C_2 in parallel combination.

$$q_2' = q_1 - q_1'$$

$$q_2' = 5000 - 3500$$

$$q_2' = 1500 \text{ PC}$$

Capacitance of C_2.

$$C_2 = \frac{q_2'}{V}$$

$$C_2 = \frac{1500}{35}$$

$$\boxed{C_2 = 42.86 \text{ PF}}$$ **Ans.**

Self Test (7):

(i) A capacitor of 200 PF is charged to a potential of 100 volts. Its plates are then connected in parallel to another capacitor and is found that potential difference between the plates falls to 60 volts. What is the capacitance of the second capacitor? **(1997)**

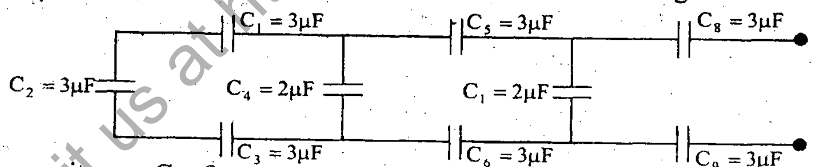
Ans. **[C_2 = 133.33 PF]**

(ii) A capacitor of 12 μF is charged to a potential difference 100 volt. Its plates are then disconnected from the source and are connected parallel to another capacitor. The potential difference in this combination comes down 60 volt. What is the capacitance of the second capacitor? **(2008)**

Ans. **[C_2 = 8μF]**

Problem # 12.18:

Find the equivalent capacitance of the combination shown in the diagram.



Required:

Equivalent capacitance, C = ?

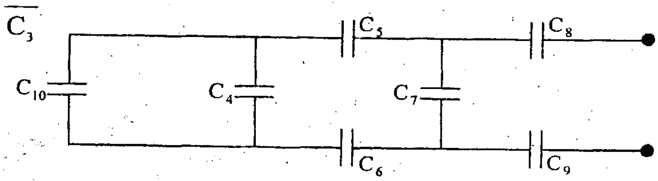
Solution:

Let C_{10} be the equivalent capacitance of C_1, C_2 and C_3 connected in series.

$$\frac{1}{C_{10}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{10}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$C_{10} = 1\mu F$$

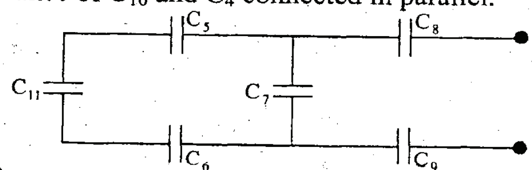


Let C_{11} be the equivalent capacitance of C_{10} and C_4 connected in parallel.

$$C_{11} = C_{10} + C_4$$

$$C_{11} = 1 + 2$$

$$C_{11} = 3\mu F$$

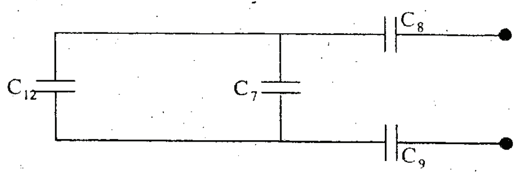


Let C_{12} be the equivalent capacitance of C_5, C_{11} and C_6 connected in series.

$$\frac{1}{C_{12}} = \frac{1}{C_5} + \frac{1}{C_{11}} + \frac{1}{C_6}$$

$$\frac{1}{C_{12}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$C_{12} = 1\mu F$$

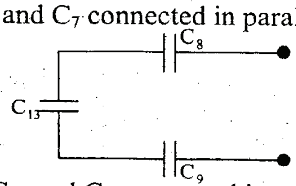


Let C_{13} be the equivalent capacitance of C_{12} and C_7 connected in parallel.

$$C_{13} = C_{12} + C_7$$

$$C_{13} = 1 + 2$$

$$C_{13} = 3\mu F$$



Let C be the equivalent capacitance of C_8 , C_{13} and C_9 connected in series.

$$\frac{1}{C_{12}} = \frac{1}{C_8} + \frac{1}{C_{13}} + \frac{1}{C_9}$$

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\boxed{C = 1\mu F} \quad \textbf{Ans.}$$

Problem # 12.19:

A parallel plate capacitor has plates 30cmx30cm separated by a distance of 2cm. By how much the capacitance changes if a dielectric slab of the same area but of thickness 1.5 is slipped between the plates. The dielectric constant of the material is 2.

Data:

$A = 30cm \times 30cm$

$A = 900cm^2 = 0.09m^2$

$d = 2cm = 0.02m$

$t = 1.5cm = 0.015m$

$\in_r = 2$

$\Delta C = ?$

Solution:

Capacitance of capacitor when air is the medium.

$$C = \frac{\in_0 A}{d}$$

$$C = \frac{8.85 \times 10^{-12} \times 0.9}{0.02}$$

$$C = 3.9825 \times 10^{-11} F$$

Capacitance of capacitor when dielectric slab of thickness t<d is slipped between its plates.

$$C' = \frac{\in_0 A}{(d - t) + t/\in_r}$$

$$C' = \frac{8.85 \times 10^{-12} \times 0.09}{(0.02 - 0.015) + \frac{0.015}{2}}$$

$$C' = \frac{7.965 \times 10^{-13}}{0.005 + 0.0075}$$

$$C' = \frac{7.965 \times 10^{-13}}{0.0125}$$

$$C' = 6.372 \times 10^{-11}$$

$$\Delta C = C' - C$$

$$\Delta C = 6.372 \times 10^{-11} - 3.9825 \times 10^{-11}$$

$$\boxed{\Delta C = 2.3895 \times 10^{-11}\ F}$$ **Ans.**

Self Test (8):

A parallel plate capacitor has the plates 10cmx10cm separated by a distance 2.5cm. It is initially filled with air. What will be the increase in its capacitance if a dielectric slab of the same area and thickness 2.5cm is placed between the two plates? **(2001)**

Ans. $\boxed{\Delta C = 3.54 \times 10^{-12} F}$

Problem # 12.20:

Three 1 PF capacitors are charged separately to the potential difference of 100, 200 and 300 volts. The capacitors are then joined in parallel. What is the resultant potential difference?

Data:

$C_1 = 1PF$ | $V_1 = 100V$

$C_2 = 1PF$ | $V_2 = 200V$

$C_3 = 1PF$ | $V_3 = 300V$

$V = ?$

Solution:

Charge of each capacitor when charged separately.

$$q_1 = C_1V_1 = 1 \times 100V = 100\ PC$$

$$q_2 = C_2V_2 = 1 \times 200V = 200\ PC$$

$$q_3 = C_3V_3 = 1 \times 300V = 300\ PC$$

Total charge in parallel combination.

$$q = q_1 + q_2 + q_3$$

$$q = 100 + 200 + 300$$

$$q = 600\ PC$$

Equivalent capacitance of parallel combination.

$$C = C_1 + C_2 + C_3$$

$$C = 1 + 1 + 1$$

$$C = 3\ PF$$

Voltage of parallel combination.

$$V = \frac{q}{C}$$

$$V = \frac{600}{3}$$

$$\boxed{V = 200V}$$ **Ans.**

Problem # 12.21:

Compare the capacitances of two identical capacitors with dielectrics inserted as shown in the diagram. The dielectric constants are K_1 and K_2.

Solution:

Let us suppose a parallel plate capacitor having plates each of area "A" separated by a distance "d" from each other. Further let us also suppose two dielectric slabs of same size but of different material. The dielectric constant of slabs are K_1 and K_2.

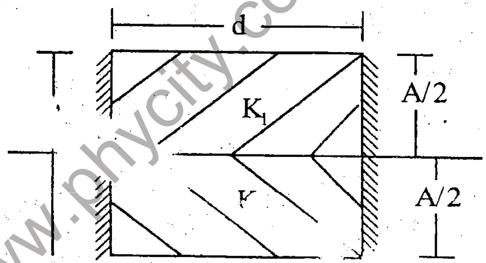
First we suppose that the dielectric slabs are placed horizontally between the plates, such that a compound capacitor of two parallel capacitors is formed, as shown the figure.

$$C_1 = \frac{\epsilon_0 K_1}{d}$$

$$C_1 = \frac{\epsilon_0 K_1 A}{2d}$$

$$C_2 = \frac{\epsilon_0 K_2 A/2}{d}$$

$$C_2 = \frac{\epsilon_0 K_2 A}{2d}$$



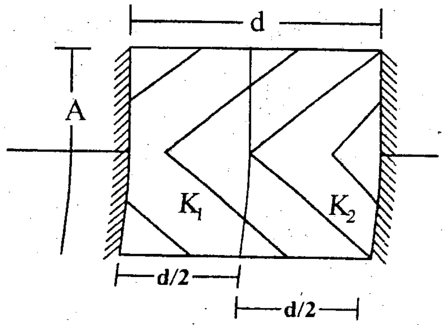
Let C' be the equivalent capacitance of a compound capacitor of two parallel capacitors.

$$C' = C_1 + C_2$$

$$C' = \frac{\epsilon_0 K_1 A}{2d} + \frac{\epsilon_0 K_2 A}{2d}$$

$$C' = \frac{\epsilon_0 A}{2d}(K_1 + K_2) \quad \text{---------------- (1)}$$

Now we suppose that the dielectric slabs are placed vertically between the plates, such that a compound capacitor of two series capacitors is formed, as shown in the figure.



$$C_1 = \frac{\epsilon_0 K_1 A}{d/2}$$

$$C_1 = \frac{2\epsilon_0 K_1 A}{d}$$

$$C_2 = \frac{\epsilon_0 K_2 A}{d/2}$$

$$C_2 = \frac{2\epsilon_0 K_2 A}{d}$$

Let "C" be the equivalent capacitance of a compound capacitor of two series capacitors.

$$\frac{1}{C''} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C''} = \frac{C_2 + C_1}{C_1 C_2}$$

$$C'' = \frac{C_1 C_2}{C_2 + C_1}$$

$$C'' = \frac{\frac{2\epsilon_0 K_1 A}{d} \times \frac{2\epsilon_0 K_2 A}{d}}{\frac{2\epsilon_0 K_1 A}{d} + \frac{2\epsilon_0 K_2 A}{d}}$$

$$C'' = \frac{\left(\frac{2\epsilon_0 A}{d}\right)^2 K_1 K_2}{\frac{2\epsilon_0 A}{d}(K_1 K_2)}$$

$$C'' = \frac{2\epsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2} \quad \text{------------- (2)}$$

Dividing eq. (2) by (1)

$$\frac{C''}{C'} = \frac{\frac{2\epsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}}{\frac{\epsilon_0 A}{2d} K_1 + K_2}$$

$$\boxed{\frac{C''}{C'} = 4\frac{K_1 K_2}{(K_1 + K_2)^2}}$$ **Ans.**

Problem # 12.22:

A capacitor 10μF and another of 20μF are connected across batteries of 600V and 1000V respectively, and then disconnected. They are then joined in parallel. What is the charge on each capacitor?

Data:

$C_1 = 10\mu F$

$C_2 = 20\mu F$

$V_1 = 600V$

$V_2 = 1000V$

Joined in parallel.

$q_1' = ?$

$q_2' = ?$

Solution:

Charge of each capacitor when charged separately.

$$q_1 = C_1V_1 = 10 \times 600 = 6000\mu C$$

$$q_2 = C_2V_2 = 20 \times 1000 = 20000\mu C$$

Total charge in parallel combination.

$$q = q_1 + q_2$$

$$q = 6000 + 20000$$

$$q = 26000\mu C$$

Equivalent capacitance of parallel combination.

$$C = C_1 + C_2$$

$$C = 10 + 20$$

$$C = 30\mu F$$

Voltage of parallel combination.

$$V = \frac{q}{C}$$

$$V = \frac{26000}{30}$$

$$V = 866.67V$$

Charge on each capacitor in parallel combination.

$$q_1' = C_1V = 10\times 866.67$$

$$\boxed{q_1' = 8666.7\ \mu C}$$ **Ans.**

$$q_2' = C_2V = 20\times 866.67$$

$$\boxed{q_2' = 17333.3\mu C}$$ **Ans.**

Self Test (9):

Two capacitors of capacitance 4μF and 6μF are charged to the potential difference of 300 volts and 400 volts respectively. They are then connected in parallel. What will be the resultant potential difference and charge on each capacitor? **(1995)**

Ans. $\boxed{V = 360\text{ volts}}$

Problem # 12.23:

Attempt the problem # 12.22 with the difference that the capacitors are joined in series after being charged as before.

Data:

$C_1 = 10\mu F$

$C_2 = 20\mu F$

$V_1 = 600V$

$V_2 = 1000V$

Joined in parallel.

$q_1' = ?$

$q_2' = ?$

Solution:

Equivalent capacitance of series combination.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{10} + \frac{1}{20}$$

$$\frac{1}{C} = \frac{3}{20}$$

$$C = 6.67\mu F$$

Voltage of series combination.

$$V = V_1 + V_2$$

$$V = 600 + 1000$$

$$V = 1600V$$

Net charge of series combination.

$$q = CV$$

$$q = 6.67 \times 1600$$

$$q = 10672\mu C$$

As C is the equivalent capacitance of C_1 and C_2 connected in series.

$$\therefore \quad q_1' = q$$

$$\boxed{q_1' = 10672\mu C} \quad \textbf{Ans.}$$

And $\boxed{q_2' = 10672\mu C}$ **Ans.**

12.18 EXTRA NUMERICALS:

Q.1 How many excess electrons must be placed on each of the two similar small spheres placed 3cm apart if the force of repulsion between the spheres is 10^{-19} N ($K = 9 \times 10^9\ Nm^2/c^2$) (2007)

DATA:

Distance between spheres = $r = 3cm = 3 \times 10^{-2}$ m

Electrostatic force = $F = 10^{-19}$ N

Number of electrons = n = ?

SOLUTION:

According to coulomb's law

$$F = K \frac{q_1 q_2}{r^2}$$

But $q_1 = q_2 = q$ (say)

$$F = K \frac{q^2}{r^2}$$

$$q^2 = \frac{Fr^2}{K}$$

$$q^2 = \frac{10^{-19}(3 \times 10^{-2})^2}{9 \times 10^9} \quad \left[\because K = \frac{1}{4\pi \epsilon_o} = 9 \times 10^9\ N.m^2/C^2\right]$$

$$q^2 = \frac{10^{-19} \times 9 \times 10^{-4} \times 10^{-9}}{9}$$

$$q^2 = 1 \times 10^{-32}\ C^2$$

∴ Taking square root both sides ∴ we get

$$q = 10^{-16}\ C \quad \text{——— (1)}$$

Now using:

$$q = ne$$

$$n = \frac{q}{e}$$

$$n = \frac{10^{-16}}{1.6 \times 10^{-19}} \quad [\because e = 1.6 \times 10^{-19}\ C]$$

n = 625 electrons

Q.2 An electron has a speed 10^6m/sec. Find its energy in electron volt ($M_e = 9.1 \times 10^{-31}$ kg) **(2004)**

DATA:

Speed of electron = v = 10^6 m/sec

Mass of electron = $M_e = 9.1 \times 10^{-31}$ kg

Energy of electron = ?

SOLUTION:

The total energy of moving Electron is the Kenetic Energy

$$E = K.E$$

OR $$E = \frac{1}{2} mv^2$$

$$E = \frac{1}{2}(9.1\times10^{-31})(10^6)^2$$

$$E = 0.5 \times 9.1 \times 10^{-31} \times 10^{12}$$

$$E = 4.55 \times 10^{-19} J$$

But

$$1\ ev = 1.6 \times 10^{-19} J$$

$$E = \frac{4.55 \times 10^{-19}}{1.6 \times 10^{-19}} ev$$

$$\boxed{E = 2.844\ ev}$$

Q.3 A charged particle of 17.7μC is placed close to a positively charged thin sheet having surface charge density 2×10^{-6} coul/m^2. Find the magnitude of force acting on the charged particle. ($\epsilon_0 = 8.85\times10^{-12}$ coul/N-m^2). (2000)

GIVEN THAT:

$q = 17.7\mu C = 17.7 \times 10^{-6}$ C

$\sigma = 2 \times 10^{-6}$ coul/m^2

REQUIRED:

F = ?

Constant

$\epsilon_o = 8.85 \times 10^{-12}$ coul/N-m^2

SOLUTION:

Electric intensity close to a charge sheet is given by

$$E = \frac{\sigma}{2\epsilon_o}$$

$$E = F/q$$

$$\frac{F}{q} = \frac{\sigma}{2\epsilon_o}$$

$$F = \frac{q\sigma}{2\epsilon_o}$$

$$F = \frac{17.7 \times 10^{-6} \times 2\ 10^{-6}}{2 \times 8.85 \times 10^{-12}}$$

$$\boxed{F = 2N}$$

Self Test (10):

(i) Calculate the force of repulsion on $+2\times10^{-8}$C charge, if it is placed before a large vertical plane whose surface charge density is $+20\times10^{-4}$ coul/m^2. ($\epsilon_0 = 8.85\times10^{-12}$ coul^2/N-m^2) **(1998)**

Ans. $\boxed{F = 2.26\ N}$

(ii) A thin sheet of positive charges attracts a light charged sphere having a charge of -5×10^{-6}C with a force of 1.695N. If $\epsilon_0=8.85\times10^{-12}$ C^2/N-m^2. Calculate the surface charge density. **(1996)**

Ans. $\boxed{\sigma = 6\times10^{-6}\ coul - m^{-2}}$

(iii) The surface charge density on a vertical metal plate is 2.5×10^{-6}c/m^2. Find force experienced by a charge of 2×10^{-10} placed infornt close to the sheet. **(2013)**