MOTION 3

Motion:

If a body changes its position with respect to surrounding then the body is said to be motion.

Rest:

If a body does not change its position with respect to its surrounding, then the body is said to be at rest.

DISPLACEMENT:

Definition:

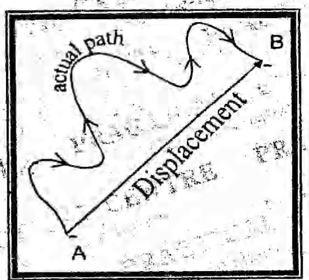
Shortest distance between two points covered along a straight line is called displacement.

It is a vector quantity and its direction is always from initial point to final point.

Explanation:

Consider a body which moves from a point A to another point B as shown in fig. The straight line $\vec{AB}$ is the shortest distance from A to B, is called displacement.

VELOCITY:



Definition:

The time rate of change of displacement is called velocity

Direction:

It is a vector quantity and its direction is along the direction of displacement.

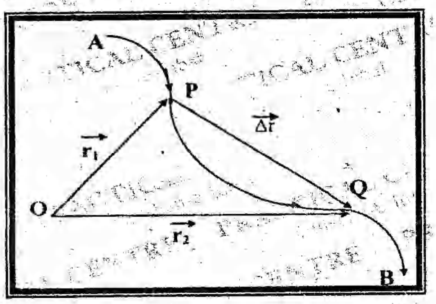
Formula:

$$\text{Average Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Unit: Its S.I.Unit is m/s.

Dimensions: The dimensions of velocity are LT^{-1}.

Explanation:



Consider a body in motion. The path of its motion is represented from A to B.

Let at time t_1, body be at point P. Its position with respect to origin O is represented by vector $\vec{r_1}$. At certain later time t_2, the body reaches to Q, such that its position with respect to origin O is represented by vector $\vec{r_2}$.

Time taken by the body to move from P to Q is $t_2 - t_1 = \Delta t$.

Change of position of the body is $\vec{r_2} - \vec{r_1} = \vec{\Delta r}$ by the definition of velocity. The average velocity of the body during interval Δt is given by:

$$\vec{V}_{av} = \frac{\vec{\Delta r}}{\Delta t}$$

Uniform Velocity:

When a body covers equal displacement in equal intervals of time, then the body is said to move with uniform velocity.

Variable Velocity:

When a body covers different displacement in equal intervals of time then the body is said to move with variable velocity.

Instantaneous Velocity:

The Velocity of a body at any instant of time is called instantaneous velocity.

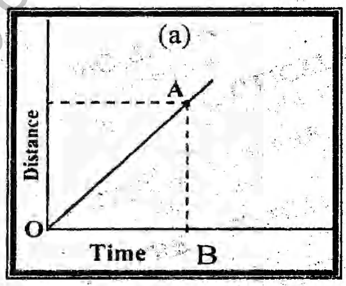
$$\vec{V}_{ins} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t}$$

Where $\lim_{\Delta t \to 0}$ shows that the time is very small that both $\overrightarrow{\Delta r}$ and Δt approach to zero.

VELOCITY FROM DISTANCE–TIME GRAPH:

For Uniform Velocity:

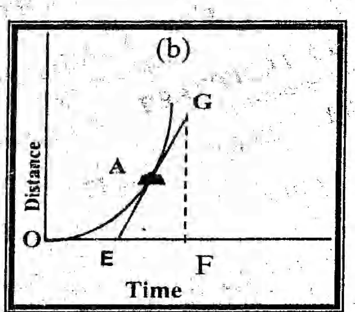
When a body moves with uniform velocity, it travels equal distances in equal intervals of time. The graph between distance and time will be a straight line, as shown in figure (a). If we take any point A on the graph and draw a perpendicular AB on the time axis, it is clear that AB represents the distance travelled and OB represents the time taken.



$$\therefore \quad \text{Velocity} = \frac{\text{Distance}}{\text{Time}} = \frac{AB}{OB}$$

For Variable Velocity:

If a body is moving with variable velocity, then graph between distance and time will not be a straight line. If the graph is a curve, as shown in figure (b) then velocity of the body at any point A can be found by drawing a tangent EG to the curve at point A. Now draw a perpendicular GF on time axis. The velocity of the body at A is given by:



$$\text{Velocity at A} = \frac{GF}{EF}$$

ACCELERATION:

Definition:

Time rate of change of velocity of a body is called its ***"Accelaration"***. Acceleration is a vector quantity and its direction is same as that of the change of velocity. i.e.

$$\text{Average Acceleration} = \frac{\text{Change of velocity}}{\text{Time}}$$

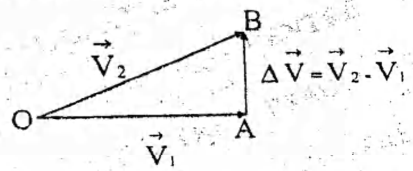
Unit: Its S.I.Unit is m/s^2.

Dimensions: The dimensions of acceleration are LT^{-2}.

Explanation:

Consider a body in motion. Let $\vec{V}_1$ be its velocity at an instant t_1 and $\vec{V}_2$ be its velocity at a later instant t_2. The average acceleration of the body during this interval is given by:

$$\vec{a}_{av} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \frac{\Delta\vec{V}}{\Delta t}$$



Uniform Acceleration:

Constant rate of change of velocity of a body is called uniform acceleration.

Instantaneous Acceleration:

The acceleration of a body at any instant of time is called instantaneous acceleration.

$$\vec{a}_{ins} = \underset{\Delta t \to 0}{\text{Limit}} \frac{\overrightarrow{\Delta V}}{\Delta t}$$

Where $\underset{\Delta t \to 0}{\text{Limit}}$ show that the time Δt is very small that both $\overrightarrow{\Delta V}$ and Δt approach to zero.

Positive Acceleration:

If the velocity of a body is increasing, then its acceleration is positive.

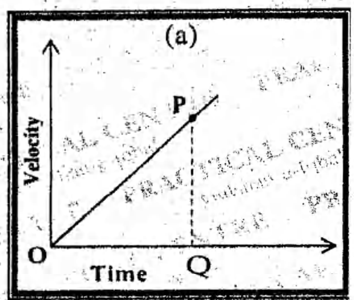
Negative Acceleration:

If the velocity of a body is decreasing then its acceleration is negative. The negative acceleration is also called retardation or deceleration.

ACCELERATION FROM VELOCITY–TIME GRAPH:

For Uniform Acceleration:

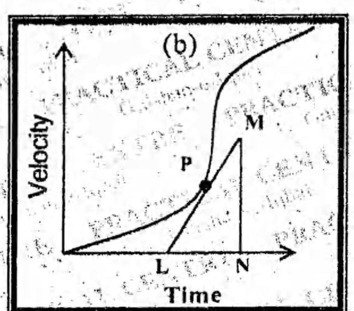
When a body moves with uniform acceleration, the graph between its velocity and time will be a straight line, as shown in figure (a). From this figure acceleration of the body is given by:



$$\text{Acceleration} = \frac{PQ}{OQ}$$

For Variable Acceleration:

If the acceleration of the body is not uniform then graph will not be a straight line, if it is curve as shown in figure (b) then acceleration at any point P is given by:



$$\text{Acceleration at P} = \frac{MN}{LN}$$

LAWS OF MOTION:

Issac Newton Studied motion of bodies and formulated the following laws.

Newton's first laws of motion:

Statement:

A body remains at rest or continues to move with uniform velocity in a straight line unless acted upon by an external unbalanced force.

Explanation:

This law consists of two parts. The fi st part states that a body cannot change its state of rest or of uniform motion in a straight line itself unless it is acted upon by some unbalanced force to change its state.

The second part of this law gives us the qualitative definition of the net force, which is stated as follows:-

Force is an agency when applied on a body, changes or tends to change its state of rest or of uniform motion.

The stated law is also known as, "law of inertia", because it points towards a very important property of matter which is called inertia.

INERTIA:

Inertia is that property of matter by virtue of which it tries to remain in its existing state. If two bodies of different masses are moving with the same velocity under identical conditions, it will be more difficult to stop or change the motion of the body of larger mass because this body has more inertia than the body having lesser mass. Thus the mass of a body is a direct measure of its inertia.

Newton's second law of motion:

Statement:

If a certain unbalanced force acts upon a body, it will accelerate the body in the direction of force. The magnitude of the acceleration is directly proportional to the magnitude of the unbalanced force.

Explanation:

Consider a body of mass "m" on which an unbalanced force "F" is acting. If "a" be the acceleration produced in the body then by the law:

$$a \propto F$$

OR $F \propto a \Rightarrow \quad F = ma$ ----------------- (1)

Where "m" is the constant of proportionality and it is the mass of body on which force is acting.

Equation (1) can also be written as

$$a = \frac{F}{m}$$

If "F" is constant then

$$a \propto \frac{1}{m}$$

Thus we can also write that acceleration is inversely proportional to the mass of the body on which constant force is acting. The second law of motion can also be written in vector form as:

$$\vec{F} = m\vec{a}$$

So, "Force is the product of mass and acceleration"

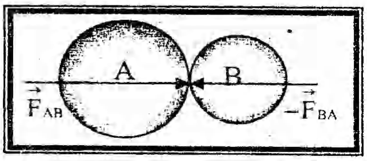
Newton's third law of motion:

Statement:

"To every action there is always an equal and opposite reaction."

Explanation:

Consider two bodies A & B which collide with each other. If $\vec{F}_{AB}$ be the force exerted by the body A on body B then body B will also exert a force $\vec{F}_{BA}$ on body A. These two forces will be equal in magnitude but opposite in direction.



$$\therefore \quad |\vec{F}_{AB}| = |\vec{F}_{BA}|$$

In the vector form

$$\vec{F}_{AB} = -\vec{F}_{B}$$

The force exerted by A on B is called ***"Action"*** and the force exerted by B on A is called ***"Reaction"***. These forces lie along the line joining the c tres of the bodies.

TENSION IN A STRING:

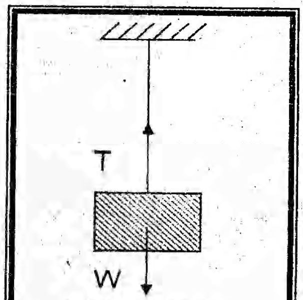
Defi..ition:

The force applied on a body through a string is called ***"Tension"***.

Explanation:

When a body of weight ***"W"*** is kept suspended by a string, the weight of the body pulls the string downwards while the string pulls the body upwards with an equal force. This force i called ***Tension in the String "T"***.

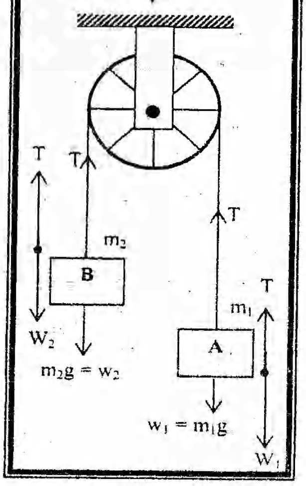
1- If the body is at rest or moves with uniform velocity then
$$T = W$$
2- If the body accelerates upward then,
$$T > W$$
3- If the body accelerates downward then,
$$T < W$$



MOTION OF BODIES CONNECTED BY A STRING:

Case-1: When both the bodies move vertically:

Consider two bodies A & B of masses m_1 & m_2 connected by an inextensible string which passes over a frictionless pulley, as shown in the figure. If $m_1 > m_2$, then the body A will accelerate down with an acceleration, ***"a"***, and the body B will move up with the same acceleration. Let "T" be the tension in the string.



Consider the downward motion of body A:

Two forces are acting on the body A.

(i) Force of gravity m_1g acting in the downward direction.

(ii) Tension "T" in the string in upward direction.

Since body A is moving downward.

Then $m_1g > T$

Net force acting on body A.

Resultant force = Downward force – Upward force

$F_1 = m_1g - T$

But as we know from Newton's 2nd Law of motion.

$F_1 = m_1a$

OR $\mathbf{m_1a = m_1g - T}$ ----------------(1)

Consider upward motion of body B:

Here also two forces are acting on the body B.

(i) Force of gravity m_2g, acting in the downward direction.

(ii) Tension "T" in the upward direction.

Since body B is moving upward.

$\therefore \quad T > m_2g$

Net force acting on body B

Resultant force = Upward force – Downward force

$\therefore \quad F_2 = T - m_2g$

From Newton's 2nd Law of motion, $F_2 = m_2a$

$\therefore \quad \mathbf{m_2a = T - m_2g}$ ----------------(2)

For the formula of acceleration.

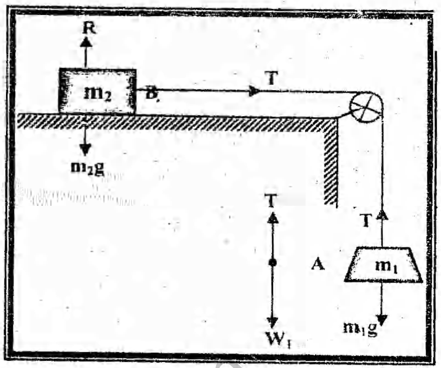
add eq. (1) & eq. (2)

$$m_1a = m_1g - T$$
$$m_2a = T - m_2g$$
$$m_1a + m_2a = m_1g - \cancel{T} + \cancel{T} - m_2g$$
$$m_1a + m_2a = m_1g - m_2g$$
$$a(m_1 + m_2) = (m_1 - m_2)g$$
$$\boxed{a = \frac{(m_1 - m_2)\,g}{m_1 + m_2}}$$

Now dividing equation (1) by equation (2), for the formula of tension.

$$\frac{m_1\cancel{a}}{m_2\cancel{a}} = \frac{m_1g - T}{T - m_2g}$$
$$\frac{m_1}{m_2} = \frac{m_1g - T}{T - m_2g}$$
$$m_1(T - m_2g) = (m_1g - T)m_2$$
$$m_1T - m_1m_2g = m_1m_2g - m_2T$$
$$m_1T + m_2T = m_1m_2g + m_1m_2g$$
$$T(m_1 + m_2) = 2m_1m_2g$$
$$\boxed{T = \frac{2m_1m_2g}{m_1 + m_2}}$$

Case-2: When one body moves vertically and the other moves on a smooth horizontal surface:



Consider two bodies A & B of masses m_1 & m_2 respectively, attached to the ends of an inextensible string which passes over a frictionless pulley, as shown in the figure.

The body "A" moves vertically downward with an acceleration "a", the body "B" moves on a smooth horizontal surface towards the pulley with the same acceleration. Let the tension in the string be "T".

Consider downward motion of body A:

There are two forces acting on it.

(i) Force of gravity m_1g acting in the direction downward

(ii) Tension "T" in the string in upward direction.

Since the body A is moving downward, so

So, $m_1g > T$

Therefore, net force will be

Resultant force = Downward force – Upward force

$F_1 = m_1g - T$

But according to Newton's 2nd Law of motion

$F_1 = m_1a$

$\mathbf{m_1a = m_1g - T}$ ----------------------(1)

Consider horizontal motion of body B towards the pulley:

There are three forces acting on it.

(i) Force of gravity m_2g which acts vertically downward.

(ii) The normal reaction "R" of the surface on the body which acts vertically upward.

(iii) The tension "T" in the string which is acting horizontally towards the pulley.

Since there is no motion of body "B" in the vertical direction.

So, along y – axis

$\Sigma Fy = 0$

$R - W_2 = 0 \Rightarrow R = W_2 \Rightarrow \boxed{R = m_2g}$

If we neglect the force of friction, the net horizontal force on the body is "T" which pulls it towards the pulley. So along x – axis

$F_2 = T$

Since the body is moving with acceleration "a", therefore by Newton's 2nd Law of motion.

$F_2 = m_2a$

$m_2a = T$ --------------(2)

Adding equations (1) & (2) For the formula for acceleration.

$$m_1a = m_1g - T$$
$$m_2a = T$$
$$m_1a + m_2a = m_1g - \cancel{T} + \cancel{T}$$
$$(m_1 + m_2)a = m_1g$$
$$\boxed{a = \frac{m_1g}{m_1 + m_2}}$$

Putting the value of "a" in eq (2), to get formula for tension.

$$T = m_2\left[\frac{m_1g}{m_1 + m_2}\right]$$

$$\boxed{T = \frac{m_1m_2g}{m_1 + m_2}}$$

LINEAR MOMENTUM:

Definition:-

"The product of mass and linear velocity of the body is called its linear momentum".

Formula:

If "m" be the mass and $\vec{V}$ be the velocity of a body then its momentum $\vec{p}$ is given by:

$$\vec{p} = m\vec{v}$$

It is a vector quantity and its direction is the same as that of the velocity.

Unit:

The S.I. Unit of momentum is Kg x $\frac{m}{s}$ or Ns.

Dimension: Dimensions of linear momentum are MLT^{-1}.

Force acting on a body is equal to the rate of change of linear momentum of the body:

Proof:-

Consider a body of mass "m" moving with a initial velocity "V_i". Let a force "$\vec{F}$" acts on the body for time "Δt" and after which its final velocity becomes "V_f".

Initial linear momentum $= P_i = mv_i$

Final linear momentum $= P_f = mv_f$

Change in momentum $= \Delta P = P_f - P_i$

According to Newton's Second Law of motion

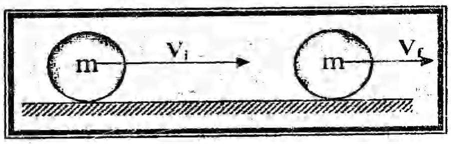
$$F = ma$$

$$F = m\left[\frac{v_f - v_i}{\Delta t}\right]$$

$$\because v_f = v_i + at$$

$$v_f - v_i = at$$

$$a = \frac{v_f - v_i}{t}$$



$$F = \frac{mv_f - mv_i}{\Delta t}$$

$$F = \frac{P_f - P_i}{\Delta t}$$

$$F = \frac{\Delta P}{\Delta t}$$

Force = The rate of change of linear momentum.

Impulsive force:

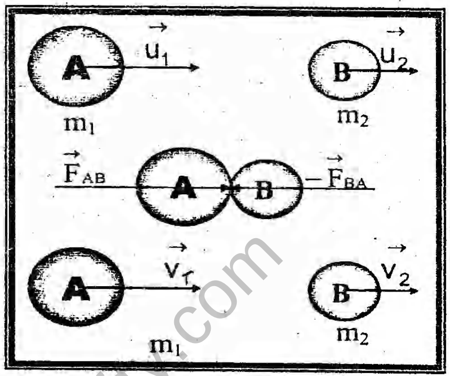
The force which acts for a very short time is called "Impulsive Force".

Law of conservation of momentum:

Statement:

"The total linear momentum of an isolated system (In the absence of external force) remains constant".

Explanation /proof:



Consider an isolated system of only two interacting spherical bodies A & B of masses m_1 & m_2 moving with velocities $\vec{u}_1$ & $\vec{u}_2$ respectively in a straight line passing through their centres along the same direction. Let the two bodies collide with each other and after which they move with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively, as shown in the figure.

The total momentum of the system before collision = $m_1u_1 + m_2u_2$ and the total momentum of the system after collision = $m_1v_1 + m_2v_2$ when the two bodies collide with each other, they come in contact for a time interval "t". During this time let the average force (F_{AB}) exerted by the body A on body B is equal to the rate of change of momentum of body B.

$$F_{AB} = \frac{m_2v_2 - m_2u_2}{t} \qquad \text{------------------(1)}$$

According to Newton's third law of motion, the body "B" will also exert a force (F_{BA}) on the body A is equal to rate of change of momentum of body A.

$$F_{BA} = \frac{m_1v_1 - m_1u_1}{t} \qquad \text{------------------(2)}$$

As the forces are equal and opposite

$$F_{AB} = -F_{BA}$$

$$\left(\frac{m_2v_2 - m_2u_2}{t}\right) = -\left(\frac{m_1v_1 - m_1u_1}{t}\right)$$

$$m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$$

$$-m_1u_1 - m_2u_2 = -m_1v_1 - m_2v_2$$

$$-(m_1u_1 + m_2u_2) = -(m_1v_1 + m_2v_2)$$

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

Total momentum before collision = Total momentum after collision

Elastic collision:

If two bodies in a system collide together, such that the total momentum and the total kinetic energy of the system remain the same before and after collision such a collision is known as ***Elastic Collision.***

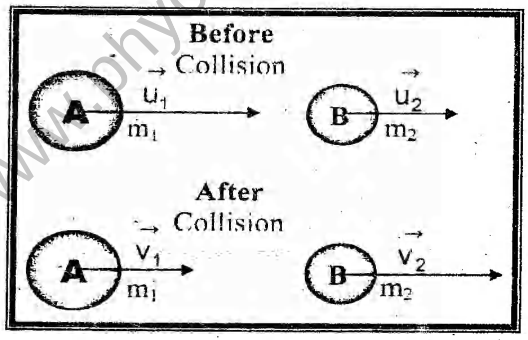
Inelastic collision:

If two bodies in a system collide together, such that the total momentum of the system remains the same before and after collision, but total kinetic energy of the system does not remain the same before and after collision, such a collision is known as an ***Inelastic Collision.***

Elastic collision in one dimension:

Consider two uniform, rigid and non rotating spheres A & B of masses m_1 & m_2 moving initially along the line joining their centers with velocities $\vec{u}_1$ & $\vec{u}_2$ respectively. If $u_1 > u_2$ then body A will collide with body B. Let the bodies move with velocities $\vec{v}_1$ & $\vec{v}_2$ respectively in the same line & direction, as shown in figure, after the elastic collision.

Now Linear momentum of the system before collision $= m_1u_1 + m_2u_2$.



Linear Momentum of the system after collision $= m_1v_1 + m_2v_2$.

By the law of conservation of momentum.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ ------- (1)}$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

OR $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ -------- (2)}$$

Kinetic energy of the system before collision.

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

Kinetic energy of the system after collision.

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

As the collision is elastic

$$\therefore \quad \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

OR $$\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$

OR $$\frac{1}{2}m_1(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ --------------- (3)}$$

Dividing equation **(3)** by equation **(2)**.

$$\frac{m_1(u_1 - v_1)(u_1 + v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 - u_2)(v_2 + u_2)}{m_2(v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2 \text{ --------------- (4)}$$

Determination of V_1

Equation (4) can be written as:

$$v_2 = u_1 + v_1 - u_2$$

Putting this value of "v_2" in equation **(1).**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2[u_1 + v_1 - u_2]$$

OR $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$

OR $m_1u_1 + m_2u_2 + m_2u_2 - m_2u_1 = m_1v_1 + m_2v_1$

OR $m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

OR $(m_1 - m_2)\, u_1 + 2m_2u_2 = (m_1 + m_2)\, v_1$

OR $V_1 = \dfrac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$

$$\mathbf{v_1 = \frac{(m_1 - m_2)\, u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}} \quad \text{------------------ (5)}$$

Determination of V_2

Equation (4) can be written as

$$v_1 = u_2 + v_2 - u_1$$

Putting this value of "v_1" in equation (1).

$$m_1u_1 + m_2u_2 = m_1[u_2 + v_2 - u_1] + m_2v_2$$

$$m_1u_1 + m_2u_2 = m_1u_2 + m_1v_2 - m_1u_1 + m_2v_2$$

$$m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1 = m_1v_2 + m_2v_2$$

$$2m_1u_1 + m_2u_2 - m_1u_2 = m_1v_2 + m_2v_2$$

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2$$

$$v_2 = \frac{2m_1u_1 + (m_2 - m_1)u_2}{m_1 + m_2}$$

$$\mathbf{v_2 = \frac{2m_1\, u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}} \quad \text{------------------ (6)}$$

SPECIAL CASES:

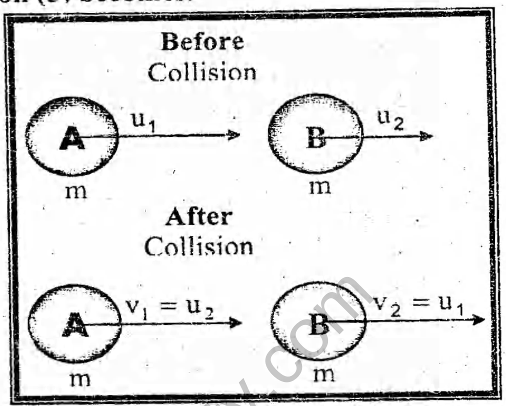
Case –1: When a body collides with second body of the same mass.

$m_1 = m_2 = m$ (say) then equation (5) becomes.

$$v_1 = \frac{(m-m)u_1}{m+m} + \frac{2mu_2}{m+m}$$

$$v_1 = 0 + \frac{2mu_2}{2m}$$

$$\boxed{v_1 = u_2}$$



Similarly equation (6) becomes:

$$v_2 = \frac{2mu_1}{m+m} + \frac{(m-m)u_2}{m+m}$$

$$v_2 = \frac{2mu_1}{2m} + 0$$

$$\boxed{v_2 = u_1}$$

Result: This shows that the bodies interchange their velocities after collision.

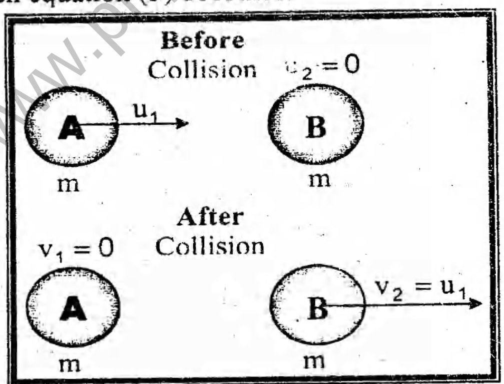
Case –2: When a body collides with second body of the same mass at rest.

$m_1 = m_2 = m$ (say) and $u_2 = 0$ then equation (5) becomes.

$$v_1 = \frac{(m-m)u_1}{(m+m)} + \frac{2m \times 0}{(m+m)}$$

$$v_1 = 0 + 0$$

$$\boxed{v_1 = 0}$$



Similarly equation (6) becomes:

$$v_2 = \frac{2m \times mu_1}{m+m} + \frac{(m-m) \times 0}{(m+m)}$$

$$v_2 = \frac{2m \times u_1}{2m} + 0$$

$$\boxed{v_2 = u_1}$$

Result: This shows that body A will stop after collision. While B will start moving with the initial velocity of A.

Case –3: When a lighter body collides with a very heavy body at rest.

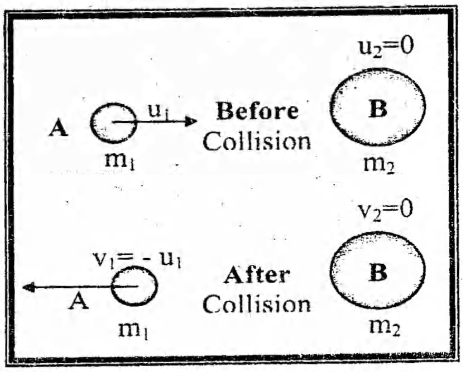
$m_1 << m_2$ and $u_2 = 0$, then m_1 can be neglected. Thus equation (5) becomes.

$$v_1 = \frac{(0-m_2)u_1}{0+m_2} + \frac{2 \times m_2 \times 0}{0+m_2}$$

$$v_1 = \frac{-m_2u_1}{m_2} = 0$$

$$= -u_1 + 0$$

$$\boxed{v_1 = -u_1}$$



similarly equation (6) becomes.

$$v_2 = \frac{2 \times 0 \times u_1}{m_2} + \frac{m_2}{m_2} \times 0$$

$$= 0 + 0$$

$$\boxed{v_2 = 0}$$

Result: This shows that body B will remain at rest while A will bounce back with the same velocity.

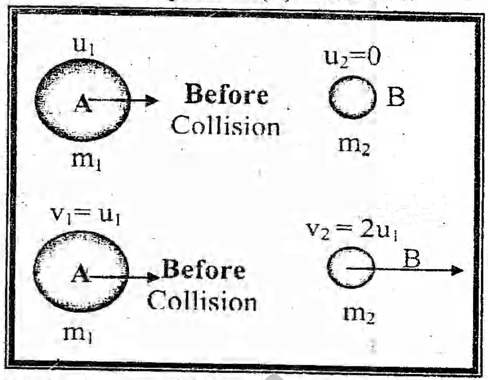
Case –4: When a very heavy body collides with a lighter body at rest.

If $m_1 >> m_2$ and $u_2 = 0$ then m_2 can be neglected. Thus equation (5) becomes.

$$v_1 = \frac{(m_1 - 0)u_1}{m_1 + 0} + \frac{2 \times 0 \times 0}{m_1 + 0}$$

$$= u_1 + 0$$

$$\boxed{v_1 = u_1}$$



Similarly equation (6) becomes.

$$v_2 = \frac{2m_1u_1}{m_1 + 0} + \frac{(0 - m_1)}{m_1 + 0}$$

$$= 2u_1 + 0$$

$$\boxed{v_2 = 2u_1}$$

Result: This shows that body A will continue to move with the same velocity while B will move with double velocity of A.

FRICTION:

When one body slides on the surface of another body, a force comes into play which opposes the motion of the body. This force is known as the ***"Force of Friction"***. The friction is due to the roughness of the material surface in contact, so if the surfaces are perfectly smooth, there is no friction to oppose the motion. The force of friction always acts parallel to the surface in contact and opposite to the direction of motion. Friction is a special property of solids.

Consider a rectangular solid body placed on a smooth horizontal surface. The forces acting on the body are.

1- Force of gravity "mg" acting vertically downwards.

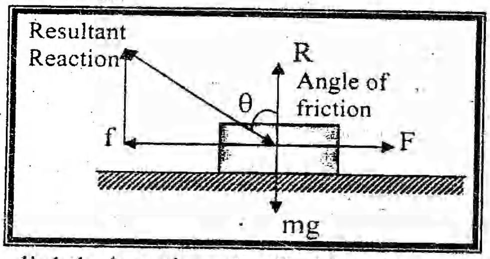
2- The reaction "R" acting vertically upwards.

As the block is at rest

$\therefore \quad R = mg$

If now a small force "F" is applied to the body parallel to the surface, an opposing force "f" comes into play which prevents the motion of the body.

If the applied force "F" is increased, gradually the opposing force "f" also increases till it attains a maximum value called ***Limiting Friction*** which depends upon the nature of the surfaces in contact and the magnitude of the normal reaction between them. If the external force be increased further, the equilibrium will be lost and the body begins to move. The friction is said to be ***Sliding*** or ***Rolling*** according as one body slides or rolls over the other.



Sliding friction is slightly less than the limiting friction. When the equilibrium is limiting, the normal reaction and the frictional force be compounded into a resultant single force, the angle which this resultant makes with the normal to the surface is called the ***"Angle of Friction"*** and the single force is called the ***Resultant Reaction.***

STATIC FRICTION:

When force is applied to move a body and the body remains at rest then friction between the bodies in contact is called ***"Static Friction".***

Limiting friction:

When one body is just at the point of sliding on the other, the friction is said to be limiting. It is maximum value of static friction.

Kinetic or dynamic friction:

When one body is actually sliding on the, other the friction is said to be kinetic or dynamic. It is less than the limiting friction.

Coefficient of friction:

The ratio of limiting friction to the normal reaction acting between two surfaces in contact is called the coefficient of friction and is usually denoted by "μ". It has no units. If "f_s" be the limiting friction & R be the normal reaction then:

$$\mu = \frac{f_s}{R} \quad \textbf{OR}$$

$$f_s = \mu R.$$

Fluid friction:

Bodies moving through the fluid i.e. in liquids or gases, experience a retarding force which is known as ***Fluid Friction OR Viscous Drag.***

Stokes studied the effect of viscous drag on small spheres falling through a liquid. He observed that these spheres experience an upward retarding force "F".

- This force is given by:

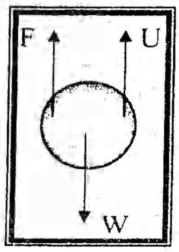
  $F = 6\,\pi\eta r v$

Where η = coefficient of viscosity

v = velocity of the sphere

r = radius of sphere

Besides this there are two other forces acting on the sphere and they are the weight of the body "W" which acts in the downward direction and the upward thrust "U" of the liquid. Resultant of these forces is (W-U) which acts in the downward direction. At constant temperature (W-U) is constant while the viscous drag "F" increases with the velocity.



Thus if a small metal sphere is allowed to fall through a liquid, it is first accelerated so that the value of "F" increases and becomes equal to (W-U), at this stage the net upward and downward forces are equal and the sphere starts moving with a uniform velocity known as ***Terminal Velocity.***

INCLINED PLANE:

Definition:

Any plane which makes certain angle with the horizontal plane is called an *Inclined Plane.* It is a kind of simple machine which helps us to do work more easily.

Explanation:

Consider an inclined plane making an angle θ with the horizontal. Let a block of weight "W" be placed on that inclined plane. There are three forces acting on it.

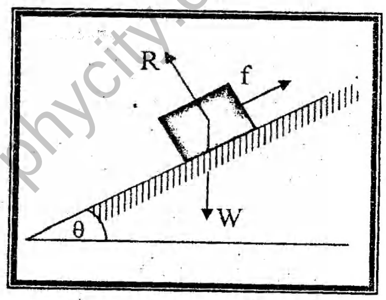
1- The force of gravity equal to its weight acting vertically downward.
2- Normal reaction "R" of the inclined plane acting perpendicular to the plane.
3- The force of friction "f" which opposes slipping down of body.

If we consider x-axis parallel to the inclined plane and y-axis perpendicular to the inclined plane, weight "W" of the block can be resolved into two components.

$W\sin\theta$ parallel to the inclined plane and

$W\cos\theta$ perpendicular to the inclined plane.

If the block is at rest then by first condition of equilibrium:



Along Y – axis

$$\Sigma F_y = 0$$
$$R - W\cos\theta = 0$$
$$R = mg\cos\theta$$

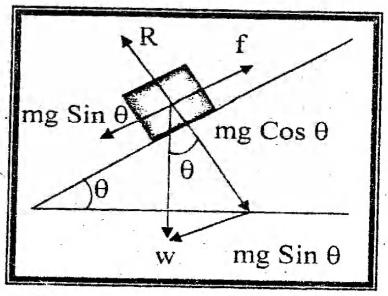
Along X – axis

At rest:

$$\Sigma F_x = 0$$
$$f - W\sin\theta = 0$$
$$f = W\sin\theta$$

For downward motion:

If the block slides down with an acceleration "a" then,



$$W\sin\theta > f$$

Net force will be

$$F = W\sin\theta - f$$
$$ma = mg\sin\theta - f$$
$$a = \frac{mg\sin\theta - f}{m}$$
$$a = \frac{mg\sin\theta}{m} - \frac{f}{m}$$
$$\boxed{a = g\sin\theta - \frac{f}{m}}$$

If the force of friction is neglected then

$$f = 0$$
$$a = g\sin\theta - \frac{0}{m}$$
$$a = g\sin\theta - 0$$

OR $\boxed{a = g\sin\theta}$

This shows that acceleration of the body is independent of mass, so all bodies sliding down a frictionless inclined plane have the same acceleration.

Particular Cases:

Case-1:

When $\theta = 0°$,

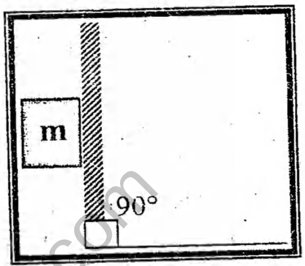
$\sin 0° = 1$

$a = g \sin 0°$ $\quad (\because \ \text{Sin } 0^{O} = 0)$

$a = g \times 0 \quad = 0$

This means that the block or any other body will have zero acceleration on a horizontal surface.

Case-2:



When $\theta = 90°$,

$\sin 90° = 1$

$a = g \sin 90°$

$a = g \times 1 \quad = g$

This means that the block or any other body falls freely.

Q. Two bodies of unequal masses (M and m) are Connected to the ends of a string which passes over a frictionless pulley, move vertically. Derive an expression to show that acceleration of the bodies is half of acceleration due to gravity if M = 3m. [2003 P.M, 2011 F]

Ans. Consider two bodies A and B of masses M and m connected by a string which passes over a frictionless pulley, as shown in the figure. If M = 3m, then the body A will accelerate down with an acceleration Say 'a' and the body B will move up with the same acceleration, let the tension in the string be "T".

Consider the Downward Motion of Body A:

Two force acting on the body A.

(i) Force of gravity 'Mg' acting in the downward direction.

(ii) Tension 'T' in the string acting in upward direction

Since body A is moving downward

$\therefore \quad Mg > T$

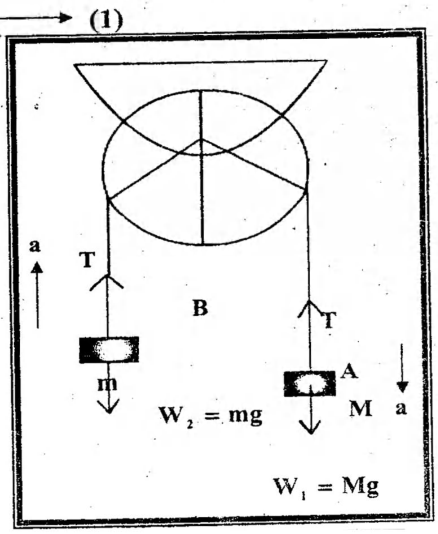
Resultant force = downward force – upward force

$F_1 = Mg - T$

But as we know that Newton's 2^{nd} Law of motion.

$F_1 = Ma$

$$\mathbf{Ma = Mg - T} \longrightarrow \mathbf{(1)}$$



Consider the Upward Motion of Body B:

Here also two forces are acting on the body B.

1) Force of gravity 'mg' acting in the downward motion

2) Tension 'T' in the string acting in upward direction

Since body B is moving upward.

$$T > mg$$

Resultant force = upward force – downward force

But According to Newton's 2nd Law of motion

$$F_2 = ma$$

$$\mathbf{ma = T - mg \longrightarrow (2)}$$

Calculation of 'a'

add eq (1) and eq (2)

$$Ma = Mg - T$$

$$ma = T - mg$$

$$Ma + ma = Mg - \cancel{T} + \cancel{T} - mg$$

$$a(M + m) = (M - m)\,g$$

$$a = \frac{(M - m)\,g}{M + m}$$

But $M = 3m$

$$a = \frac{(3m - m)\,g}{3m + m}$$

$$a = \frac{(2m)}{4m}\,g$$

$$\boxed{a = \frac{1}{2}\,g}$$