

11.46: SOLVED NUMERICALS OF BOOK:

PROBLEM # 11.1

(i) The normal human body temperature is 98.4 °F. What is the temperature on Celsius scale?

DATA:

$$T_F = 98.4 \text{ }^\circ\text{F}$$

$$T_C = ?$$

SOLUTION:

$$T^\circ\text{C} = \frac{5}{9} (T^\circ\text{F} - 32)$$

$$T^\circ\text{C} = \frac{5}{9} (98.4 - 32)$$

$$T^\circ\text{C} = \frac{5}{9} \times 66.4$$

$$\boxed{T^\circ\text{C} = 36.88 \text{ }^\circ\text{C}}$$

(ii) At what temperature do the Fahrenheit and Celsius scale coincide.

SOLUTION:

$$\text{Let } T^\circ\text{F} = T^\circ\text{C} = X$$

$$T^\circ\text{C} = \frac{5}{9} (T^\circ\text{F} - 32)$$

$$X = \frac{5}{9} (X - 32)$$

$$9X = 5X - 160$$

$$4X = -160$$

$$\boxed{X = -40}$$

PROBLEM # 11.2

A steel rod has a length of exactly 0.2cm at 30 °C. What will be its length at 60 °C?

DATA:

$$L_1 = 0.2 \text{ cm}$$

$$T_1 = 30 \text{ }^\circ\text{C}$$

$$T_2 = 60 \text{ }^\circ\text{C}$$

$$\alpha = 11 \times 10^{-6} \text{ K}^{-1}$$

$$L_2 = ?$$

SOLUTION:

$$\Delta T = 60 - 30 = 30 \text{ }^\circ\text{C} = 30 \text{ K}$$

$$L_2 = L_1 \{1 + \alpha \Delta T\}$$

$$L_2 = 0.2 \{1 + 11 \times 10^{-6} \times 30\}$$

$$L_2 = 0.2 \{1 + 0.00033\}$$

$$L_2 = 0.2 \times 1.00033$$

$$\boxed{L_2 = 0.200066 \text{ cm}}$$

Self Test (1)

(i) A steel bar is 10m in length at -2.5 °C. What will be the change in its length when it is at 25°C? (β for steel = $3.3 \times 10^{-8} \text{ K}^{-1}$) (2008)

PROBLEM # 11.3

Find the change in volume of an aluminum sphere of 0.4m radius when it is heated from 0°C to 100°C.

DATA:

$$\Delta V = ?$$

$$r = 0.4 \text{ m}$$

$$T_1 = 0 \text{ }^\circ\text{C}$$

$$T_2 = 100 \text{ }^\circ\text{C}$$

$$\alpha = 24 \times 10^{-6} \text{ K}^{-1}$$

SOLUTION:

$$V_1 = \frac{4}{3} \pi r^3$$

$$V_1 = \frac{4}{3} (3.14) (0.4)^3$$

$$V_1 = 0.268 \text{ m}^3$$

$$\beta = 3 \alpha$$

$$\beta = 3 \times 24 \times 10^{-6}$$

$$\beta = 7.2 \times 10^{-5} \text{ K}^{-1}$$

$$\Delta T = 100 - 0 = 100 \text{ }^\circ\text{C} = 100 \text{ K}$$

$$\text{Using } \Delta V = \beta V_1 \Delta T$$

$$\Delta V = 7.2 \times 10^{-5} \times 0.268 \times 100$$

$$\boxed{\Delta V = 0.0019 \text{ m}^3}$$

PROBLEM # 11.4

Calculate the root mean square speed of hydrogen molecule at 800 K.

DATA: $T = 800 \text{ K}$
 V_{rms} of $\text{H}_2 = ?$
 $K = 1.38 \times 10^{-23} \text{ JK}^{-1}$

SOLUTION:

$$n = \frac{m}{M}$$

$$\frac{N}{N_A} = \frac{m}{M}$$

$$\frac{1}{6.02 \times 10^{23}} = \frac{m}{2}$$

$$m = 3.32 \times 10^{-24} \text{ g}$$

or $m = 3.32 \times 10^{-27} \text{ kg}$

Using $V_{\text{rms}} = \sqrt{\frac{3KT}{m}}$

$$V_{\text{rms}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 800}{3.32 \times 10^{-27}}}$$

$$\boxed{V_{\text{rms}} = 3158.5 \text{ m/s}}$$

Self Test (2)

(i) Find the root mean square velocity of a hydrogen gas molecule at 100°C . Take the mass of the hydrogen molecule $3.32 \times 10^{-27} \text{ Kg}$ and $K = 1.38 \times 10^{-23} \text{ J/K}$ (2002 P. E)
Ans. 21567 m/s

(ii) Find the root mean square velocity of a hydrogen molecule at 7°C take the mass of hydrogen molecule to be $3.32 \times 10^{-27} \text{ Kg}$ and Boltzman constant, (1998)
 $K = 1.38 \times 10^{-23} \text{ J-K}^{-1}$.

Ans. $v_{\text{rms}} = 1868.6 \text{ m/s}$

Find r.m.s speed of the nitrogen molecules at 27°C . Given the mass of nitrogen molecule to be $4.67 \times 10^{-26} \text{ Kg}$. $K = 1.38 \times 10^{-23} \text{ J/K}$. (1996)

Ans. $v_{\text{rms}} = 515.7 \text{ m/s}$

PROBLEM # 11.5

(a) Determine the average value of the Kinetic energy of the Particles of an ideal gas at 0°C and at 50°C .

DATA: (K.E) = ?
(i) $T = 0^\circ\text{C} + 273 = 273 \text{ K}$
(ii) $T = 50^\circ\text{C} + 273 = 323 \text{ K}$

SOLUTION:

$$(K.E) = \frac{3}{2} KT$$

(i) $(K.E) = \frac{3}{2} \times 1.38 \times 10^{-23} \times 273$

$$\boxed{K.E = 5.65 \times 10^{-21} \text{ J}}$$

(ii) $(K.E) = \frac{3}{2} \times 1.38 \times 10^{-23} \times 323$

$$\boxed{K.E = 6.68 \times 10^{-21} \text{ J}}$$

(b) Determine the kinetic energy per mole of the particles of an ideal gas at 0°C and at 50°C .

DATA: K.E per mole = ?
(i) $T = 0^\circ\text{C} + 273 = 273 \text{ K}$
(ii) $T = 50^\circ\text{C} + 273 = 323 \text{ K}$

SOLUTION:

$$K.E \text{ per mole} = \frac{3}{2} RT$$

(i) $K.E \text{ per mole} = \frac{3}{2} \times 8.31 \times 273$

$$\boxed{K.E \text{ per mole} = 3402.9 \text{ joule/mole}}$$

(ii) $K.E \text{ per mole} = \frac{3}{2} \times 8.31 \times 323$

$$\boxed{K.E \text{ per mole} = 4026.2 \text{ joule/mole}}$$

PROBLEM # 11.6

A 2kg iron block is taken from a furnace where its temperature was 650°C and placed on a large block of ice at 0 °C. Assuming that all the heat given up by the iron is used to melt the, how much ice is melted?

DATA:

mass of iron block = $m_1 = 2\text{kg}$

Temperature of iron block = $T_1 = 650\text{ }^\circ\text{C}$

mass of ice melted = $m_2 = ?$

Temperature of ice = $T_2 = 0\text{ }^\circ\text{C}$

Specific heat capacity of iron = $C_1 = 499.8\text{ J/kg. K}$

Latent heat of fusion of ice = $H_f = 336000\text{ J/kg}$

SOLUTION:

$\Delta T = 650 - 0$

$\Delta T = 650\text{ }^\circ\text{C} = 650\text{ K}$

According to the law of heat exchange Heat lost by iron block = Heat gained by ice

$m_1 C_1 \Delta T = m_2 H_f$

$$m_2 = \frac{m_1 C_1 \Delta T}{H_f}$$

$$m_2 = \frac{2 \times 499.8 \times 650}{336000}$$

$m_2 = 1.93\text{ kg}$

PROBLEM # 11.7

In a certain process 400J of heat is supplied to a system and at the same time 150J of work is done by the system. What is the increase in the internal energy of the system.

DATA:

$\Delta Q = 400\text{J}$

$\Delta W = 150\text{J}$

$\Delta U = ?$

SOLUTION:

$\Delta U = \Delta Q - \Delta W$

$\Delta U = 400 - 150$

$\Delta U = 250\text{J}$

PROBLEM # 11.8

There is an increase of internal energy by 400J when 800J of work is done by the system. What is the amount of heat supplied during this process?

DATA:

$\Delta U = 400\text{J}$

$\Delta W = 800\text{J}$

$\Delta Q = ?$

SOLUTION:

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = 400 + 800$

$\Delta Q = 1200\text{J}$

PROBLEM # 11.9

A heat engine performs 200J of work in each cycle and has efficiency of 20 percent. For each cycle of operation

- (a) How much heat is absorbed? and (b) How much is expelled?

DATA:

$$\Delta W = 200\text{J}$$

$$E = 20\%$$

(a) $Q_1 = ?$

(b) $Q_2 = ?$

SOLUTION:

(a) $E = \frac{\Delta W}{Q_1} \times 100$

$$\frac{20}{100} = \frac{200}{Q_1}$$

$$\boxed{Q_1 = 1000\text{J}}$$

(b) $\Delta W = Q_1 - Q_2$

$$200 = 1000 - Q_2$$

$$\boxed{Q_2 = 800\text{J}}$$

Self Test (3)

- (i) A heat engine performing 400J of work in each cycle has an efficiency of 25%. How much heat is absorbed and rejected in each cycle. (2010)

Ans. $Q_1 = 1600\text{J}, Q_2 = 1200\text{J}$

PROBLEM # 11.10

A heat engine operates between two reservoirs at temperatures of 25°C and 300 °C. What is the maximum efficiency for this engine?

DATA:

$$T_1 = 300^\circ\text{C} + 273 = 573\text{ k}$$

$$T_2 = 25^\circ\text{C} + 273 = 298\text{ k}$$

$$E = ?$$

SOLUTION:

$$E = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$E = \left(1 - \frac{298}{573}\right) \times 100$$

$$E = (1 - 0.52) \times 100$$

$$E = 0.48 \times 100$$

$$\boxed{E = 48\%}$$

Self Test (4)

- (i) Find the efficiency of a carnot engine working between 100°C and 50°C.

Ans. $E = 13.4\%$

(1999)

- (ii) An ideal heat engine operates in carnot cycle between temperatures 227°C and 127°C and it absorbs 600Joules of heat energy, find the (i) work done per cycle (ii) Efficiency of the heat engine. (2001)

PROBLEM # 11.11

The low temperature reservoir of a Carnot engine is at 7°C and has an efficiency of 40%. If it is desired to increase the efficiency upto 50%. By how much degrees the temperature of hot reservoir be increased.

DATA:

$$T_2 = 7^{\circ}\text{C} + 273 = 280\text{k}$$

$$E_1 = 40\%$$

$$E_2 = 50\%$$

$$\text{Increase in temp. of hot body} = \Delta T = ?$$

SOLUTION:

$$E_1 = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$40 = \left(1 - \frac{280}{T_1}\right) \times 100$$

$$\frac{40}{100} = \frac{T_1 - 280}{T_1}$$

$$\frac{2}{5} = \frac{T_1 - 280}{T_1}$$

$$5(T_1 - 280) = 2T_1$$

$$5T_1 - 1400 = 2T_1$$

$$5T_1 - 2T_1 = 1400$$

$$3T_1 = 1400$$

$$T_1 = 466.6\text{ k}$$

$$E_2 = \left(1 - \frac{T_2}{T_1'}\right) \times 100$$

$$50 = \left(1 - \frac{280}{T_1'}\right) \times 100$$

$$\frac{50}{100} = \frac{T_1' - 280}{T_1'}$$

$$\frac{1}{2} = \frac{T_1' - 280}{T_1'}$$

$$2(T_1' - 280) = T_1'$$

$$2T_1' - 560 = T_1'$$

$$T_1' = 560\text{ k}$$

Now increase in temp. of hot body is

$$\Delta T = T_1' - T_1$$

$$\Delta T = 560 - 466.6$$

$$\Delta T = 93.4\text{ k}$$

Self Test (5)

- (i) The low temperature reservoir of a Carnot engine is at 5°C and has efficiency of 40%. It is desired to increase its efficiency to 50%. By what degrees should the temperature of the hot reservoir be increased? (2013)
- (ii) The low temperature reservoir of a Carnot engine is at -3°C and has efficiency of 40%. It is desired to increase the efficiency to 50%. By how many degrees should the temperature of the hot reservoir be increased? [Similar To Q.11.11 of Book Also Similar To Q.2 (D) 2003 P.M] (2004)

11.47: SOLVED NUMERICALS OF PAPERS

- Q.1 A Carnot engine whose low temperature reservoir is 200K has an efficiency of 50%. It is desired to increase this to 75%. By how many degree must the temperature of low temperature reservoir be decreased if the temperature of the higher temperature reservoir remains constant. (2012)

Data:

$T_2 = 200\text{K}$

$E = 50\%$

If $E = 75\%$

Decrease in temperature of cold body = $\Delta T_2 = ?$

Solution:

As we know that

$$E = \left(1 - \frac{T_2}{T_1}\right) \times 100 \dots\dots\dots(a)$$

When E = 50%

$$(a) \Rightarrow 50 = \left(1 - \frac{200}{T_1}\right) \times 100$$

$$\frac{50}{100} = 1 - \frac{200}{T_1}$$

$$0.5 = 1 - \frac{200}{T_1}$$

$$\frac{200}{T_1} = 0.5$$

$$T_1 = \frac{200}{0.5}$$

$$T_1 = 400\text{K} \dots\dots(1)$$

When E = 75%

$$(a) \Rightarrow 75 = \left(1 - \frac{T_2'}{400}\right) \times 100$$

$$\frac{75}{100} = 1 - \frac{T_2'}{400}$$

$$0.75 = 1 - \frac{T_2'}{400}$$

$$\frac{T_2'}{400} = 0.25$$

$$T_2' = 0.25 \times 400$$

$$T_2' = 100\text{K} \dots\dots(2)$$

Now Decrease in temp. of cold body

$$\Delta T_2' = T_2 - T_2' = 200 - 100$$

$$\Delta T_2' = 100\text{K}$$

Self Test (6)

- (i) A Carnot engine whose low temperature reservoir is at 200K has an efficiency of 50%. It is desired to increase this to 75%. By how much degrees must the temperature be decreased, if higher temperature of the reservoir remains constant.

Ans. $\Delta T_2 = 100\text{K}$

(1997)

- Q.2 A 200gm piece of metal is heated to 150°C and then dropped into an aluminium calorimeter of mass 500gm containing 500gm of water initially at 25°C. Find the final equilibrium temperature of the system if the specific heat of metal is 128.100 J/kg K, specific heat of aluminum is 903 J/kg K, while the specific heat of water 4200 J/kg K. (2011)

Data:**For Metal**

Mass of metal = $m_1 = 200\text{gm} = \frac{200}{1000}\text{kg} = 0.2\text{kg}$

Temp. of metal = $T_1 = 150^\circ\text{C}$

Special Heat of metal = $C_1 = 128.1\text{ J/kg K}$

For Calotimeter

Mass of calorimeter = $m_2 = 500 = \frac{500}{1000}\text{kg} = 0.5\text{kg}$

Temp. of calorimeter = $T_2 = 25^\circ\text{C}$

Specific Heat of calorimeter = $C_2 = 903\text{ J/kg K}$

For Water

Mass of water = $m_3 = 500\text{gm} = \frac{500}{1000}\text{kg} = 0.5\text{kg}$

Temp. of water = $T_3 = 25^\circ\text{c}$

Secific Heat of water = $C_3 = 4200\text{ J/kg K}$

Final Temperature of Mixture = $T = ?$

Solution:

A_1 given that

$m_2 = m_3 = m = 0.5\text{ kg}$ (1)

$T_2 = T_3 = t = 25^\circ\text{c}$ (2)

According to Law of Heat exchang

Heat gain by calorimeter & water = Heat loss by Metal

$\Delta Q_2 + \Delta Q_3 = \Delta Q_1$

$m_2 C_2 \Delta T_2 + m_3 C_3 \Delta T_3 = m_1 C_1 \Delta T_1$

or

$m_2 C_2 (T - T_2) + m_3 C_3 (T - T_3) = m_1 C_1 (T_1 - T)$

$m C_2 (T - t) + m C_3 (T - t) = m_1 C_1 (T - T)$

$m (T - t) (C_2 + C_3) = m_1 C_1 (T - T)$

$\frac{T - t_2}{T_1 - T} = \frac{m_1 C_1}{m (C_2 + C_3)}$

$\frac{T - 25}{150 - T} = \frac{0.2 \times 128.1}{0.5 (903 + 4200)}$

$\frac{T - 25}{150 - T} = \frac{25.62}{2551.5}$

$\frac{T - 25}{150 - T} = 0.01004115$

$T - 25 = 0.01 (150 - T)$

$T - 25 = 1.5 - 0.01T$

$T + 0.01T = 1.5 + 25$

$1.01T = 26.5$

$T = \frac{26.5}{1.01}$

T = 26.23°C

$$\left. \begin{aligned} \Delta Q &= mc \Delta T \\ \Delta T_2 &= T - T_2 \\ \Delta T_3 &= T - T_3 \\ \Delta T_1 &= T_1 - T \end{aligned} \right\}$$

Self Test (7)

- (i) A 100gm copper block is heated in boiling water for ten minutes and then it is dropped into 150gm of water at 30°C in a 200gm calorimeter. If the temperature of water is raised to 33.6°C, determine the specific heat of the material of the calorimeter. (for copper $C_1 = 386\text{ JKg}^{-1}\text{ C}^{-1}$). (2005)

- Q.3) A scientist stores 22 gm of a gas in a tank at 1200 atmospheres. Overnight the tank develops slight leakage and the pressure drops to 950 atmospheres. Calculate the mass of the gas escaped. (2009)

DATA:

$m_1 = 22\text{gm} = 22 \times 10^{-3}\text{ kg}$

$P_1 = 1200\text{ atmosph } P_2 = 950\text{ atmosph}$

Mass escaped = $\Delta m = ?$

SOLUTION:

According to Boyle's low with mass effect

$\frac{P_1 V_1}{m_1} = \frac{P_2 V_2}{m_2}$

There

$V_1 = V_2 = V$

Or

$\frac{P_1}{m_1} = \frac{P_2}{m_2}$

$m_2 = \frac{m_1 P_2}{P_1} = \frac{22 \times 10^{-3} \times 950}{1200}$

$m_2 = 17.5 \times 10^{-3}\text{ kg}$

Now required escaped mass is

$$\frac{P_1 V}{m_1} = \frac{P_2 V}{m_2}$$

$$\Delta m = m_1 - m_2$$

$$\Delta m = 22 \times 10^{-3} - 17.5 \times 10^{-3}$$

$$\Delta m = 4.5 \times 10^{-3} \text{ kg}$$

Or $\Delta m = 4.5 \text{ gram}$

Q.4 In an isobaric process 2000 J of heat energy is supplied to a cylinder, the piston of area $2.0 \times 10^{-2} \text{ m}^2$ moves through 40 cm under a pressure of $1.01 \times 10^5 \text{ N/m}^2$. Calculate the increase in internal energy of the system. (2009)

DATA:

$$\Delta Q = 2000 \text{ Joule}$$

$$A = 2 \times 10^{-2} \text{ m}^2$$

$$\Delta Y = h = 40 \text{ cm} = 40 \times 10^{-2} \text{ m}$$

$$P = 1.05 \times 10^5 \text{ N/m}^2$$

$$\Delta U = ?$$

SOLUTION:

As we know that

$$\Delta Q = \Delta U + \Delta w$$

$$\Delta U = \Delta Q - \Delta w \text{ -----(1)}$$

But

$$\Delta w = P \Delta V$$

Or

$$\Delta w = P A h \quad [\because \Delta V = A h]$$

$$\Delta w = 1.05 \times 10^5 \times 2 \times 10^{-2} \times 40 \times 10^{-2}$$

$$\Delta w = 840 \text{ Joule} \text{ -----(2)}$$

Now(1) \Rightarrow

$$\Delta Q = 2000 - 840$$

$$\Delta Q = 1160 \text{ Joule}$$

Self Test (8)

- (i) 1200J of heat energy are supplied to the system at constant pressure. The internal energy of the system is increased by 750J and the volume by 4.5m³. Find the work done against the pressure and the pressure on the piston. (2003, 1994)
- (ii) When 2000J of heat energy is supplied to a gas in a cylinder at constant pressure of $1.01 \times 10^5 \text{ N/m}^2$, the piston of area of cross-section $2 \times 10^{-2} \text{ m}^2$ moves through 0.5m. Calculate the work done and the increase in the internal energy of the system. (2002 P.M)
- (iii) In an isobaric process, when 2000J of heat energy is supplied to a gas in a cylinder, the piston moves through 0.1m under a constant pressure of $2 \times 1.01 \times 10^5 \text{ N/m}^2$. If the area of the piston is $5 \times 10^{-2} \text{ m}^2$, calculate the work done and the increase in the internal energy of the system. (1996)

Ans. $\Delta U = 990 \text{ J}$

Q.5 A Carnot engine performs 2000J of work and rejects 4000J of heat to the sink. If the difference of temperature between the source and the sink is 85°C , find the temperatures of the source and the sink. (2008)

DATA:

$$\Delta W = 2000 \text{ J}$$

$$Q_2 = 4000 \text{ J}$$

$$\Delta T = 85^\circ \text{C} = 85 \text{ K}$$

$$\text{Temp. of Heat Source} = T_1 = ?$$

$$\text{Temp. of Heat Sink} = T_2 = ?$$

SOLUTION:

As $\Delta W = Q_1 - Q_2$

Or $Q_1 = \Delta W + Q_2$

$$Q_1 = 2000 + 4000$$

$$Q_1 = 6000 \text{ J} \text{ -----(1)}$$

As $E = \frac{\Delta W}{Q_1} \times 100$

$$E = \frac{2000}{6000} \times 100$$

$$E = 33.33\% \text{ -----(2)}$$

As $E = \left(1 - \frac{T_2}{T_1}\right) \times 100$

$$\frac{33.33}{100} = \frac{T_1 - T_2}{T_1} \quad [\because T_1 - T_2 = \Delta T]$$

$$0.3333 = \frac{\Delta T}{T_1}$$

$$T_1 = \frac{85}{0.3333}$$

$$T_1 = 255.025 \text{ K}$$

As $\Delta T = T_1 - T_2$

$$T_2 = T_1 - \Delta T$$

$$T_2 = 255.025 - 85$$

$$T_2 = 170.025 \text{ K}$$

Self Test (9)

(i) A Heat engine performs 1000J of work and at the same time rejects, 4000J of heat energy to the cold reservoir. What is the efficiency of the engine? If the difference of temperature between the sink and the source of engine is 75°C, find the temperature of its source. (2002 P.M)

Q.6 Calculate density of hydrogen gas, considering it to be an ideal gas, when the root mean square velocity of hydrogen molecules is 1850 m/sec at 0°C and 1 atmospheric pressure. (1-atmosphere = 1.01 x 10⁵ N/m²) (2007,2004)

DATA:

$V_{rms} = 1850 \text{ m/sec}$

$T = 0^\circ\text{C} = 273\text{K}$

$P = 1 \text{ atmosphere} = 1.01 \times 10^5 \text{ N/m}^2$

Density = $\rho = ?$

SOLUTION:

As we know that

$$V_{rms} = \sqrt{\frac{3P}{\rho}}$$

$$(V_{rms})^2 = \frac{3P}{\rho}$$

$$\rho = \frac{3P}{(V_{rms})^2}$$

$$\rho = \frac{3 \times 1.01 \times 10^5}{(1850)^2}$$

$$\rho = \frac{3.03 \times 10^5}{3.4225 \times 10^6}$$

$$\rho = 0.08853 \text{ kg/m}^3$$

or

$$\rho = 8.853 \times 10^{-2} \text{ kg/m}^3$$

Q.7 A heat engine performs work at the rate of 500 kilowatt. The efficiency of the engine is 30% calculate the loss of heat per hour. (2007)

DATA:

Power = $P = 500 \text{ KW} = 5 \times 10^5 \text{ watt}$ (1KW = 10³ watt)

Efficiency = $E = 30\%$

$t = 1 \text{ hour} = 3600 \text{ sec}$

Heat loss per hour = $Q_2 = ?$

SOLUTION:

$$P = \frac{\text{Work}}{t} = \frac{\Delta W}{t}$$

$$\Delta W = P \times t$$

$$\Delta W = 5 \times 10^5 \times 3600$$

$$\Delta W = 1.8 \times 10^9 \text{ J} \text{-----(1)}$$

As

$$E = \frac{\Delta W}{Q_1} \times 100$$

$$Q_1 = \frac{\Delta W}{E} \times 100$$

$$Q_1 = \frac{1.8 \times 10^9}{30} \times 100$$

$$Q_1 = 6 \times 10^9 \text{ J} \text{-----(2)}$$

Now

$$\Delta W = Q_1 - Q_2$$

or

$$Q_2 = Q_1 - \Delta W$$

$$Q_2 = 6 \times 10^9 - 1.8 \times 10^9$$

$$Q_2 = 4.2 \times 10^9 \text{ J}$$

- Q.8 A brass ring of 20cm diameter is to be mounted on a metal rod of 20.02cm diameter at 20°C. To what temperature should the ring be heated? (for brass $\alpha = 1.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$) (2005)

DATA:

Diameter of ring = $d_1 = 20\text{cm}$.
Diameter of rod = $d_2 = 20.02\text{cm}$.
Initial temperature = $T_1 = 20^\circ\text{C}$
For brass = $\alpha = 1.9 \times 10^{-5}/^\circ\text{C}$
Final Temperature = $T_2 = ?$

SOLUTION:

As we know that

$$\Delta L = \alpha L_1 \Delta T \text{ -----(1)}$$

Consider length is equal to diameter therefore

$$\Delta L = \Delta d = d_2 - d_1$$

$$L_1 = d_1$$

$$\therefore (1) \Rightarrow \Delta d = \alpha d_1 \Delta T$$

or

$$\Delta T = \frac{d_2 - d_1}{\alpha d_1}$$

$$\Delta T = \frac{20.02 - 20}{1.9 \times 10^{-5} \times 20}$$

or

$$T_2 - T_1 = \frac{0.02}{3.8 \times 10^{-4}}$$

$$T_2 - 20 = 52.63$$

$$T_2 = 52.63 + 20$$

$$T_2 = 72.63 \text{ K}$$

Self Test (10)

- (i) A cylinder of diameter 1.00cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.9770cm at 30°C. To what temperature must the plate be heated? (α for steel = $1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$). (2002 P.E)

11.48: NUMERICALS FOR SELF PRACTICE:

YEAR 2000:

Calculate the volume occupied by a gram-mole of a gas at 10°C and pressure of one atmosphere.

$$(R=8.313 \text{ J/mol-K, } 1\text{atm}=1.01 \times 10^5 \text{ N/m}^2)$$

Ans. $V = 23.3 \text{ litre}$

YEAR 1998:

540 calories of heat is required to vaporize 1g of water at 100°C. Determine the entropy change involved in vaporizing 5g of water. (One calorie = 4.2 Joules).

Ans. $\Delta S = 30.4 \text{ J} - \text{K}^{-1}$

YEAR 1997:

A glass flask is filled to the mark with 60cm³ of mercury at 20°C. If the flask and its contents are heated to 40°C, how much mercury will be above the mark?

(For glass $\alpha=9 \times 10^{-6}/^\circ\text{C}$ and for mercury $\beta=182 \times 10^{-6}/^\circ\text{C}$).

Ans. $\Delta V_2 = 0.186 \text{ cm}^3$

YEAR 1995:

A meter bar of steel is corrected at 0°C and another at -2.5°C. What will be the difference between their length at 30°C. ($\alpha = 12 \times 10^{-6}/^\circ\text{C}$).

Ans. $\Delta L_2 = 3 \times 10^{-5} \text{ m}$

1994/2003 P.E:

1200J of heat energy are supplied to the system at constant pressure. The internal energy of the system is increased by 750J and the volume by 4.5m^3 . Find the work done against the pressure and the pressure on the piston.

Ans. $P = 100 \text{ N/m}^2$

YEAR 1992:

A cylinder contains an ideal gas below a frictionless piston fitted in it. If the gas in the cylinder is supplied 3000J of heat and the piston rises by 0.35m, while the internal energy of the gas increases by 400J. Calculate the work done by the piston.

Ans. $\Delta W = 2600 \text{ J}$

YEAR 1991:

One gram-mole of a gas occupies a volume of 24.93m^3 at pressure of 500 N/m^2 , find the temperature of the gas in Celsius. ($R=8.313 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$)

Ans. $T = 1226 \text{ }^\circ\text{C}$

YEAR 1990:

At certain temperature the average Kinetic energy of hydrogen molecule is $6.2 \times 10^{-21} \text{ J}$. If the mass of hydrogen molecule is $3.1 \times 10^{-27} \text{ Kg}$, find (i) the temperature and (ii) root mean square velocity of hydrogen molecule.

[Given $K = 1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$]

Ans. $v_{\text{rms}} = 2000 \text{ m/s}$

YEAR 1989:

An iron ball has a diameter of 5cm and is 0.01mm too large to pass through a hole in a brass plate when the ball and the plate are at a temperature of 30°C . At what temperature, will the ball just pass through the hole?

Ans. $T_2 = 40.3^\circ\text{C}$

YEAR 1987:

The efficiency of a heat engine is 50%. If the temperature of the cold reservoir is 300K, find the temperature of hot reservoir.

Ans. $T_1 = 600\text{K}$

YEAR 1986:

10 Kg of water fall through a distance of 854m and all the energy is effective in heating the water. To what the temperature will the water be raised if it was initially at 20°C .

Ans. $T_2 = 22^\circ\text{C}$

YEAR 1985:

What is the root mean square velocity of hydrogen molecule at 300K? Take the mass of hydrogen molecule to be $3.32 \times 10^{-27} \text{ Kg}$ and Boltzman constant = $1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$.

Ans. $v_{\text{rms}} = 1934 \text{ m/s}$