WAVE MOTION AND SOUND 8

This chapter is divided into three parts.

(a) Simple Harmonic Motion (b) Waves (c) Sound

SIMPLE HARMONIC MOTION

VIBRATORY MOTION:

If a body moves to and fro about its mean position, its motion is said to be "vibratory" e.g. motion of the prongs of a tuning fork when strucked, motion of the string of sitar when plucked, motion of mass attached to an elastic spring, motion of pendulum, motion of the membrane of a drum etc.

(i) Periodic Motion:

The motion of a body which repeats itself in equal intervals of time, is called periodic motion.

(ii) Vibration:

One complete round trip of a body about its means position, is called one vibration.

(iii) Time Period 'T'

The time required to complete one vibration, is called time period. It is denoted by T.

(iv) Frequency 'υ'

The number of vibrations which are completed in one second, is called frequency. It is denoted by 'υ'.

(v) Displacement (x)

It is the distance of vibrating body at any instant from the mean position it is denoted by 'x'.

(vi) Amplitude x_0

The maximum displacement of a vibrating body from the mean position is called amplitude. It is denoted by 'x_0'.

Hook's Law:

The extension 'x' produced in the spring is directly proportional to the applied force 'F' provided the elastic limit is not violated.

$$F \propto x$$

$$\boxed{F = kx}$$

Where K is a constant of proportionality and is called spring constant.

Restoring Force:

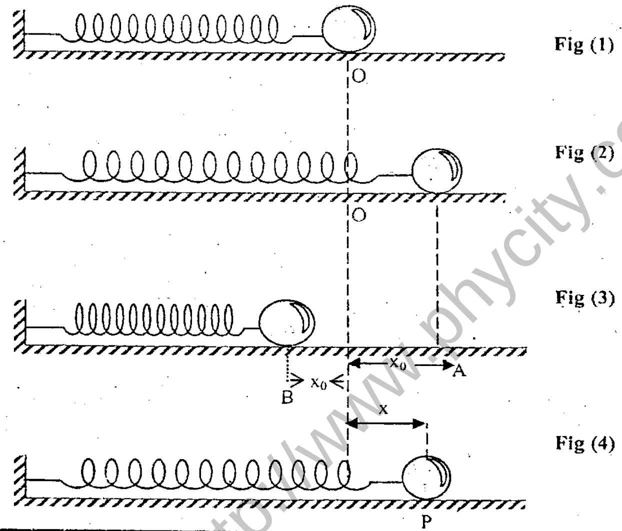
It is the force which brings the body back towards the mean position. This force is equal in magnitude but opposite to the applied force and given by

$$F = -kx$$

MOTION UNDER ELASTIC RESTORING FORCE OR MASS ATTACHED WITH SPRING:

Consider a body of mass "m" attached to a horizontal helical spring as shown in the fig (1). The whole system is placed on a horizontal frictionless surface. If the spring is stretched or compressed through a small displacement 'x' from the equilibrium position then according to Hooke's law force 'F' is proportional to the displacement x.

$F \propto x$ *OR* $F = k\,x$



Where k = Force constant of spring due to elasticity, the spring exerts force which is equal and opposite to the applied force. This force is called "RESTORING FORCE" and is given by.

$$F = -k\,x \longrightarrow \quad (1)$$

The negative sign indicates that the force exerted by the spring on the body is always directed opposite to the displacement.

The work done in displacing the body from O to A will be stored in the spring as P.E. If the body is now released, it will move towards O. At O, all the P.E is converted into K.E. and due to inertia the body continues to move and reaches B, where all the K.E is converted to P.E. the spring is now compressed and pushes the body towards O. In this way the body continues to move between A & B thus its motion is vibratory.

ACCELERATION OF THE BODY:

Let at any instant "t" the body is at point "P" which is at a distance "x" from O, as shown in fig. Then restoring force on the body will be,

$$F = -kx \rightarrow \quad (2)$$

If "a" be the acceleration of the body at P then the Newton's second law of motion is: $F = ma \rightarrow (3)$

Comparing eq. (2) and (3)

$$ma = -kx$$

$$\boxed{a = -\frac{k}{m}x} \longrightarrow (4)$$

Eq. (4) gives the acceleration of the body.

Since $\frac{k}{m}$ is constant

$$\therefore a = -(\text{constant})x$$

$$\textbf{or} \quad a \propto -x$$

i.e acceleration $\propto -$ displacement.

This shows that the acceleration of a vibrating body is directly proportional to the instantaneous displacement & is always directed towards equilibrium position. Such a motion is called "SIMPLE HARMONIC MOTION" (SHM).

SIMPLE HARMONIC MOTION:

If a body is performing vibratory motion so that acceleration of the vibrating body is directly proportional to –ve of its displacement from mean position or equilibrium position.

$$a \propto -x$$

i.e acceleration $\propto -$ displacement

CONDITIONS OF SIMPLE HARMONIC MOTION:

Following are the conditions of S.H.M.

1. It should be in vibratory motion.
2. The acceleration of the body (vibrating) should be proportional to –ve of it's displacement from mean position.
3. The acceleration should be directed towards mean position.
4. The system must be frictionless.

EXAMPLES OF S.H.M:

Following are the few examples of S.H.M.

1. Motion of the mass attached to a spring.
2. Motion of a string of sitar when plucked.
3. Motion of a pendulum.
4. Motion of the prongs of tuning fork when strucked.
5. Motion of the membrane of a drum when beaten.

Q:A particle/body is moving in a circle, show that its projection performs S.H.M. Derive expression for its.

(1) Acceleration (2) Displacement (3) Time period and (4) Frequency.

Consider a point mass "m" moving in a circle of radius "x_o" with constant angular velocity "ω". As the point mass "m" rotates along the circumference of the circle, its projection Q moves back and forth along the diameter AB. Thus Q performs vibratory motion.

Let at any instance "t" the point mass be at P and its projection Q is at a distance "x" from O. The magnitude of centripetal acceleration of P is given by.

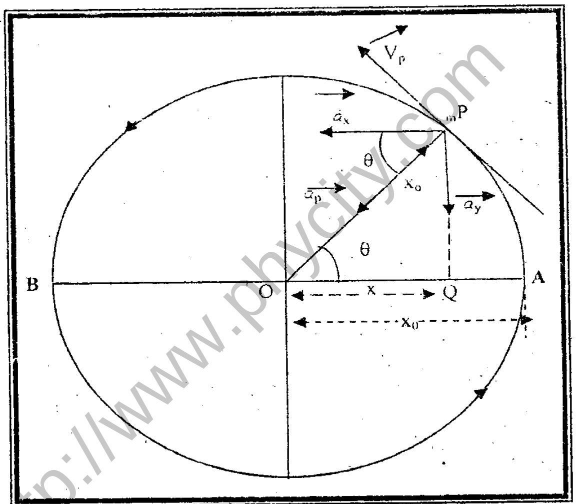
Here.

$$a_c = \frac{V^2}{r}$$

$a_c = a_p$

$V = V_p$

$r = x_0$



$$\therefore \quad a_p = \frac{v_p^2}{x_o}$$

But $v_p = x_o\omega$

$$a_p = \frac{x_o^2\omega^2}{x_o}$$

Or

$$\boxed{a_p = x_o\omega^2}$$

Resolve this acceleration into two components.

(i) $a_p \cos\theta = x_o\omega^2 \cos\theta$ (Horizontal Component)

(ii) $a_p \sin\theta = x_o\omega^2 \cos\theta$ (Vertical Component)

As the projection "Q" vibrates along the diameter AB about O, therefore acceleration will be equal to the horizontal component.

Acceleration of $Q = a_x = -x_o\omega^2\cos\theta$

Negative sign is taken because it is directed towards mean position.

$$\text{In } \Delta OQP, \quad \cos\theta = \frac{OQ}{OP} = \frac{x}{x_o}$$

$$\therefore \quad a_x = -x_o\omega^2\frac{x}{x_o}$$

$$\boxed{a_x = -\omega^2 x} \longrightarrow (1)$$

As ω is constant, $\therefore \omega^2 = \text{Constant}$

$$\therefore a_x = -(\text{Constant})x$$

$$\boxed{a_x - \propto x}$$

Thus acceleration of Q is proportional to displacement and is directed towards the mean position.

Hence motion of Q is S.H.M.

Equation (1) gives the "Instantaneous Acceleration" of Q.

MAXIMUM ACCELERATION:

The acceleration will be maximum "a_{max}" at extreme position where $x = x_o$ ∴ Eq (1) becomes

$$\boxed{a_{max} = -\omega^2 x_o}$$

MINIMUM ACCELERATION:

The acceleration will be minimum "a_{min}" at mean position where

$x = o$ eq. (1) becomes $a_{min} = -(\omega^2)\, o$

$$\boxed{a_{min} = o}$$

DISPLACEMENT:

Suppose at time t = 0 the point mass is at C and the angle which the OC makes with the x-axis is "Φ"then it is known as "INTIAL PHASE ANGLE"

Let at any instant "t" the point mass is at P. The angle between OC & OP is "ωt" while the total angle which OP makes with x-axis is $\Phi + \omega t = \theta$. The displacement "x" of the image Q from O can be found by

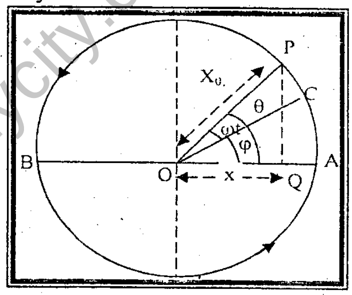
Considering Δ OQP, here

$$\text{Cos } \theta = \frac{OQ}{OP} = \frac{x}{x_o}$$

Or $\quad x = x_o \cos\theta$

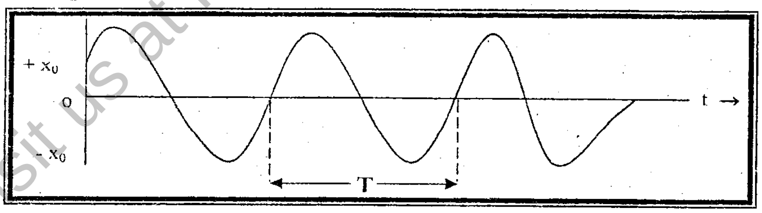
But $\quad \theta = \Phi + \omega t$

∴ $\quad \boxed{x = x_o \cos(\Phi + \omega t)}$



"x" will be position when displacement is to the right & negative when displacement is to the left of the mean position.

As the point mass circulates, displacement changes its direction, thus if a graph is plotted between displacement "x" & time "t" it will be a sinusoidal curve as shown in the figure.



TIME PERIOD:

The time in which the point mass "P" Completes its one round of the circle, its projection "Q" Completes one vibration.

The value of x at time t is equals to the value of x at time (t + T) and we know that the phase increased by 2π radians as t increases by T, then

$$\omega t + \phi + 2\pi = (t + T) + \phi$$

$$\omega T = 2\pi$$

or $$\boxed{T = \frac{2\pi}{\omega}}$$

FREQUENCY:

The number of vibrations or rotations completed by a body in one second is its frequency. It is denoted by "υ" Frequency and time period are related by the formula $\upsilon = \frac{1}{T}$

Putting the value of "T" from eq. (2)

$$\upsilon = \frac{1}{\frac{2\pi}{\omega}}$$

$$\boxed{\upsilon = \frac{\omega}{2\pi}} \longrightarrow (3)$$

VELOCITY OF PROJECTION / PARTICLE / BODY EXECUTING S.H.M.

Consider a point mass "m" moving in a circle of radius "x_o" with constant angular velocity "ω" let at any instant "t" the point mass is at P and its projection Q is at distance "x" from O. As the point mass circulates, its projection Q moves to and fro about O along the diameter AB. Hence Q performs vibratory motion.

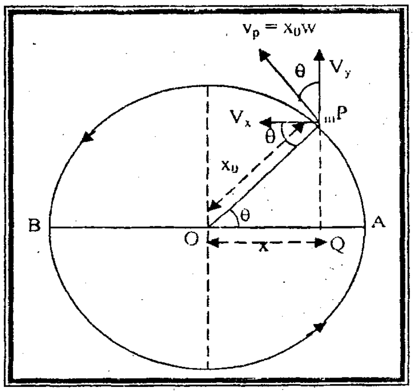
The velocity of the point mass at P is given by $v_p = x_o\,\omega$ [$v = r\omega$]

Resolve this velocity into two components.

(i) $x_o\,\omega\,\sin\theta$ (Horizontal Component parallel to the diameter)

(ii) $x_o\,\omega\,\cos\theta$ (Vertical Component perpendicular to the diameter)

As Q moves along x-axis therefore its velocity will be.



$$V_x = x_o\omega\sin\theta$$

As $\sin^2\theta + \cos^2\theta = 1$

Or $\sin^2\theta = 1 - \cos^2\theta$

Or $\sin\theta = \sqrt{1-\cos^2\theta}$

$$\therefore V_x = x_o\omega\sqrt{1-\cos^2\theta}$$

Consider ΔOQP

$$\cos\theta = \frac{x}{x_o}$$

$$v_x = x_o\omega\sqrt{1-\left(\frac{x}{x_o}\right)^2}$$

$$= x_o\omega\sqrt{1-\frac{x^2}{x_o^2}}$$

$$v_x = x_o\omega\sqrt{\frac{x_o^2 - x^2}{x_0^2}} = x_0\,\omega\frac{\sqrt{x_0^2 - x^2}}{x_0}$$

$$\boxed{v_x = \omega\sqrt{x_0^2 - x^2}} \longrightarrow (i)$$

Equation (i) gives the "INSTANTANEOUS VELOCITY" of Q.

MAXIMUM VELOCITY:

The velocity of Q will be maximum at mean position where x = o

∴ Eq(i)becomes

$$v_{max} = \omega\sqrt{x_o^2 - o} = \omega\sqrt{x_o^2}$$

$$\boxed{v_{max} = x_o\,\omega}$$

MINIMUM VELOCITY:

The velocity of Q will be minimum at extreme position where $x = x_o$

∴ Eq (i) becomes.

$$V_{min} = \omega\,(x_o - x_o)^2 = \omega^2 \times o$$

$$\boxed{V_{min} = 0}$$

TIME PERIOD OF A MASS ATTACHED WITH SPRING:

If a body of mass "m" is attached to one end of an elastic spring and performs S.H.M then.

$$a = -\frac{k}{m}x \longrightarrow \textbf{(a)}$$

Acceleration on a diameter of the projection of a point mass moving in a circle is given by. $a = -\,\omega^2 x \longrightarrow$ **(b)**

Comparing eq. (a) & (b)

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}} \rightarrow (c)$$

The time period of a body performing S.H.M is given by. $T = \frac{2\pi}{\omega}$

Putting the value of "ω" from equation (c) $T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$ $\boxed{\therefore T = 2\pi\sqrt{\frac{m}{k}}} \longrightarrow$ **(d)**

FREQUENCY:

The frequency of the body is the reciprocal of its time period.

i.e. $\nu = \frac{1}{T}$

Putting the value of "T" from e.q. (d)

$$\nu = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}} \longrightarrow \textbf{(e)}$$

INSTANTANEOUS VELOCITY:

The instantaneous velocity of the body performing S.H.M is given by.

$$V_x = \omega\sqrt{x_0^2 - x^2}$$

Putting the value of "ω" from e.q. (c)

$$\boxed{V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}} \longrightarrow (e)$$

MAXIMUM VELOCITY:

The velocity will be maximum at mean position where $x = 0$

∴ e.q. (e) becomes

$$V_{max} = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - (0)^2} = \sqrt{\frac{k}{m}}\sqrt{x_0^2}$$

$$\boxed{V_{max} = x_0\sqrt{\frac{k}{m}}} \longrightarrow (f)$$

MINIMUM VELOCITY:

The velocity will be minimum at extreme position where $x = x_0$

∴ e.q (e) becomes

$$V_{min} = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x_0^2} = \sqrt{\frac{k}{m}} \times 0$$

$$\boxed{V_{min} = 0}$$

RELATIONSHIP BETWEEN INSTANTANEOUS AND MAXIMUM VELOCITY:

Equation (e) can be written as.

$$V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2\left(1 - \frac{x^2}{x_0^2}\right)}$$

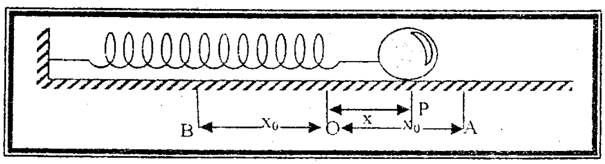
$$V_x = x_0\sqrt{\frac{k}{m}}\sqrt{1 - \frac{x^2}{x_0^2}}$$

But from equation (f) $x_0\sqrt{\frac{k}{m}} = V_{max}$

$$\therefore V_x = V_{max}\sqrt{1 - \frac{x^2}{x_0^2}}$$

ENERGY OF A BODY EXECUTING S.H.M:

Consider a body of mass "m" attached to one end of a helical spring whose other end is fixed. The whole system is placed on a horizontal frictionless surface. When the body is at equilibrium position O, the force acting on it is zero. When the body is moved to an extreme position A or B and then released, it moves to and fro around mean position. Let at any instant "t" the body is at P which is at a distance "x" from O.



INSTANTANEOUS KINETIC ENERGY:

The instantaneous speed of the body at distance "x" from the mean position is given by:

$$V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

As $K{\cdot}E = \frac{1}{2}mV_x^2$

$\therefore$ $K{\cdot}E = \frac{1}{2}m\left[\sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}\right]^2$

$K{\cdot}E = \frac{1}{2}m \times \frac{k}{m} \times (x_0^2 - x^2)$

$$\boxed{K{\cdot}E = \frac{1}{2}k\,(x_0^2 - x^2)}$$

MAXIMUM K·E:

The K·E of the body is maximum at mean position where x = 0

As $K{\cdot}E = \frac{1}{2}k(x_0^2 - x^2)$

$(K{\cdot}E)_{max} = \frac{1}{2}k[x_0^2 - (o)^2]$

$$\boxed{(K{\cdot}E)_{max} = \frac{1}{2}kx_0^2} \longrightarrow (1)$$

MINIMUM K·E:

The K·E of the body is minimum at two extreme position were $x = x_o$

As $K{\cdot}E = \frac{1}{2}k(x_0^2 - x^2)$

$(K{\cdot}E)_{min} = \frac{1}{2}k\left[x_0^2 - x_0^2\right]$

$(K{\cdot}E)_{min} = \frac{1}{2}k \times 0$

$$\boxed{(K{\cdot}E)_{min} = 0}$$

INSTANTANEOUS P·E:

When the body is at mean position where x = 0

$F_1 = k\,x$

$F_1 = k \times 0$

$$\boxed{F_1 = 0}$$

When the body is at displacement 'x' from the mean position $F_2 = Kx$

Average force on the body

$$Fav = \frac{F_1 + F_2}{2}$$

$$Fav = \frac{0 + kx}{2}$$

$$Fav = \frac{1}{2}kx$$

Work done in moving the body through a displacement 'x' is given by

Work = (Force) (Displacement)

$$Work = \left(\frac{1}{2}kx\right)(x)$$

$$Work = \frac{1}{2}kx^2$$

This work is stored in the body as its P·E

$$\boxed{P \cdot E = \frac{1}{2}kx^2}$$

MAXIMUM P·E:

The P·E of the body is maximum at two extreme position where $x = x_o$

As $P \cdot E \frac{1}{2}kx^2$

$$\boxed{(P \cdot E)_{max} \frac{1}{2}kx_o^2} \longrightarrow (2)$$

MINIMUM P·E:

The P·E of the body is minimum at mean position where $x = o$

As $P \cdot E \frac{1}{2}kx^2$

$$(P \cdot E)_{min} \frac{1}{2} K \times 0$$

$$\boxed{(P \cdot E)_{min} = 0}$$

The total energy of the body at any point can be found by adding its P.E & K.E at that point.

Thus total energy at P= P.E at P + K.E at P

$$= \frac{kx^2}{2} + \frac{kx_o^2}{2} - \frac{kx^2}{2}$$

$$= \frac{1}{2}kx_o^2 \longrightarrow (3)$$

From equations (1), (2), & (3) it can be seen that the total energy of the body executing S.H.M remains constant.

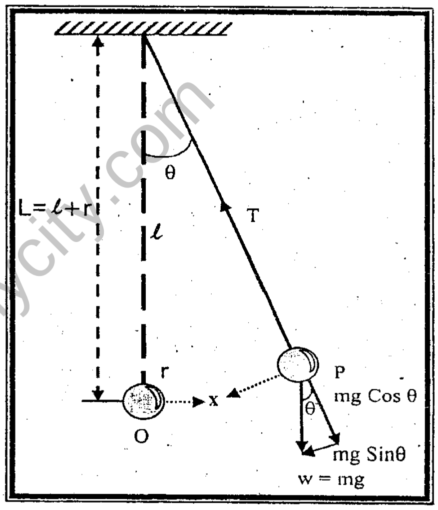
SIMPLE PENDULUM:

An ideal simple pendulum consists of a spherical bob suspended from a light, flexible and inextensible string tied to a rigid and frictionless support. When the bob is displaced from its equilibrium position, it begins to perform oscillatory motion.

Consider a simple pendulum of length "L" consisting of a bob of mass "m". If the pendulum is disturbed from its mean position it starts vibrating between two extreme positions. Let at any instant "t" the bob is at point P and at this point two forces are acting on the bob.

1. The force of gravity = $F_g = mg$

2. Tension in the string = T



the force of gravity is the weight of the body and can be resolved into two components.

(i) $(F_g)_{||} = mg \cos\theta$, along the string

(ii) $(F_g)_{\perp} = mg \sin\theta$, perpendicular to the string.

Since there is no motion along the string

$\therefore\ T = mg \cos\theta$

The only force responsible for the motion of the bob is mg sin θ as this force is directed towards the mean position i.e acts as a restoring force, therefore we shall assign a negative sign to it.

$\therefore\quad F = -mg \sin\theta \quad (1)$

If "a" be the acceleration of the bob at P, then by Newton's second law of motion.

$F = ma \longrightarrow (2)$

Comparing eq. (1) & (2)

$ma = -mg \sin\theta$

$a = -g \sin\theta$

If "θ" is small then $\sin\theta \cong \theta$

$\therefore\quad a = -g\theta$

But

$$\theta \text{ (in radian)} = \frac{\text{length of arc}}{\text{radius}}$$

$$\theta = \frac{x}{L}$$

Then $$a = -g\frac{x}{L}$$

$$\boxed{a = -\frac{g}{L}x}$$

As $\frac{g}{L}$ is Constant

$$\therefore \quad \boxed{a \propto -x}$$

Thus the acceleration of simple pendulum is directly proportional to its displacement and is directed towards its mean position. Hence the motion of the simple pendulum is S.H.M.

TIME PERIOD OF SIMPLE PENDULUM:

The time required to complete one oscillation by a simple pendulum is called its time period. The acceleration of the simple pendulum is given by.

$$a = -\frac{g}{L}x \longrightarrow \text{(i)}$$

The acceleration of a body executing S.H.M is given by.

$$a = -\omega^2 x \longrightarrow \text{(ii)}$$

Comparing equation **(i)** & **(ii)** we get

$$\omega^2 = \frac{g}{L}$$

$$\omega = \sqrt{\frac{g}{L}} \longrightarrow \text{(iii)}$$

The time period of a body executing S.H.M is given by.

$$T = \frac{2\pi}{\omega}$$

Putting the value of "ω" from e.g. **(iii)**

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}}$$

$$\boxed{\therefore T = 2\pi\sqrt{\frac{L}{g}}}$$

This shows that the time period of a simple pendulum depends upon its length and acceleration due to gravity and it is independent of the mass of the bob.

FREQUENCY OF SIMPLE PENDULUM:

Frequency is reciprocal of time period i.e.

$$\nu = \frac{1}{T}$$

But $$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\therefore \nu = \frac{1}{2\pi\sqrt{\frac{L}{g}}}$$

$$\boxed{\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$$

GENERAL FORMULA FOR THE TIME PERIOD:

The general formula for time period of a body performing S.H.M is

$$\boxed{T = \frac{2\pi}{\text{Constant}}}$$

1. In the case of circle the constant is ω^2

$$\therefore T = \frac{2\pi}{\sqrt{\omega^2}} \qquad \left\{\begin{array}{l} a = -\omega^2 x \\ a = -(\text{Constant})x \end{array}\right\}$$

$$T = \frac{2\pi}{\omega}$$

2. In the case of the body attached to an elastic spring the constant is $\frac{k}{m}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{k}{m}}} \qquad \left\{\begin{array}{l} a = -\frac{k}{m}x \\ a = -(\text{Constant})x \end{array}\right\}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In the case of simple pendulum the constant is $\frac{g}{L}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{g}{L}}} \qquad \left\{\begin{array}{l} a = -\frac{g}{L}x \\ a = -(\text{constant})x \end{array}\right\}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Second's Pendulum:

A pendulum which completes one vibration in two seconds is called second's pendulum.

Its time period = T = 2 Seconds

Frequency = $\upsilon = \frac{1}{T} = \frac{1}{2} = 0.5\text{Hz}$

WAVES

WAVE PULSE

A single isolated disturbance which travels through a medium or space is known as a "WAVE PULSE"

TRAVELLING OR PROGRESSIVE WAVES:

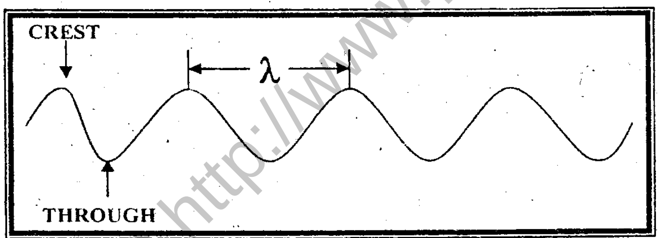
The waves which changes their position with the passage of time are called "traveling or progressive waves" e.g. water waves, sound waves, heat waves, light waves", etc.

TRANSVERSE WAVES:

Transverse waves are those in which particles of a medium vibrate perpendicular to the direction of propagation of waves. These waves consist of crests and troughs e.g. water waves, waves in string, electromagnetic waves etc. are all transverse waves.

CREST:

The portion of the transverse wave which is higher than the equilibrium position of the medium is called a crest.



TROUGH:

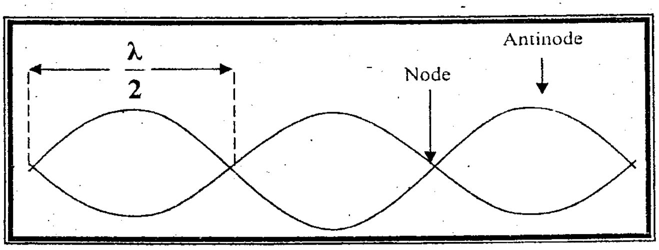
The position of the transverse wave which is lower than equilibrium position of the medium is called a trough.

STATIONARY WAVES OR STANDING WAVES:

When two similar waves move along the same line in a medium but in opposite direction these waves will combine according to the principle of super position and special kind of waves are formed, called stationary waves or standing waves.

NODE:

The point of a standing wave where displacement is minimum or zero is called a "node"



ANTINODE:

The point of a standing wave where displacement is maximum is called an "Antinode"

The distance between two consecutive nodes or antinodes is equal to the half of the wave length. $(\lambda/2)$

The distance between a node and an antinode is equal to quarter of the wave length. $(\lambda/4)$

WAVE LENGTH:

The distance between any two consecutive points on a wave that behave identically is called wavelength. Wave length can also be defined as the distance between two consecutive crests or troughs, compressions or rarefactions. It is denoted by "λ"

TIME PERIOD:

The time required to complete one vibration or one rotation is called "time period". It is denoted by T.

FREQUENCY:

The number of vibrations or rotations completed by a body in one second is called its frequency. It is denoted by "ν"

The frequency and time period are related by $\nu = \frac{1}{T}$

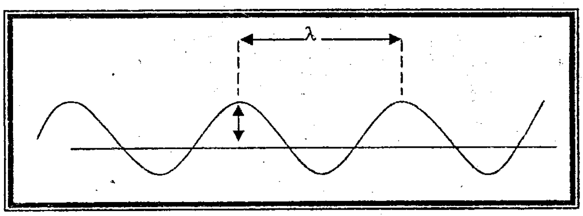
Consider a wave train passing through a point P. If "ν" be the frequency and V be the velocity of wave and λ be the wave length of wave then.

S= Vt Becomes $\lambda = VT$

$$V = \frac{\lambda}{T}$$

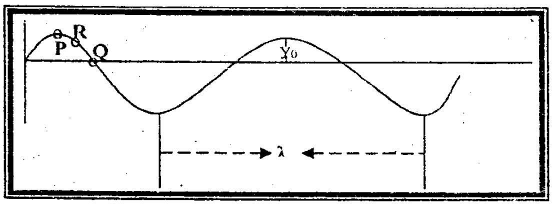
$$V = \frac{1}{T}\lambda$$

$$V = \nu\lambda$$



ENERGY IN WAVES:

A wave travelling through a medium carries energy with it which can be found by considering a wave produced in a string as shown in the figure. The points P, Q& R represent various segments of the wave.



The energy of P is entirely potential because its displacement from the mean position is maximum. The energy of the segment Q is entirely kinetic because its velocity is maximum. The energy of the segment R is partially potential and partially kinetic.

Consider the segment Q, if its velocity is V_{max} then its energy "ΔE" is given by. $\triangle E = \frac{1}{2} \triangle m\ V^2_{max}$

Where $\triangle m$ = mas of the segment. Q

If "y_o be amplitude & "ω" be the frequency then $V_{max} = y_o\omega$ $\because V = r\omega$

$$\therefore \triangle E = \frac{1}{2} \triangle m\ (y_o\omega)^2$$

If "m" be the mass of whole wave then its energy "E" is given by.

$$E = \frac{1}{2} m\ y_o^2\omega^2$$

If "μ" be the linear density of the wave of string then $\mu = \frac{m}{\lambda}$

Or $m = \mu\lambda$

$$\therefore \boxed{E = \frac{1}{2}\mu\lambda y_o^2\omega^2}$$

where λ = wave length.

POWER TRANSMITED BY WAVE:

Energy transmitted in one second is called power i.e.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

Power "P" transmitted by a wave is given by

$$P = \frac{E}{T} = \frac{1}{2}\frac{\mu\lambda y_o^2\omega^2}{T}$$

If "V" be the velocity of the wave then

$$\frac{\lambda}{T} = V$$

$$\therefore \boxed{P = \frac{1}{2}\mu\ V\ y_o^2\omega^2}$$

The above relation shows that the power transmitted by any harmonic wave mechanical or electromagnetic) is proportional to

1. The square of the frequency
2. The square of the amplitude

ANALYTICAL TREATMENT OF STANDING WAVES:

Consider two sinusoidal waves with the same amplitude, frequency and wave length but traveling in opposite directions. These waves can be written as.

$y_1 = A_o \sin(kx - \omega t)$

$y_2 = A_o \sin(kx + \omega t)$

Where

y_1 = Displacement of wave traveling to the Right.

y_2 = Displacement of wave traveling to the Left (Reflected wave).

If

y = Resultant wave function then

$y = y_1 + y_2$

$y = A_o \sin(kx - \omega t) + A_o \sin(kx + \omega t)$

$y = A_o \{\sin(kx - \omega t) + \sin(kx + \omega t)\}$

As we know that

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

So

$$y = A_o 2\sin\left(\frac{kx - \omega t + kx + \omega t}{2}\right)\cos\left(\frac{kx - \omega t - kx - \omega t}{2}\right)$$

$y = (2A_o \sin kx) \cos \omega t$

Where $2A_o \sin kx$ = Amplitude of standing wave for any value of x.

POSITIONS OF ANTI NODES:

There will be Anti Nodes when maximum amplitude is equals to $2A_o$

It happens when

$\sin kx = 1$

i.e. $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots, \frac{n\lambda}{4}$

$n = 1, 3, 5, \ldots\ldots$

POSITIONS OF NODES:

There will be Nodes when maximum amplitude is equals to zero. It happens when.

$\sin kx = 0$

i.e. $kx = \pi, 2\pi, 3\pi, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots, \frac{n\lambda}{2}$

Where $n = 1, 2, 3, \ldots\ldots$

This shows that two successive Nodes are separated by $\frac{\lambda}{2}$ and Anti Nodes and Nodes are separated by $\frac{\lambda}{4}$.

TRANSVERSE STATIONARY WAVES IN ASTRETCHED STRING

Consider a stretched string of length "L" fixed at both ends as shown in fig (1). If the string is plucked upward or downward then stationary waves will be produced in it. The string has a number of natural patterns of vibrations called "natural modes" each of these modes has a characteristic frequency called "FUNDAMENTAL FREQUENCY" or the FIRST HARMONIC. The higher frequencies are called OVERTONES.

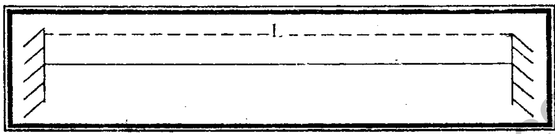


Fig (1)

FUNDAMENTAL FREQUENCY OR FIRST HARMONIC:

If the string is displaced at its middle point and released, it vibrates in the one loop, as shown in fig (2). In this case an antinode is formed at the middle while node at the ends of the string. If λ_1 is the wave length of wave formed then it is given by.

$$\frac{\lambda_1}{2} = L \text{ Or } \lambda_1 = 2L \quad (1)$$

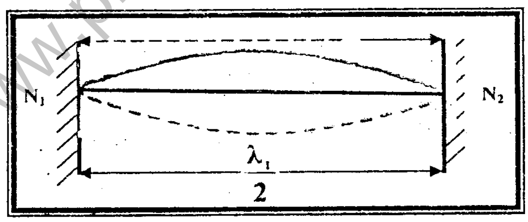


Fig (2)

If v_1 be the fundamental frequency then

$$V = v_1\lambda_1$$

$$\upsilon_1 = \frac{V}{2L} \longrightarrow (2)$$

The velocity "V" of the wave depends upon the tension in the string and linear density of the string.

If 'm' is the total mass of the string, then the velocity of the wave along the string is given by

$$V = \sqrt{\frac{TL}{m}}$$

$$\frac{m}{L} = \mu = \text{Linear density}$$

$$V = \sqrt{T/\mu}$$

$$\upsilon_1 = \frac{\sqrt{T/\mu}}{2L}$$

$$\upsilon_1 = \frac{1}{2L}\sqrt{T/\mu}$$

SECOND HARMONIC OR FIRST OVERTONE:

If the string is plucked at one quarter of its length it will vibrate in two loops, as shown in the figure (3). In this case there are two antinodes and three nodes. If λ_2 be the length of the wave formed then.

$$\frac{\lambda_2}{2} + \frac{\lambda_2}{2} = L$$

$$\frac{2\lambda_2}{2} = L$$

$$\lambda_2 = \frac{2L}{2} \longrightarrow (3)$$

If "ν_2" be the frequency of the vibrating string then. $\nu_2 = \frac{v}{\lambda_2}$

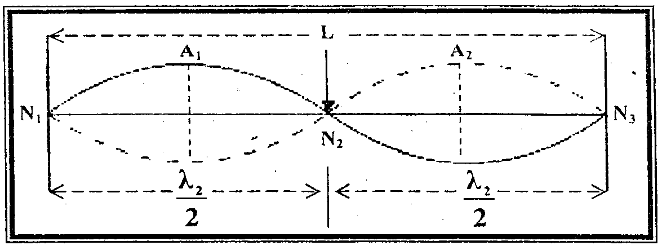


Fig (3)

Or $\upsilon_2 = \frac{V}{\frac{2L}{2}}$

$$\upsilon_2 = \frac{2V}{2L}$$

But from eq. **(2)** $\upsilon_1 = \frac{V}{2L}$

$$\boxed{\therefore\ \upsilon_2 = 2\upsilon_1 \rightarrow (4)}$$

THIRD HARMONIC OR SECOND OVERTONE:

If the same string is plucked from a suitable place, it can vibrate in three loops as shown in fig (4). In this case there are three antinodes and four nodes. The length of the string will be.

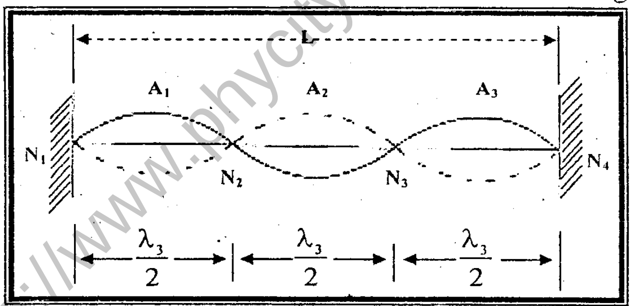


Fig (4)

$$\frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = L$$

$$\frac{3\lambda_3}{2} = L$$

$$\lambda_3 = \frac{2L}{3} \rightarrow (5)$$

If "υ_3" be the frequency of the vibrating string then.

$$\upsilon_3 = \frac{V}{\lambda_3}$$

$$\upsilon_3 = \frac{V}{\frac{2L}{3}}$$

$$\upsilon_3 = \frac{3V}{2L}$$

But from eq (2) $\frac{v}{2L} = \upsilon_1$

$$\therefore \boxed{\upsilon_3 = 3\upsilon_1} \longrightarrow \textbf{(6)}$$

$$\upsilon_4 = 4\upsilon_1$$

$$\upsilon_5 = 5\upsilon_1$$

FOR n LOOPS:

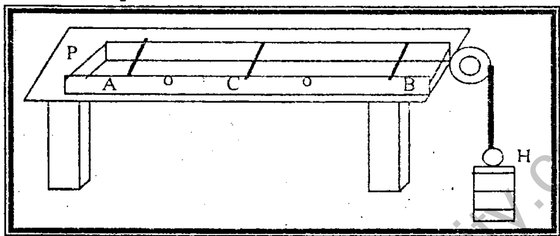
If the string vibrates in n loops then like equation (1), (3) & (5) we can write

$$\upsilon_n = n\upsilon_1$$

Where n = 1, 2, 3, 4,...........

SONOMETER:

A sonometer consists of a sounding box with two fixed bridges A & B and one movable bridge C. A vibrating string is stretched over the bridges. End of the string is tied to a peg P at one end of the sounding box and the other end passes over a pulley and carries a hanger on which weights can be slipped. Increasing or decreasing the number of weights on the hanger can alter the stretched force or the tension "T" in the string. The string can be made to vibrate in many loops. The lowest or fundamental frequency will be obtained if the string over the two bridges AC or BC vibrates in one loop.



The laws of transverse vibrations of strings can be verified by sonometer for which we have to confine ourselves to the fundamental mode of vibration. If V be the frequency of vibration of fundamental mode, then it is given by.

$$\nu = \frac{V}{\lambda} \rightarrow (1)$$

Where $\lambda =$ wavelength $= 2L$

And $V =$ velocity of the wave $= \sqrt{\frac{T}{\mu}}$

Where $T =$ tension on the string &
$\mu =$ mass per unit length or linear density of the string.

By putting the values of "V" and "λ"in e.g. (1), we get $\nu = \frac{v}{\lambda}$

$$\nu = \frac{\sqrt{T/\mu}}{\lambda L}$$

$$\boxed{\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}} \longrightarrow (2)$$

From e.q. (2) we can write the following relations which are the laws of transverse vibrations of string.

LAWS OF TRANSVERSE VIBRATIONS OF STRING:

1: $\nu \propto \frac{1}{L}$ i.e frequency produced in the string is inversely proportional to its length for a given tension and linear density.

2. $\nu \propto \sqrt{T}$ i.e for a given length and material, the frequency is directly proportional to the square root of the tension i.e. stretching force.

3. $\nu \propto \frac{1}{\sqrt{\mu}}$ i.e for a given length and tension, the frequency is inversely proportional to the square root of the linear density of the string.

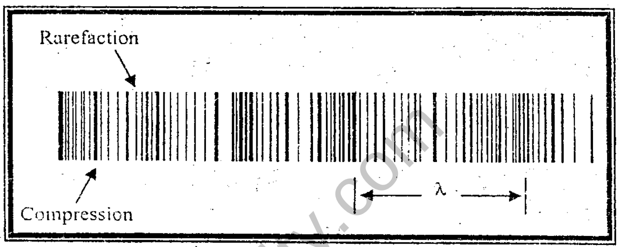
SOUND

LONGITUDINAL WAVES:

Longitudinal waves are those in which particles of the medium vibrate along the direction of propagation of waves.

These waves consist of compressions and rarefactions. The waves in a helical spring and sound waves are the examples of longitudinal waves. Sound waves can travel through any material medium i.e gases. Liquids and solids with a speed which depends upon the properties of the medium. Sound waves can be divided into there categories.

1. Audible sound waves,
2. Infrasonic waves and
3. Ultrasonic waves.



AUDIBLE WAVES:

The sound waves whose frequency lies between 20 hertz and 20,000 hertz are called audible waves. Musical instruments, human vocal chords, feathers of birds& insects etc, can produce audible waves.

INFRASONIC WAVES:

The sound waves whose frequency is less then 20 hertz are called "infrasonic waves". Earth quake waves are the examples of infrasonic waves.

ULTRASONIC WAVES:

The sound waves whose frequency is greater than 20,000 hertz are called "ultrasonic waves" these waves can be produced by inducing vibrations in a quartz crystal with an applied alternating electric field.

SPEED OF SOUND WAVES

NEWTON'S FORMULA FOR THE SPEED OF SOUND WAVES:

Sound waves are compressional waves, which propagate through a compressive medium such as air. The speed of such compressional waves depends upon the compressibility and the inertia of the medium. The compressible medium which has bulk modulus "B" & density (inertial property) "ρ" then the speed of sound "V" in the medium is given by.

$$V = \sqrt{\frac{B}{\rho}} \longrightarrow (1)$$

This formula is known as "Newton's formula" for the speed of sound waves.

The Bulk modulus is given by. $B = \frac{\text{stress}}{\text{volumetric strain}}$

As stress = F/A = ΔP = change in pressure, &

$$\text{volumetric strain.} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

$$\therefore \quad B = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

The ratio ($\Delta P/\Delta V$) is always negative because volume decreases as pressure increases and vice versa. This shows that "B" is always positive.

Sound waves are mechanical waves and the speed of all mechanical waves can be expressed in a general form as.

$$V = \sqrt{\frac{\text{Elastic property}}{\text{inertial property}}}$$

Newton assumed that the compressions and rarefactions take place at constant temperature. This process is called "ISOTHERMAL PROCESS" for constant temperature Newton used Boyle's law and by using Boyle's law he found that

B = P From eq. (1) $v = \sqrt{\frac{P}{\rho}}$

Erorr: speed of sound in air at O°C as calculated by Newton's formula is 281m/s.

But experimental value is 332 m/s.

There is an error of 16%.

LAPLACE'S CORRECTION:

Laplace corrected Newton's formula for the speed of sound. He said that during compression, temperature increases & during a rarefaction temperature decreases. Thus the temperature of the medium does not remain constant. The compression and rarefactions occur "adiabatically. Under this condition

$$\mathbf{B = \gamma P}$$

Where $\gamma = \frac{Cp}{Cv} = \frac{\text{Molar specific heat at constant. pressure}}{\text{Molar specific heat at constant. volume}} = 1.42$ **(For air)**

Thus eq. (1) becomes. $V = \sqrt{\frac{\gamma P}{\rho}} \longrightarrow (2)$

This is called Laplace correction.

SPEED OF SOUND AND TEMPERATURE:

By general gas equation

$PV = nRT$

$P = \frac{nRT}{V}$

Thus eq (2) becomes $V = \sqrt{\frac{\gamma nRT}{\rho V}}$

As ρ = density = $\frac{M}{V}$

Then = $\sqrt{\frac{\gamma nRT}{m}}$

Where R = Universal gas constant = 8.314 J/mole x K

T = absolute temperature of the gas

M = Molecular mass of gas (for air M=28.8 gm/mol)

n = number of moles

If $n = 1$ mole then

$$V = \sqrt{\frac{\gamma RT}{M}}$$

This shows that the speed of sound depends upon temperature. At 0^0C the speed of sound in air is 332m/s. At any other temperature T_K, the speed of sound in air can be obtained by multiplying this result (i.e 332 m/s) by $\sqrt{\frac{T_K}{273}}$ thus

$$V = 332\sqrt{\frac{T_K}{273}}$$

MUSICAL SOUND AND NOISE:

The sound that produces pleasant effect on ear is known as musical sound and that which produces a jarring effect is know as noise. For example the clamming of a door or the rumbling of a truck is considered as a noise whereas the sound produced by a piano or guitar is regarded as musical sound. If a graph is plotted between frequency and amplitude it will be irregular, non symmetric, random fluctuations for noise as shown in fig **(1)** for musical sound it will be regular symmetric fluctuation periodic pattern as shown in fig **(2)**.

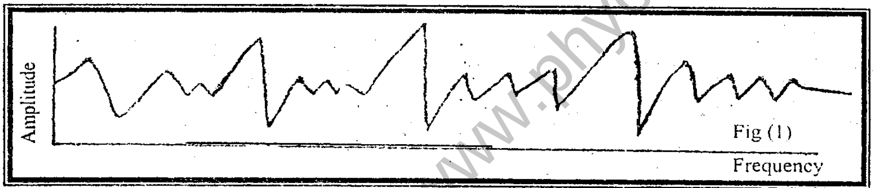


Fig (1)

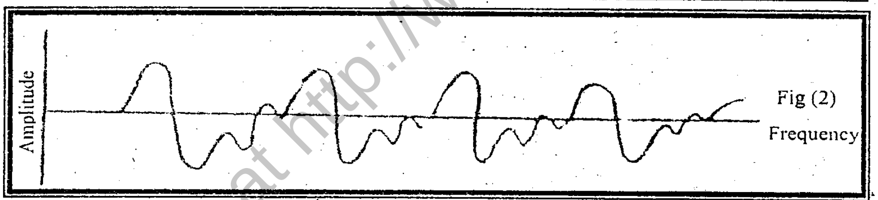


Fig (2)

CHARACTERISTICS OF MUSICAL SOUND:

Following are the characteristics of musical sound.

Intensity and loudness

Pitch or frequency

Quality or timber.

INTENSITY:

The energy of the sound waves passing every second through a unit area perpendicular to the direction of propagation of sound is called "intensity of sound." Its unit is watt/m^2.

Intensity of the sound is purely a physical quantity and can be measured accurately. It does not depend upon the auditory sensation of the ear.

LOUDNESS:

The magnitude of auditory sensation produced by the sound is called "loudness." It depends upon the intensity of the sound as well as the auditory sensation of the ear.

WEBER FECHNER LAW:

Loudness and intensity of sound are related with each other by a law called "Weber Fechner Law" according to law "the loudness is proportional to the logarithm of intensity" If "L" be the loudness and "I" be the intensity then by this law.

$$L \propto \log I$$

$$L = c \log I \longrightarrow (1)$$

Similarly if "L_0" be the loudness of faintest audible sound & "I_0" be its intensity then like equation (1) we can write

$$L_0 = c \log I_0 \longrightarrow (2)$$

INTENSITY LEVEL:

Intensity level is defined as difference in loudness of two sound.
Subtracting equation **(2)** from equation **(1)**

$$L - L_0 = c \log I - c \log I_0$$

$$L - L_0 = c \left[\log I - \log I_0\right]$$

$$L - L_0 = c \log \left(\frac{I}{I_0}\right)$$

$(L - L_0)$ is the difference of two loudness & is called as "INTENSITY LEVEL(β)"

$$\therefore \text{ Intensity level} = \beta = c \log\left(\frac{I}{I_0}\right) \longrightarrow \textbf{(3)}$$

When "c" is "a" constant depending upon the units, & "I_0" is the intensity of faintest audible sound. $I_0 = 10^{-12}\ \text{watt}/\text{m}^2$

Unit:

The unit of intensity level is "Bel" (β):
Intensity level of sound is 1 Bel if $I = 10 I_0$

$$\therefore\ 1 = c \log \frac{10 I_o}{I_o}$$

$$1 = c \log 10$$

$$1 = c \times 1$$

$$\therefore\ c = 1 \text{ bel}$$

Bel is a large unit. The smaller unit is "decibel" (dB). 1 dB $= \frac{1}{10}$ Bel

When intensity level is measured in decibel C = 10 dB

Thus intensity level in decibel is given by **(3)** = $\beta = 10 \log \left[I/I_o\right]$

This is called dB scale for intensity level of sound.

If $I = 10{,}000 I_0 = 10^4 I_0$ then

$$\beta = 10 \log \frac{10^4 I_0}{I_0}$$

$$\beta = 10 \times 4 = 40 \text{ dB}$$

POWER LAW:

A power law between loudness and intensity level of the sound is $L = K\left(\frac{I}{I_0}\right)^{0.3} \rightarrow (3)$

Where "K" is an arbitrary constant which can be evaluated by defining the unit of loudness which is "sone".

1 sone = 40 dB at 1000 Hz for this $I = 10^4 I_0$

This e.g. (3) becomes.

$$1 \text{ sone} = K\left(\frac{10^4\ I_0}{I_0}\right)^{0.3} = K(10^4)^{0.3} = K\,10^{1.2}$$

$$K = \frac{1}{10^{1.2}} = \frac{1}{16}$$

Now equation (3) becomes. $\boxed{L = \frac{1}{16}\left(\frac{I}{I_0}\right)^{0.3}}$

This is called sone scale for loudness of sound.

PITCH:

It is defined as the sensation that sound produces in the ear of a listener. It is related to the frequency of sound. The greater the frequency greater the pitch & lower the frequency lower the pitch. The sound produced by various physical instruments usually depends upon the natural resonant frequency of the source. If the frequency is greater, sound will be shrill. If frequency is less, sound wave will be grave. The sound of a sparrow is shrill because its frequency or pitch is high while that of a Lamb is grave because its frequency or pitch is low.

QUALITY:

The characteristic of sound by which two sounds of same pitch & intensity can be distinguished is called "quality or timbre".

Quality of sound depends upon the resultant waveforms. The resultant waveform of any sound is reobtained by combining the amplitudes of fundamental & the harmonics of the given sound. Figure (1) shows fundamental, second harmonics & their resultant wave form. Figure (2) shows fundamentals third harmonic & their resultant wave form.

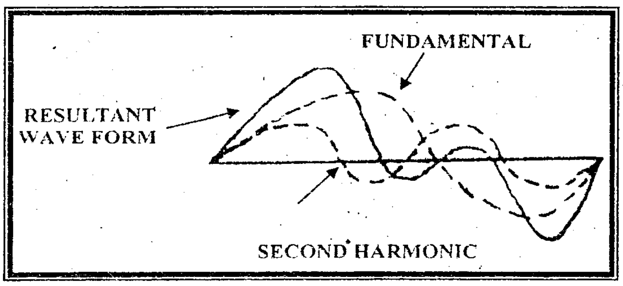


Fig (1)

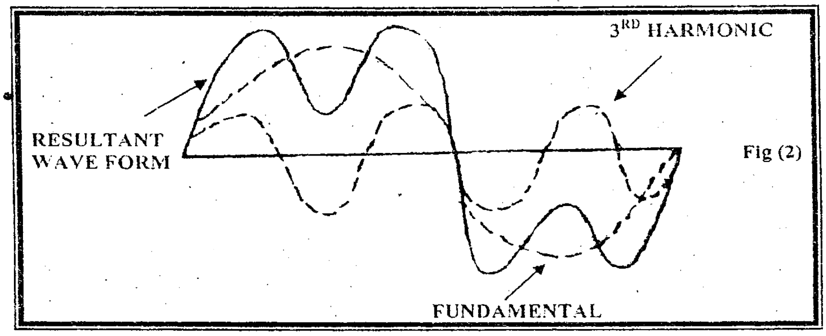


Fig (2)

PRINCIPLE OF SUPER POSITION:

According to the principle of super position when two or more waves travel, in the same medium the net displacement of the medium caused by the resultant wave at any point is equal to the algebraic sum of the displacement of all waves. i.e.

$Y = Y_1 + Y_2 + Y_3 + \text{--------} Y_n$.

BEATS:

When two sound waves of same amplitude & slightly different frequencies reach the ear, we do not hear two different sounds but a single sound which rises & falls in intensity periodically. This phenomenon is known as "BEATS".

Let us suppose that sound waves are produced in a big hall these sound waves spread out, scatter and strike the surfaces of the walls, the floor & the ceiling of the hall. Some of the sound energy is absorbed by these surfaces & some is reflected and travel back and when two sound waves are in phase, constructive interference takes place & loud sound is heard. When compression of one falls on rarefaction of the other, destructive interference takes place therefore no sound is heard.

One rise & one fall and then again rise of sound is known as one beat. The number of beats produced per second is called "beat frequency". It is equal to the difference in frequency of the two waves. It is denoted by "υ_b" if the frequency of the two sound waves are ν_1 & ν_2 then $\upsilon_b = \upsilon_1 - \upsilon_2$ the maximum beat frequency that the human ear can detect is about 7. When the beat frequency is greater then 7.we can not hear them clearly.

USES:

The phenomenon of beats is used:-

1. To adjust the frequency of various musical instruments
2. To find unknown frequency by tuning variable known frequency.

DERIVATION:

Consider two waves of equal amplitude "A_0" travelling through a medium in the same direction having slightly different frequencies. If "ν_1" be the frequency of one wave then its instantaneous displacement "Y_1" at any instant "t" is given by.

$$\therefore \quad Y_1 = A_0 \cos 2\pi \upsilon_1 t$$

Similarly if "ν_2" be the frequency of the other wave then its displacement "Y_2" at the instant "t" is given by. $Y_2 = A_0 \cos 2\pi\upsilon_2 t$

If "Y" be the resultant displacement caused by the two waves then.

$$Y = Y_1 + Y_2$$

$$Y = A_0 \cos 2\pi \nu_1 t + A_0 \cos 2\pi \nu_2 t$$

$$Y = A_0 (\cos 2\pi \nu_1 t + \cos 2\pi \nu_2 t)$$

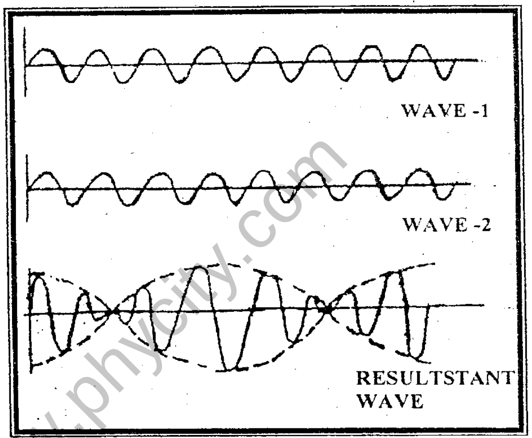
$$[\cos\alpha + \cos\beta = 2 \cos \frac{\alpha-\beta}{2} \cos\frac{\alpha+\beta}{2}]$$

$$Y = A_0[2\cos(\frac{2\pi\nu_1 t - 2\pi\nu_2 t}{2})\cos(\frac{2\pi\nu_1 t + 2\pi\nu_2 t}{2})]$$

$$Y = 2A_0 \cos 2\pi (\frac{\nu_1 - \nu_2}{2})t \cos 2\pi (\frac{\nu_1 + \nu_2}{2})t$$

In the above equation $\left(\frac{\nu_1 + \nu_2}{2}\right)$ is the " effective frequency" & $\left[2A_0\cos2\pi\left(\frac{\nu_1 - \nu_2}{2}\right)t\right]$ is the "amplitude" "A" i.e

$$A = 2A_0 \cos2\pi\left(\frac{\nu_1 - \nu_2}{2}\right)t$$



The amplitude varies with frequency $\frac{\nu_1 - \nu_2}{2}$ The amplitude will be maximum when $\cos 2\pi \left(\frac{\nu_1 - \nu_2}{2}\right)t$ have its maximum value

i.e $\cos 2\pi \left(\frac{\nu_1 - \nu_2}{2}\right) t = \pm 1$

It means that variation of frequency $\frac{\upsilon_1 - \upsilon_2}{2}$ occurs twice in on second, so beat frequency is given by.

$$\upsilon_b = 2\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)$$

$$\upsilon_b = \upsilon_1 - \upsilon_2$$

This shows that the number of beats per second will be equal to the difference of frequencies of two second waves.

ACOUSTICS:

The study of production and properties of sound is called "acoustics". The term "acoustics" is also used to describe the way in which sound is reproduced in practical situations, the reproduced sound may be indistinct & confusing and may also be clear & distinct.

Recombine to form undesirable echoes. These echoes interfere with the original sound waves coming directly from source. This result in indistinct and unintelligible sound and hence give rise to bad Acoustics.

CONDITIONS FOR GOOD ACOUSTICS:

1. The deciphered of each syllable should be small so that the succeeding syllable can be heard clearly.
2. The loudness of each syllable should be sufficiently large.
3. Echoes should be just sufficient to maintain the continuity of sound.
4. The hall should have some open windows. Sound absorbing soft porous materials like cloth, cork, asbestos etc. Or heavy curtains should be placed in the hall at various places so as to avoid much reflection.
5. For good acoustics of a hall reverberation should not be too small otherwise that will be away instantaneously & will give dead effect to the hall.

DOPPLER'S EFFECT:

When a source of sound or a listener, or both are in motion relative to the medium (air), the frequency & hence the pitch of the sound, as heard by the listener will seem to be changed this phenomena referred to as DOPPLER'S EFFECT.

Doppler's effect can be explained by considering three possibilities.

1. When the listener is moving & the source is at rest
2. When the source is moving and the listener is at rest
3. When both the source and listeners are moving

CASE-1 (A)

WHEN THE LISTENER IS MOVING TOWARDS A STATIONARY SOURCE

Consider a source emitting sound waves of frequency "υ". If "V" be the speed of sound waves then the wavelength "λ" of the waves is given by.

$$\lambda = \frac{V}{\nu} \rightarrow (1)$$

If listener is at rest then the waves will approach him with speed "V" but if listener is moving towards the source with speed "V_0" the sound waves approach him with a speed of $V + V_0$ thus he receives more waves per unit time as compared to that when he was at rest i.e. frequency increases. Thus if υ' be the changed frequency then it is given by.

$$\upsilon' = \frac{V + V_0}{\lambda}$$

Putting the value of "λ" from e.q. (1) $\upsilon' = \dfrac{V + V_0}{\frac{V}{\upsilon}}$

$$\boxed{\nu' = \left(\frac{V + V_0}{V}\right)\nu} \longrightarrow (2)$$

This shows that $\upsilon' > \upsilon$

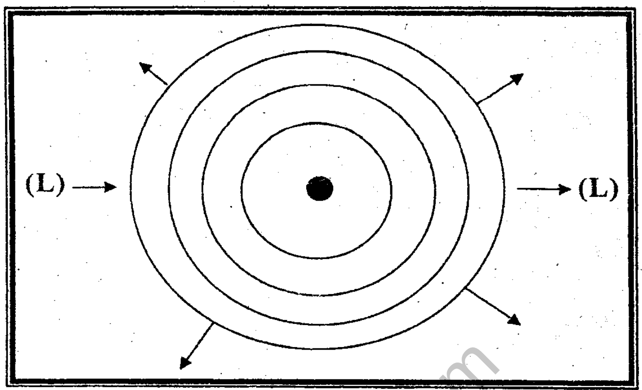
$$\nu' = \left(1+\frac{V_0}{V}\right)\nu$$

$$\nu' = \nu + \left(\frac{V_0}{V}\right)\nu$$

$$\nu' - \nu = \frac{V_0}{V}\nu$$

$$\nu - \nu = \Delta\nu$$

$$\boxed{\Delta\nu = \frac{V_0}{V}\nu}$$



This gives change in frequency of sound heard by the listener.

CASE-1 (B)

WHEN THE LISTENER MOVES AWAY FROM THE Stationary SOURCE

When the listener moves away from the stationary source with speed "V_0" the sound wave approach him with speed $V - V_0$ thus he receives less sound waves per unit time as compared to that when he was at rest. i.e frequency decreases if ν^1 be the changed frequency then it is given by.

$$\nu' = \frac{V - V_0}{\lambda}$$

Putting the value of "λ" from equation (1)

$$\nu' = \frac{V - V_0}{\frac{V}{\nu}}$$

$$\boxed{\nu' = \left(\frac{V - V_0}{V}\right)\nu} \longrightarrow (3)$$

$$\nu' = (1 - \frac{V_0}{V})\nu$$

$$\nu' = \nu - \frac{V_0}{V}\nu$$

$$\nu' - \nu = -\frac{V_0}{V}\nu$$

$$\nu' - \nu = \Delta\nu$$

$$\boxed{\Delta\upsilon = -\frac{V_0}{V}\upsilon}$$

This gives change in frequency of sound heard by the listener.

CASE-2 (A)

SOURCE MOVES TOWARDS A STATIONARY LISTENER

If the source moves towards a stationary listener then wavelength received by the listener will decrease by $\frac{V_s}{V}$ if λ' be the wavelength (decreased) of the waves received by the listener then it is given by.

$$\lambda' = \frac{V}{\nu} - \frac{V_s}{\nu} = \frac{V - V_s}{\nu} \rightarrow (4)$$

As the wavelength decreases, frequency increases. If "υ'" be the apparent frequency then it is given by $\upsilon' = \frac{V}{\lambda'}$ putting the value of λ' from eq (4)

$$\upsilon' = \frac{V}{\frac{V - V_s}{\nu}} \qquad \boxed{\upsilon' = \left(\frac{V}{V - V_s}\right)\nu} \longrightarrow (5)$$

CASE-2 (B):

SOURCE MOVES AWAY FROM THE LISTENER

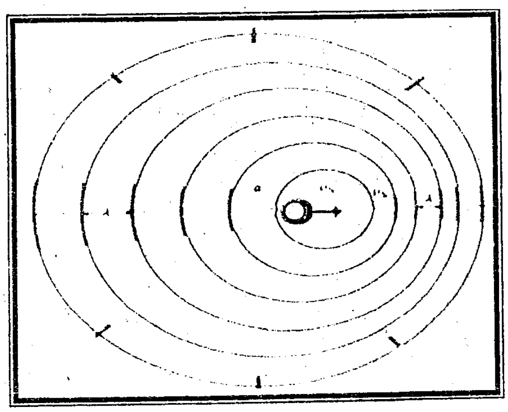
If the source moves away from the listener with velocity V_S, then the wavelength of the waves received by the listener will increase by. $\frac{V_s}{V}$ if λ be the increased wavelength then it is given by. $\lambda' = \frac{V}{\nu} + \frac{V_s}{\nu} = \frac{V + V_s}{\nu} \rightarrow (6)$

As the wavelength increases, frequency decreases. If "υ'" be the apparent frequency then it is given by. $\upsilon' = \frac{V}{\lambda'}$

Putting the value of λ' from e.q. (6)

$$\upsilon' = \frac{V}{\frac{V + V_s}{\nu}} \qquad (L)$$

$$\therefore \boxed{\upsilon' = \left(\frac{V}{V + V_s}\right)\nu} \longrightarrow (7)$$



CASE-3(A):

SOURCE & LISTENER ARE MOVING TOWARDS EACH OTHER

If the listener is moving with velocity v_0 towards the source of sound then frequency changes from ν to υ' and is given by. $\upsilon' = \left(\frac{V + V_0}{V}\right)\nu \rightarrow (8)$

If the source of sound is also moving with velocity Vs towards two listeners then frequency further changes υ' to υ'' and is given by.

$$\nu'' = \left(\frac{V}{V - Vs}\right)\nu' \rightarrow (9)$$

Putting the value of υ' from e.q. (8) in e.g. (9)

$$\nu'' = \left(\frac{V}{V - V_s}\right)\left(\frac{V + V_0}{V}\right)\nu$$

$$\therefore \quad \boxed{\nu'' = \left(\frac{V + V_0}{V - Vs}\right)\nu}$$

CASE-3(B):

SOURCE & LISTENER ARE MOVING AWAY FROM EACH OTHER

If the listener is moving with velocity V_0 away from the source of sound then frequency changes from ν to υ' & is given by.

$$\upsilon' = \left(\frac{V - V_0}{V}\right)\nu \rightarrow (10)$$

If the source of sound is also moving with velocity Vs away from the listener then frequency further changes.

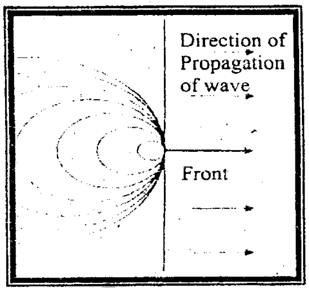
From υ' to υ'' **& is given by.** $\upsilon'' = \left(\frac{V}{V + V_s}\right)\nu \rightarrow (11)$

Putting the value of ν^1 from e.q (10) in e.g (11)

$$\therefore \quad \boxed{\nu'' = \left(\frac{V - V_o}{V + V_s}\right)\nu}$$

SPECIAL CUSSES:-
WHEN VELOCITY OF SOURCE IS EQUAL TO THE VELOCITY OF SOUND:

When velocity of source is equal to the velocity of sound i.e $V_S = V$ then the source & sound waves move intact with each other and high pressure is formed. As the process continues the pile region extends further and further from the source in the direction perpendicular to its motion due to which a thick wave front or sheet is formed as shown in the figure. This sheet exerts tremendous pressure on the source.



WHEN VELOCITY OF SOURCE IS GREATER THAN THE VELOCITY OF SOUND:

When velocity of sound exceeds the wave speed i.e $V_s > V$ the source runs ahead of outgoing waves and in such a case the waves pile up and produce the shape of a cone with the moving object at its apex, as shown in figure.

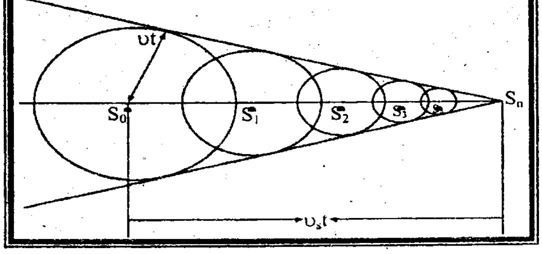
To explain this situation consider the source at S_o when t = 0. after time "t" the wave front will cover the distance "vt" while the source will cover a distance "V_st" at this instant the source is at "S_n" & the waves just beginning to be generated at this point have wave of zero radius.

If the source were at "S_1" the mid point of "S_o" & "S_n" then after time $\frac{T}{2}$ it will cover distance $\frac{V_st}{2}$ while the wave point will cover distance $\frac{Vt}{2}$ if the source were at S_2 the mid point of S_1 & S_n then after time t/4 it will cover distance $\frac{V_st}{4}$ while the wave front will cover distance $\frac{Vt}{4}$ if the lines are drawn to join S_n & the wave fronts centered on S_0, S_1 & S_2, a cone shaped wave front is formed as shown in the figure. If θ be the angle between the direction of motion of source and the cone shaped wave front then.

$$\text{Sin } \theta = \frac{Vt}{V_st} = \frac{V}{V_s}$$

In aerodynamics the ratio $\frac{V_s}{V}$ is referred to as the "Mach number".

Along the cone shaped wave front, that is produced by supersonic object, the air is highly compressed. This moving sheet of high pressure air is called SHOCK WAVE. This shock wave will result in loud explosion of sonic boom which can damage buildings if the source is near to it.



APPLICATION OF THE DOPPLER EFFECT:

(i) Tracking a Satellite:

The Doppler effect provides a method for tracking a satellite. Suppose the satellite is emitting a radio signal (i.e an electromagnetic wave) of constant frequency fs. The frequency f_L of the signal received on the earth decreases as the satellite is passing. The received signal is combined with a constant signal generated in the receiver to produce beat. The beat frequency produces an audible note whose pitch changes as the satellite passes overhead.

(ii) Radar gun:

The Doppler effect is used in measuring the speed of automobile by traffic police. A "radar gun" is fixed on police car. An electromagnetic signal is emitted by the radar gun in the direction of the automobile whose speed is to be checked. The wave is reflected from the moving automobile and received back. The reflected wave is then mixed with the locally generated original signal and beats are produced. The frequency shift is measured using beats and hence the speed of the automobile is determined.

(iii) Radars:

Radars (Radio detection and ranging) are commonly used for civil and military purposes at civilian airports and military air-bases respectively, to detect the presence of any aircraft (for or friend) in the airspace by evaluating the range. The Doppler Radar is based on the principle of the Doppler Effect, is extensively used in the detection of aircraft speed and direction.

(iv) VOR:

VOR (very high frequency Omni Range) is a guiding system usually installed at the airports to guide the incoming aircrafts toward to location of the airport. Nowadays big and modern airports for example Quiad-e-Azam International Airport, Islamabad Airport, are equipped with Doppler VOR whose principle is once again based on the Doppler effect and provides air effective and better guiding system to the aircraft. The electromagnetic signal used here has frequency in the VHF range (30MHz – 300MHz).

(v) Homosonde:

X – rays have been a major diagnostic tool of medicine, but recent years have seen the emergence of an alternate tool which is inherently safer then x – rays: ultra sound. Unlike x – rays, ultra sound radiations have not been reported to damaged living cells so far. Measurements of internal reflections of ultra sound have facilitated the diagnosis of breast cancer and taking the heartbeats of fetuses and newborns.

An ultrasonic instrument called Homosonde uses the Doppler effect of ultrasonic waves reflected from moving masses in the patient. The device is very sensitive for detecting blood flow and measures faint heartbeats in a very noisy environment where the use of stethoscope may not be reliable.

(vi) Sonar:

In similar manner, we can detect the motion of an objects under-water (for example a submarine) by employing sonar (ultrasonic waves) based on effective use of Doppler effect.

(vii) Red Shift:

The Doppler effect for light is important in astronomy. Spectral analysis of light emitted by the elements in distant stars shows shift in the wavelength compared to light from the same element on earth. These shifts can be interpreted as Doppler shifts due to the motion of the stars. The shift is nearly always toward the longer wavelength, or red end of the spectrum, and is therefore called the red shift. Such observations have provided practically all the evidence for "expanding universe" cosmological theories, which represents the universe as having evolved from a great explosion several billion years ago in a relatively small region of space.