SCALARS:

Physical quantities having magnitude only but no direction are called Scalars. Scalars are completely described by

i) a number
ii) a suitable unit

Scalars can be added, subtracted, multiplied and divided by simple arithmetical rules. For example: mass, distance, time, speed, temperature, energy, work, volume and density etc.

VECTORS:

Physical quantities having both magnitude and direction and also obey commutative law of vector addition are called vectors. Vectors are completely described by

i) a number
ii) a suitable unit
iii) a certain direction

Vectors can not be added, subtracted, multiplied and divided by simple arithmetical rules. For example: Displacement, velocity, acceleration, force, angular velocity, torque, weight, electric field strength etc.

Representation of a Vector:

A vector is represented graphically by a directed line segment or an arrow head line segment. The length of the line, according to the scale chosen, represents the magnitude of the vector while arrow head indicates the direction. To represent a vector we need:

1. A suitable scale.
2. Reference axes i.e. x, y and z-axis, or horizontal and vertical directions or the directions of north, south, east and west.

Types of Vectors:

1) **Unit vector** 2) **Free vector** 3) **Position vector**
4) **Negative vector** 5) **Null vector**

1) Unit Vector:

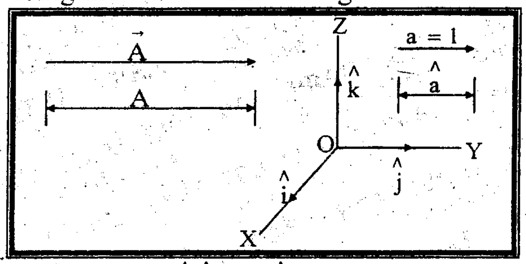
A unit vector is that whose magnitude is unity i.e. equal to 1 and has any given direction only.

A unit vector can be obtained by dividing the vector with its magnitude i.e.

$$\text{unit vector} = \frac{\text{Vector}}{\text{magnitude of vector}}$$

If unit vector of a vector $\vec{A}$ is $\hat{a}$ then

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$



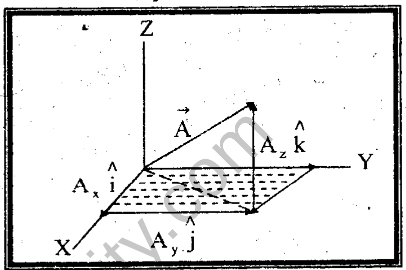
Unit vectors along x , y and z-axis are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

Rectangular Components of a Vector in Terms of Unit Vectors:

Consider a vector $\vec{A}$ starting from the origin "O" of a rectangular coordinate system, as shown in the figure.

Its rectangular components in the direction of positive x,y and z-axis are $A_x\hat{i}$, $A_y\hat{j}$ and $A_z\hat{k}$ respectively.

Conversely the sum of rectangular component vectors produces the original vector $\vec{A}$ i.e.



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

The magnitude of $\vec{A}$ is given by formula:

$$|\vec{A}| = A = \sqrt{Ax^2 + Ay^2 + Az^2}$$

2) Free Vector:

A vector which can be displaced parallel to itself and applied at any point is said to be a free vector.

3) Position Vector:

A vector which represents the position of a point with reference to a fixed point (i.e. origin) is called ***POSITION VECTOR***.

If the fixed point is taken as origin O and the coordinates of point P are x and y as shown in fig (1) then by Pythagoras theorem, the magnitude of position vector $\vec{r}$ is given by:

$$r^2 = x^2 + y^2$$

$$|\vec{r}| = r = \sqrt{x^2 + y^2}$$

In three dimensional rectangular coordinate system.

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

The components of a position vector are known as "Co-ordinates".

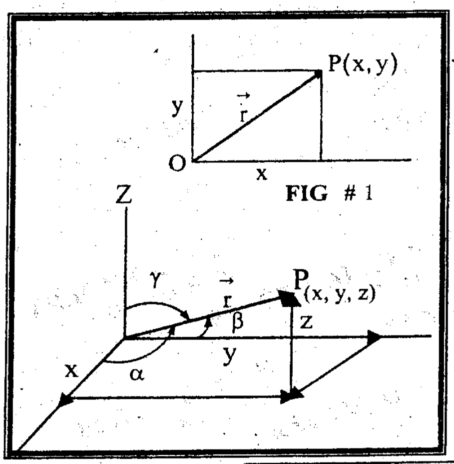
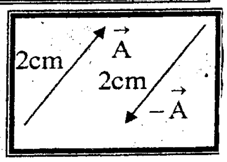


FIG # 1

4) Negative Vector:

A vector having the same magnitude as that of the given vector but opposite in direction is called a negative vector of the given vector. e.g. Negative vector of a vector is as shown in the figure.



5) Null Vector:

A vector whose magnitude is zero and has no direction or it may have all directions is said to be a ***NULL VECTOR***. It is denoted by $\vec{O}$.

A null vector can be obtained by adding two or more vectors.

Direction Cosines:

If $\vec{r}$ makes angle α, β & γ with x, y, and z axes respectively then.

$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

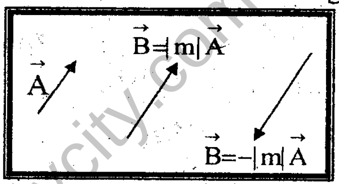
$$\cos\beta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Multiplication of a Vector by a Number:

When vector $\vec{A}$ is multiplied by a number "m", a new vector is generated whose magnitude is |m| times the magnitude of $\vec{A}$. If $\vec{B}$ is a new vector then its magnitude is given by

$$\vec{B} = |m|\vec{A}$$



The direction of $\vec{B}$ is the same as that of vector $\vec{A}$ if "m" is positive.
The direction of vector $\vec{B}$ is opposite to that of $\vec{A}$ if "m" is negative.

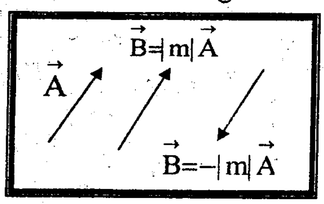
The multiplication of a vector by one or more numbers (say m, n) is governed by the following rules:

1.	$m\vec{A} = \vec{A}\,m$	Commutative law for multiplication.
2.	$m(n\vec{A}) = (mn)\vec{A}$	Associative law for multiplication.
3.	$(m+n)\vec{A} = m\vec{A} + n\vec{A}$	Distributive law.
4.	$m(\vec{A}+\vec{B}) = m\vec{A} + m\vec{B}$	Distributive law.

Division of a Vector by a Number (Non-Zero):

When a vector $\vec{A}$ is divided by a number "n", a new vector is generated whose magnitude is $\left|\frac{1}{n}\right|$ times the magnitude of $\vec{A}$. If $\vec{B}$ is a new vector then its magnitude is given by $\vec{B} = \left|\frac{1}{n}\right|\vec{A}$. If we write $\left|\frac{1}{n}\right| = m$ then

$$\vec{B} = |m|\vec{A}$$



The direction of the new vector $\vec{B}$ is the same as that of $\vec{A}$ if "n" is positive.
The direction of the new vec' ... site to that of $\vec{A}$ if "n" is negative.

Addition of Vectors by Geometrical Method:

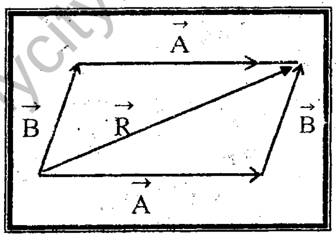
There are three ways of addition of vectors by geometrical methods.

1. Law of parallelogram of vectors.
2. Law of Triangle of vectors.
3. Head -To- Tail Rule:

1. The Law of Parallelogram of Vectors:

According to this law, "if two vectors $\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by two sides of a parallelogram drawn from a point, their resultant $\vec{R}$ will be completely represented by the diagonal passing through the same point.

The single vector $\vec{R}$ is called resultant of vector $\vec{A}$ and $\vec{B}$. The vector $\vec{A}$ and $\vec{B}$ are known as components of resultant vector $\vec{R}$.



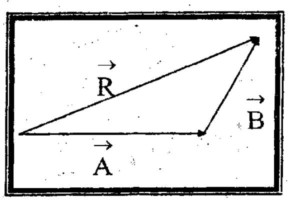
The parallelogram law may be used to find the resultant of any two vectors. In the above figure.

$$\vec{R} = \vec{A} + \vec{B}$$

Law of Triangle of Vectors:

If $\vec{A}$ and $\vec{B}$ are two sides of a triangle taken in order then their resultant $\vec{R}$ will be represented by the third side taken in opposite order. i.e.

$$\vec{R} = \vec{A} + \vec{B}$$

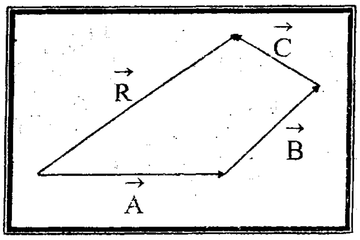


Head-To-Tail Rule:

Two or more vectors can be added by head-to-tail rule. According to the rule "The vectors are drawn one by one in such a way that the head of the one vector coincides with the tail of the other. The resultant is given in magnitude and direction by a line starting from tail of first vector to the head of the last vector". i.e.

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

Where $\vec{R}$ is the resultant of vector $\vec{A}$, $\vec{B}$ and $\vec{C}$

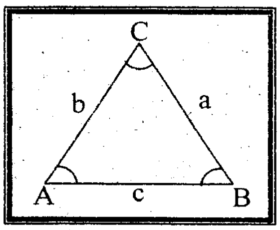


The magnitude and the direction of the resultant vector can be found by scale & protector.

The magnitude and the direction of the resultant vector can also be found by using trigonometric formula i.e. law of cosine and law of sine.

LAW OF COSINE:

In a triangle ABC, side opposite to $\angle A = a$, side opposite to $\angle B = b$ and side opposite to $\angle C = c$, as Shown in figure:



$a^2 = b^2 + c^2 - 2bc \cos \angle A$

$b^2 = a^2 + c^2 - 2ac \cos \angle B$

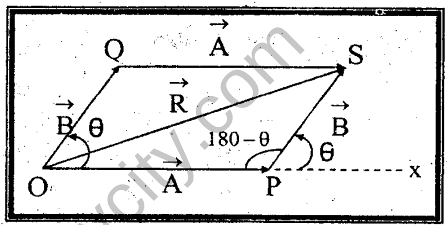
$c^2 = a^2 + b^2 - 2ab \cos \angle C$

These equations are known as "Law of Cosine".

If this law is applied on triangle OPS, we get:-

$R^2 = A^2 + B^2 - 2AB \cos \angle OPS$

OR $R = \sqrt{A^2 + B^2 - 2AB \, Cos\angle OPS}$



If $\angle SPX = \theta$ then

$\angle OPS = 180 - \theta$

Cos $\angle OPS = Cos(180 - \theta) = - Cos\theta$

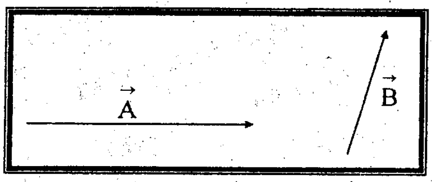
Thus equation (1) becomes.

$$R = \sqrt{A^2 + B^2 + 2AB \, Cos\theta}$$

LAW OF SINE:

In triangle ABC

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$

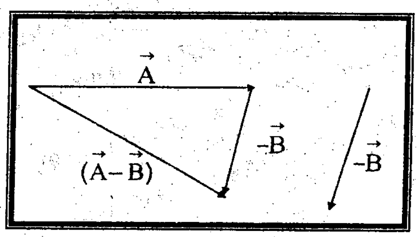


In triangle OPS

$$\frac{R}{Sin\ \angle OPS} = \frac{A}{Sin\ \angle OSP} = \frac{B}{Sin\ \angle POS}$$

Subtraction of Vectors:

Consider two vectors $\vec{A}$ and $\vec{B}$. If we want to subtract vector $\vec{B}$ from vector $\vec{A}$ then it can be done by making vector (i.e.– $\vec{B}$) and adding it with vector $\vec{A}$. The resultant is $(\vec{A} - \vec{B})$, as shown in the figure.



PROPERTIES OF VECTOR ADDITION:

1. Commutative law of vector addition:

Addition of vectors is commutative i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Proof:-

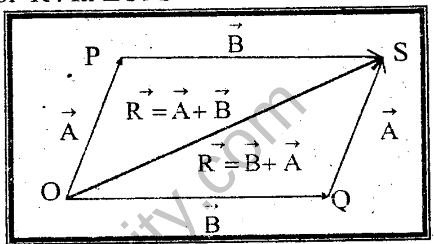
Consider two vectors $\vec{A}$ & $\vec{B}$ which can form a parallelogram. If we add these vectors by head-to-tail rule, we get a resultant vector $\vec{R}$. In ΔOPS

$$\vec{R} = \vec{A} + \vec{B} \text{ -------(1)}$$

In ΔOQS $\quad \vec{R} = \vec{B} + \vec{A}$ -------(2)

Comparing equation (1) & (2)

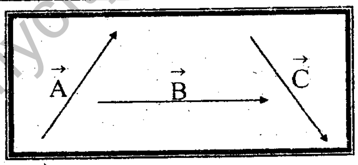
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



2. Associative law of vector addition:

Addition of vector is associative i.e.

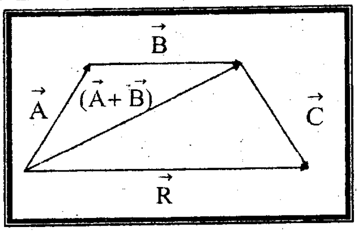
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$



Proof:

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, If we add $\vec{A}$ & $\vec{B}$ vectors by head-to-tail rule we get a resultant vector $(\vec{A} + \vec{B})$. Now add vector $\vec{C}$ to vector $(\vec{A} + \vec{B})$ by head-to-tail rule, we get a resultant vector

$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} \longrightarrow \textbf{(1)}$$

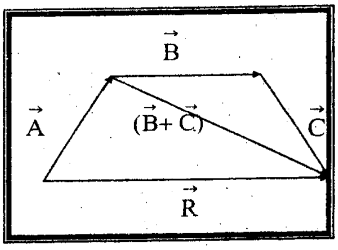


On the other hand if we add $\vec{B}$ and $\vec{C}$ first, we get a resultant vector $(\vec{B} + \vec{C})$. Now add vector $(\vec{B} + \vec{C})$ to vector $\vec{A}$ by head-to-tail rule. We get a resultant vector $\vec{R}$.

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) \longrightarrow \textbf{(2)}$$

Comparing equation 1 & 2

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{A})$ Hence Proved.



RESOLUTION OF A VECTOR:

Definition:

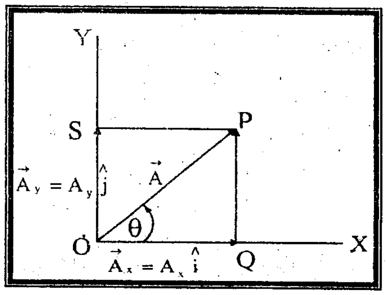
The process of dividing a vector into its rectangular components, is called resolution of a vector.

Explanation:

Consider a vector $\vec{A}$ represented by line $\overrightarrow{OP}$ and which makes an angle θ with the x-axis. To resolve this vector, draw perpendicular $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ on x and y axis respectively, as shown in the figure the line $\overrightarrow{OQ}$ is along x-axis and is denoted by $\vec{A}_x$ and the line $\overrightarrow{OS}$ is along y-axis so it is denoted by $\vec{A}_y$.

It can be seen by head-to-tail rule that:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



Since $\vec{A}_x$ and $\vec{A}_y$ are mutually perpendicular. Therefore they are referred as RECTANGULAR COMPONENTS of vector $\vec{A}$. The magnitudes of $\vec{A}_x$ and $\vec{A}_y$ can be found by considering ΔOPQ. Here

$$\text{Cos}\theta = \frac{OQ}{OP} = \frac{A_x}{A}$$

OR $\boxed{A_x = A\,\text{Cos}\theta}$

Similarly, $$\text{Sin}\theta = \frac{QP}{OP} = \frac{OS}{OP}$$

$$\text{Sin}\theta = \frac{A_y}{A}$$

$$\boxed{A_y = A\,\text{Sin}\theta}$$

Where A, A_x and A_y are the magnitudes of vector $\vec{A}$, $\vec{A}_x$ and $\vec{A}_y$ respectively.

The vector $\vec{A}$ can be written in terms of its components and rectangular unit vector as,

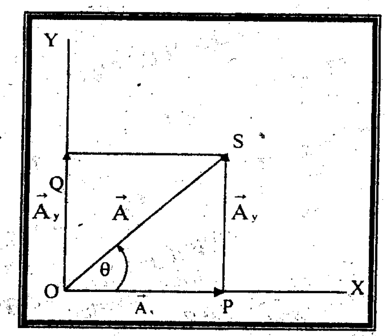
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Composition of Vector by Its Rectangular Components:

Definition:

The process by which a vector can be made from its rectangular components is called composition of a vector.

Explanation:



Consider two rectangular components $\vec{A}_x$ and $\vec{A}_y$ of a vector $\vec{A}$. If we add these components by head-to-tail rule, we get a resultant vector $\vec{A}$ of which $\vec{A}_x$ and $\vec{A}_y$ are the components, as shown in the figure.

The magnitude of $\vec{A}$ can be found by considering ΔPOS. By Pythagoras theorem

$$(OS)^2 = (OP)^2 + (PS)^2$$

$$A^2 = (A_x)^2 + (A_y)^2$$

$$\boxed{A = \sqrt{(A_x)^2 + (A_y)^2}}$$

The direction of $\vec{A}$ with respect to + x-axis can be found by formula:

$$\tan\theta = \frac{PS}{OP}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

Addition of Vectors by Rectangular Components Method:

Consider two vectors $\vec{A}_1$ and $\vec{A}_2$ making angles θ_1 & θ_2 with the x-axis respectively, as shown in fig.

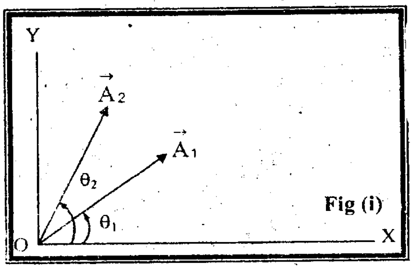


Fig (i)

1. Resolve $\vec{A}_1$ into its rectangular components $\vec{A}_{1x}$ and $\vec{A}_{1y}$, the magnitude of these components are given by:

   $A_{1x} = A_1 \cos\theta_1$

   $A_{1y} = A_1 \sin\theta_1$

2. Move the vector $\vec{A}_2$ parallel to itself so that its initial point lies on the terminal point of $\vec{A}_1$. Now resolve $\vec{A}_2$ into its rectangular components $\vec{A}_{2x}$ and $\vec{A}_{2y}$.

   The magnitude of these components are given by:

   $A_{2x} = A_2 \cos\theta_2$

   $A_{2y} = A_2 \sin\theta_2$

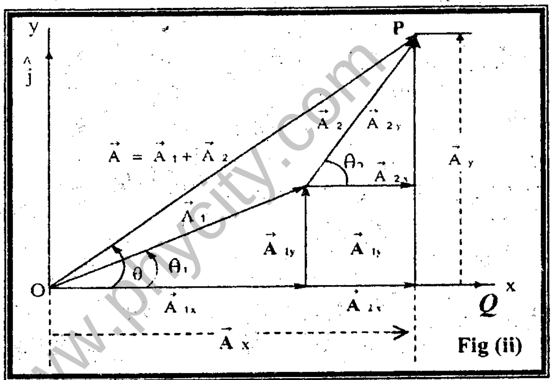


Fig (ii)

3. The resultant vector along x- axis:

   $\vec{A}_x = \vec{A}_{1x} + \vec{A}_{2x}$

   $A_x\hat{i} = A_{1x}\hat{i} + A_{2x}\hat{i}$

   $A_x\hat{i} = (A_{1x} + A_{2x})\hat{i}$

   OR $A_x = A_{1x} + A_{2x}$

   OR $A_x = A_1 \cos\theta_1 + A_2 \cos\theta_2$

4. The resultant vector along y-axis:

   $\vec{A}_y = \vec{A}_{1y} + \vec{A}_{2y}$

   $A_y\hat{j} = A_{1y}\hat{j} + A_{2y}\hat{j}$

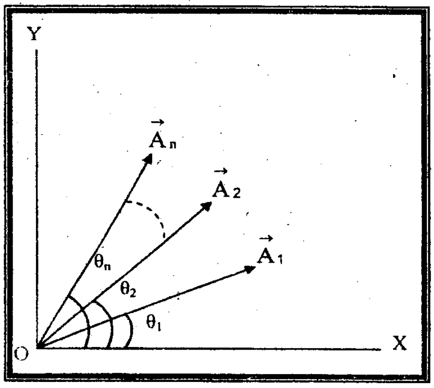
   OR $A_y\hat{j} = (A_{1y} + A_{2y})\hat{j}$

   $A_y = (A_{1y} + A_{2y})$

   OR $A_y = A_1 \sin\theta_1 + A_2 \sin\theta_2$

5. If there are "n" vectors $\vec{A}_1, \vec{A}_2$ -----, $\vec{A}_n$ making angles θ_1, θ_2 -----, θ_n with the x-axis respectively then we shall resolve these rectangular components.

$\vec{A}_{1x}, \vec{A}_{2x}$, -----, $\vec{A}_{nx}$ and $\vec{A}_{1y}, \vec{A}_{2y}$, ----, $\vec{A}_{ny}$.



Thus equation ① becomes

$$\vec{A}_x = \vec{A}_{1x} + \vec{A}_{2x} + \text{-----} + \vec{A}_{nx}$$

Similarly eq ② become

$$\vec{A}_y = \vec{A}_{1y} + \vec{A}_{2y} + \text{-----} + \vec{A}_{ny}$$

The magnitude of the resultant vector $\vec{A}$ can be found by Pythagoras theorem i.e.

$$A^2 = (A_x)^2 + (A_y)^2$$

OR $$A = \sqrt{(A_x)^2 + (A_y)^2}$$

7. The direction of the resultant vector can be found by formula $\tan\theta = \frac{A_y}{A_x}$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

Product of Two Vectors:

When two vectors are multiplied with each other, answer may be a scalar or a vector quantity.

Thus product of two vectors is divided into two categories.

1) Scalar product or dot product.
2) Vector product or cross product.

THE SCALAR PRODUCT OR DOT PRODUCT:

When two vectors are multiplied with each other and the answer is a scalar quantity then such a product is called "Scalar product".

A dot (.) is placed between the vectors which are multiplied with each other that's why it is also called "dot product". i.e. Scalar = vector .vector

Examples:

1. The product of force $\vec{F}$ and displacement $\vec{S}$ is work " W ".

   i.e. $W = \vec{F} \cdot \vec{S}$

2. The product of force $\vec{F}$ and velocity $\vec{V}$ is power "P".

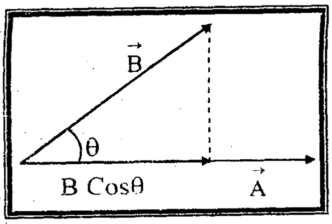
   i.e. $P = \vec{F} \cdot \vec{V}$

3. The product of electric intensity $\vec{E}$ and area vector $\vec{A}$ is electric flux ϕ_e.

   i.e. $\phi_e = \vec{E} \cdot \vec{A}$

Explanation:-

The dot product is defined as "The product of magnitudes of the vectors and the cosine of the angle between them."



Consider two vectors $\vec{A}$ & $\vec{B}$ making an angle θ with each other:-

i.e. $\vec{A}.\vec{B} = AB\cos\theta$

Where B Cosθ is the component of $\vec{B}$ along vector $\vec{A}$ and

$0 \leq \theta \leq \pi$

Special Casses of Scalar Product:

(i) If vector $\vec{A}$ is parallel to $\vec{B}$ then their scalar product is maximum.

$\vec{A}\,.\,\vec{B} = AB\cos 0^\circ = AB\,.\,(1) = AB$

(ii) Scalar product of same vectors is equal to square of their magnitude.

$\vec{A}\,.\,\vec{B} = \vec{A}.\vec{A} = AA\cos 0^\circ = A^2 \times 1 = A^2$.

(iii) If two vectors are opposite to each other then their scalar product will be negative.

$\vec{A}\,.\,\vec{B} = AB\cos\theta = AB\cos 180^\circ = AB(-1) = -AB$

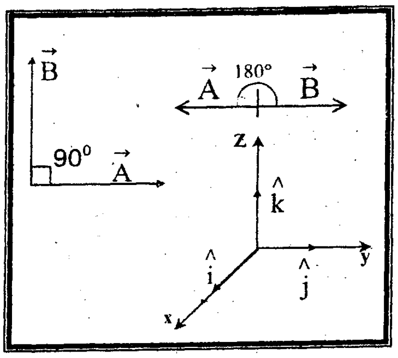
(iv) If $\vec{A}$ is perpendicular to $\vec{B}$ then their scalar product is minimum.

$\vec{A}\,.\,\vec{B} = AB\cos 90^\circ = AB \times 0 = 0.$

(v) For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, the scalar product of same unit vectors is 1 and for different unit vectors is zero.

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$



Commutative Law for Dot Product:

The dot product is commutative i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$.

Proof:-

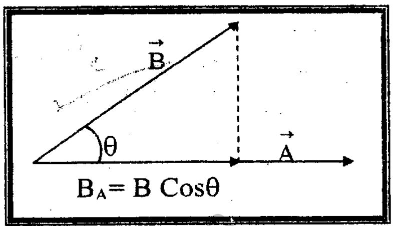
Consider two vectors $\vec{A}$ & $\vec{B}$ making an angle θ with each other. Their dot product is given by.

$$\vec{A}.\vec{B} = AB_A = AB\ \text{Cos}\theta \quad \text{------------ (1)}$$

Where $B_A = B\ \text{Cos}\theta$ is the projection of $\vec{B}$ onto the direction of $\vec{A}$. On the other hand.

$$\vec{B}.\vec{A} = BA_B = BA\ \text{Cos}\theta \quad \text{------------ (2)}$$

Where $A_B = A\ \text{Cos}\theta$ is the projection of $\vec{A}$ on to the direction of $\vec{B}$.



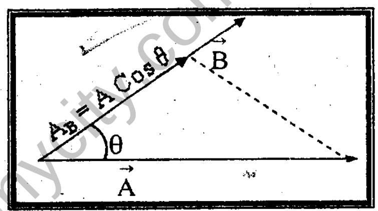
Comparing equation **(1)** & **(2)**.

$$AB\ \text{Cos}\theta = BA\ \text{Cos}\theta$$

OR $$AB_A = BA_B$$

OR $$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

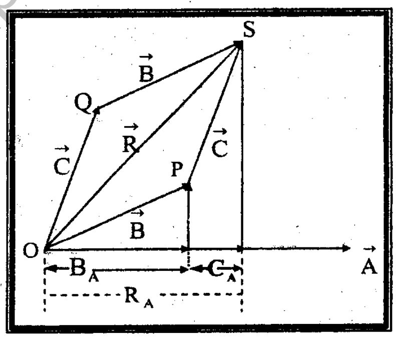
Thus scalar product of two vectors does not change by changing the order of the vectors.



Distributive Law for Dot Product:

The dot product is distributive i.e.

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$



Proof:

Consider three vectors $\vec{A}, \vec{B}$ & $\vec{C}$. By head-to-tail rule, we get:-

$$\vec{R} = \vec{B} + \vec{C}$$

OR $$\vec{R} = \vec{C} + \vec{B}$$

Now draw perpendiculars from P and S on the direction of $\vec{A}$. The dot product of $\vec{A}$ and $\vec{R}$ is given by

$$\vec{A}.\vec{R} = AR_A$$

But $$\vec{R} = \vec{B} + \vec{C} \text{ and } R_A = B_A + C_A$$

$$\therefore \quad \vec{A}.(\vec{B} + \vec{C}) = A(B_A + C_A)$$

$$\vec{A}.(\vec{B} + \vec{C}) = AB_A + AC_A$$

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

$$\therefore \quad \vec{A}.\vec{B} = AB_A,\ \vec{A}.\vec{C} = AC_A$$

Where B_A & C_A are the projections of $\vec{B}$ and $\vec{C}$ onto the direction of $\vec{A}$.

THE VECTOR PRODUCT OR THE CROSS PRODUCT:

When two vectors are multiplied with each other and the answer is also a vector quantity then such a product is called ***"VECTOR PRODUCT"***.

A cross (x) is placed between the vectors which are multiplied with each other that's why it is also known as "cross product" i.e.

Vector = Vector x Vector

Examples:

1. The product of position vector "$\vec{r}$" of force and force "$\vec{F}$" is Torque "$\vec{\tau}$". i.e.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

2. The product of angular velocity $\vec{\omega}$ and radius vector $\vec{r}$ is tangential velocity $\vec{V}_t$.

i.e. $\vec{V}_t = \vec{\omega} \times \vec{r}$

3. The product of angular acceleration $\vec{\alpha}$ and radius vector $\vec{r}$ is tangential acceleration $\vec{a}_t$ i.e. $\vec{a}_t = \vec{\alpha} \times \vec{r}$

4. An electric charge moving with velocity $\vec{V}$ in a magnetic field of induction $\vec{B}$ experiences a force $\vec{F}$ which is given by:

$$\vec{F} = q(\vec{V} \times \vec{B})$$

Explanation:-

The cross product is defined by the relation.

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{C} = |\vec{A}||\vec{B}| \sin\theta \, \vec{u}$$

Where $\vec{u}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

Direction:-

The direction of $\vec{C}$ or $\vec{V}$ can be found by "Right Hand Rule"

Properties of Vector Product:-

(i) The vector product does not obey commutative law

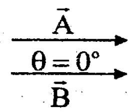
$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ or $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

(ii) The vector product obeys distributive law

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(iii) If vector $\vec{A}$ is parallel to $\vec{B}$ then their vector product is zero

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$



$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin 0^\circ$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \times 0$$

$$|\vec{A} \times \vec{B}| = 0$$

In case of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$\hat{i} \times \hat{i} = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{k} = 0$$

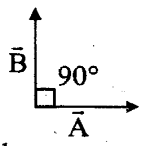
(iv) If vector $\vec{A}$ is perpendicular to $\vec{B}$ then their vector product is maximum

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$ In case of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\theta = 90°$ $\hat{i} \times \hat{j} = \hat{k}$

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin 90°$ $\hat{j} \times \hat{k} = \hat{i}$

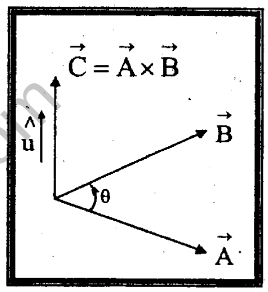
$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|$ $\hat{k} \times \hat{i} = \hat{j}$ $\because \sin 90° = 1$



Q. Prove that vector product is not commutative.

Consider two vectors $\vec{A}$ and $\vec{B}$ in a certain plane. Their vector product is given by:

$$\vec{C} = \vec{A} \times \vec{B}$$



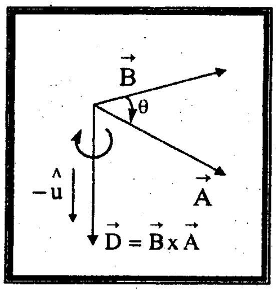
Where $\vec{C}$ is a new vector. Its magnitude is given by $C = AB \sin\theta$. The direction of $\vec{C}$ can be found by right hand rule for cross product or right handed screw turning rule.

The cross product of $\vec{A}$ & $\vec{B}$ can also be given by:

$$\vec{C} = \vec{A} \times \vec{B} = (AB \sin\theta)\hat{u} \text{ ------------------(1)}$$

Where $\hat{u}$ is a unit vector in the direction of $\vec{C}$.

Similarly a right-handed screw turning from $\vec{B}$ to $\vec{A}$ gives the direction of unit vector $\vec{D}$.



$$\vec{D} = \vec{B} \times \vec{A} = (BA \sin\theta)(-\hat{u})$$

OR $$\vec{D} = \vec{B} \times \vec{A} = -(BA \sin\theta)\hat{u}$$

OR $$-\vec{D} = -\vec{B} \times \vec{A} = (BA \sin\theta)\hat{u} \text{ ------------------(2)}$$

Comparing equations (1) & (2)

$$(AB \sin\theta)\hat{u} = (BA \sin\theta)\hat{u}$$

OR $$\vec{C} = -\vec{D}$$

OR $$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

Thus $$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

This shows that the vector product is not commutative.

Q. Give Physical interpretation of vector product.

OR

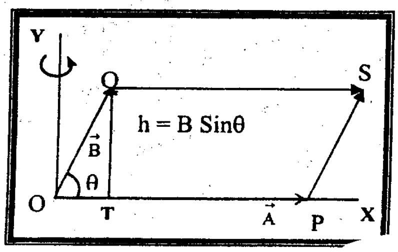
Prove that area of parallelogram formed by two vectors is equal to magnitude of vector product.

Consider two vectors $\vec{A}$ & $\vec{B}$ which are the adjacent sides of a parallelogram OPSQ, as shown in the figure. From figure, the area of a parallelogram = Ah

But $h = B \sin\theta$

Area of parallelogram = $AB \sin\theta$ where θ is the angle between $\vec{A}$ & $\vec{B}$.

area or parallelogram = $|\vec{A} \times \vec{B}|$



The direction of the vector area can be found by right-handed screw turning rule.

DIFFERENCES:

SCALARS

1) Physical quantities having magnitude only but no direction are called Scalars.

2) Scalars are completely described by
   (i) a number
   (ii) a suitable unit

3) Scalars are added, subtracted, multiplied and divided by simple arithmetical rules.

VECTORS

1) Physical quantities having both magnitude and direction are called Vectors.

2) Vectors are completely described by
   (i) a number
   (ii) a suitable unit
   (iii) a certain direction

3) Vectors can not be added, subtracted, multiplied and divided by simple arithmetical rules.

SCALAR PRODUCT

1) If the product of two vectors is a scalar quantity, the product is called scalar product or dot product.

2) The dot product is defined by the relation $\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta$

3) The scalar product obeys commutative law $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

4) If two vectors are perpendicular to each other, their scalar product is zero.
   $\vec{A}.\vec{B} = 0$

VECTOR PRODUCT

1) If the product of two vectors is a vector quantity, the product is called vector product or cross product.

2) The cross product is defined by the relation $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta\, \hat{u}$

3) The vector product does not obey commutative law $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

4) If two vectors are parallel to each other, their vector product is zero.
   $\vec{A} \times \vec{B} = 0$