PROBLEMS:

Q.1. An object 4cm tall is placed near the axis of a thin converging lens. If the focal length of the lens is 25cm. Where will the image be formed and what will be the size of the image? Sketch the principal ray diagram.

Give Data:

Size of object = h_o = 4cm

Focal length of convex = f = 25cm

Object distance = p = 33.33cm.

To Find:

Image distance = q = ?

Size of image = h_i = ?

Given problem is not correct because object distance is not given. The data is that p = 33.33cm

Solution:

Using the lens formula:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{25} = \frac{1}{33.33} + \frac{1}{q}$$

$$\frac{1}{25} - \frac{1}{33.33} = \frac{1}{q}$$

$$\frac{33.33 - 25}{25 \times 33.33} = \frac{1}{q}$$

$$q = \frac{25 \times 33.33}{8.33}$$

q = 100cm **Ans.**

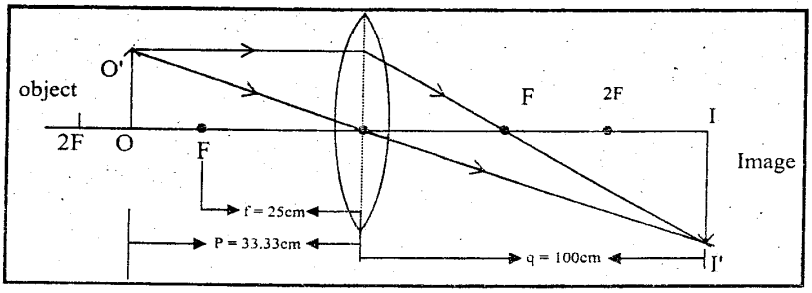
Magnification produced by the lens:

$$\frac{h_i}{h_o} = \frac{q}{p}$$

$$h_i = \frac{q}{p} h_o$$

$$h_i = \frac{100}{33.33} \times 4$$

h_i = 12 cm **Ans.**



Q.2. A convex lens has a focal length of 10cm. Determine the image distances when an object is placed at the following distances from the lens.
50cm, 20cm, 15cm, 10cm, and 5cm

Give Data:

Focal length of lens = f = 10cm

(i) Object distance = p = 50cm (ii) Object distance = p = 20cm
(iii) Object distance = p = 15cm (iv) Object distance = p = 10cm
(v) Object distance = p = 5cm

To Find:

Image distance = q = ?
(iv) object distance = p = 10cm

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{10} = \frac{1}{10} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{10} \times \frac{1}{10} = \frac{0}{10}$$

$$q = \frac{10}{0} \qquad \because \frac{10}{0} = \infty$$

$\boxed{q = \infty}$ **Ans.**

(1) SELF TEST:

(i) p = 50 q = ? (ii) p = 20cm q = ? (iii) p = 15cm q = ? (iv) p = 5cm q = ?

Ans.(i) q = 12.5cm (ii) q = 20cm (iii) q = 30cm (iv) q = 10cm

(v) A converging lens of focal length 20cm form image of an object placed at a distance of (a) 30cm (b) 10cm from the lens. Find the image distance and the magnification in each case. (2008 Failures)

Ans.(a) q = 60cm (b) q = –20cm
M = 2 M = –2

Q.3. Two converging lenses of focal lengths 40cm and 50cm are placed in contact. What is the focal length of thin lens combination? What is the power of the combination in diopters?

Given Data:

Focal length of first lens = f_1 = 40cm
Focal length of second lens = f_2 = 50cm

To Find:

Focal length of combination = f = ?
Power of combination = P = ?

Solution:

Using the relation:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{40} + \frac{1}{50}$$

$$\frac{1}{f} = \frac{5+4}{200}$$

$$\frac{1}{f} = \frac{9}{200}$$

$$f = \frac{200}{9}$$

$$f = 22.2$$

$\boxed{f = 22.2\text{cm}}$ **Ans.**

$$P = \frac{100}{f}$$

$$P = \frac{100}{22.2}$$

$\boxed{P = 4.5 \text{ diopters}}$ **Ans.**

SELF TEST:

Two converging lenses of focal lengths 30cm and 60cm are placed in contact. What is the focal length of this combination? Calculate the power of the combination in diopters. (2011)

(2) Self Test:

Q.6. Two lenses are in contact, a converging lens of focal length 30cm and diverging lens of focal length -10cm. What is the focal length and power of the combination? Ans: [f = –15cm P= – 6.66 dioptre]

Q.4. A converging lens of focal length 20cm is placed infront of a converging lens of focal length 4cm. What is the distance between the lenses if parallel rays entering the first lens leave the second lens as parallel rays?

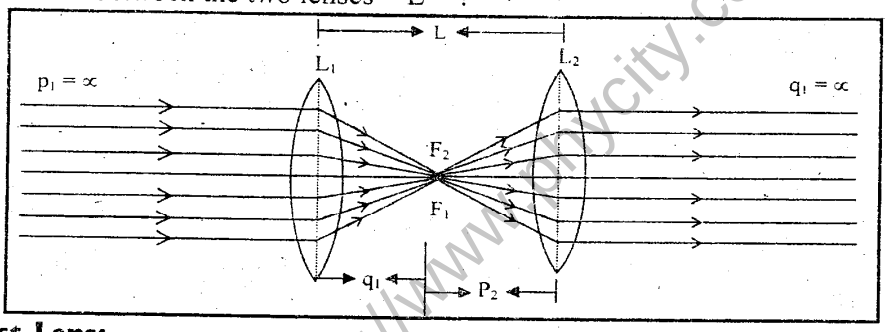
Given Data:

Focal length of first lens = $f_1 = 20cm$

Focal length of second lens = $f_2 = 4cm$

To Find:

Distance between the two lenses = L = ?



For First Lens:

The rays from the object are parallel.

Therefore the object is at infinity

$p_1 = \infty$

$q_1 = ?$

Using the lens formula:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{20} = \frac{1}{\infty} + \frac{1}{q_1}$$

$$\frac{1}{20} = 0 + \frac{1}{q_1} \qquad \because \frac{1}{\infty} = 0$$

$$\frac{1}{20} = \frac{1}{q_1}$$

$$\boxed{q_1 = 20cm}$$

For Second Lens:

The rays become parallel after passing through it, therefore the final image is at infinity from it.

$q_2 = \infty$

$p_2 = ?$

Using the Lens formula:

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$$

$$\frac{1}{4} = \frac{1}{p_2} + \frac{1}{\infty}$$

$$\because \quad \frac{1}{\infty} = 0$$

$$\frac{1}{4} = \frac{1}{p_2} + 0$$

$$p_2 = 4\text{cm}$$

Hence the distance between the two lenses is

$$L = q_1 + p_2 = 20 + 4$$

L = 24cm. Ans.

(3) Self Test:

(i) A parallel beam of light is diverged by a lens of focal length -12.5cm and then made parallel once more by a convex lens of focal length 50cm. How far are the two lenses apart.

Ans.[L = 37.5cm]

(ii) A parallel beam of light is diverged by a concave lens of focal length-10.5cm and then it is made parallel by using a concave lens of focal length 30.5cm. How far are lenses apart **(2005 Failures)**

Ans.[L = 20cm]

Q.7. Moon light passes through a converging lens of focal length 19 cm. Which is 20.5cm from a second converging lens of focal length 2cm. Where is the image of the moon produced by the combination?

Given Data:

Focal length of first lens = $f_1 = 19\text{cm}$

Focal length of second lens = $f_2 = 2\text{cm}$

Distance between the two lenses = $L = 20.5\text{cm}$

To Find:

Image distance = ?

For First Lens:

The rays from moon are parallel therefore

$$p_1 = \infty$$

$$q_1 = ?$$

Using Lens Formula:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{19} = \frac{1}{\infty} + \frac{1}{q_1}$$

$$\frac{1}{19} = 0 + \frac{1}{q_1} \qquad \because \quad \frac{1}{\infty} = 0$$

$$\frac{1}{19} = \frac{1}{q_1}$$

$$\boxed{q_1 = 19\text{cm}}$$

For Second Lens:

$P_2 = L - q_1$
$P_2 = 20.5 - 19$
$P_2 = 1.5$ cm.
$q_2 = ?$

Using Lens Formula:

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$$

$$\frac{1}{2} = \frac{1}{1.5} + \frac{1}{q_2}$$

$$\frac{1}{2} - \frac{1}{1.5} = \frac{1}{q_2}$$

$$\frac{1.5 - 2}{2 \times 1.5} = \frac{1}{q_2}$$

$$-\frac{0.5}{3} = \frac{1}{q_2}$$

$$q_2 = -\frac{3}{0.5}$$

$q_2 = -6$cm (Image distance from second lens)
Image distance from first lens = 20.5 – 60 = 14.5cm

Q.8. Find the distance at which an object should be placed infront of a convex lens of focal length 10 cm to obtain an image of double its size?

Given Data:

Focal length of lens = f = 10cm
Magnification = M = 2

To Find:

Object distance = p = ?

Solution:

For real image we know that	For virtual image
M = q/p	M = q/p
2 = q/p	–2 = q/p
q = 2p	q = –2p

Using Lens Formula:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{10} = \frac{1}{p} + \frac{1}{2p}$$

$$\frac{1}{10} = \frac{2+1}{2p}$$

$$\frac{1}{10} = \frac{3}{2p}$$

$$2p = 30$$

$$\boxed{p = 15cm}$$ **Ans.**

Using Lens formula:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{10} = \frac{1}{p} + \frac{1}{-2p}$$

$$\frac{1}{10} = \frac{1}{p} - \frac{1}{2p}$$

$$\frac{1}{10} = \frac{2-1}{2p}$$

$$\frac{1}{10} = \frac{1}{2p}$$

$$2p = 10cm$$

$$\boxed{p = 5cm}$$ **Ans.**

Q.9. A compound microscope has a 12 mm focal length objective and a 75 mm focal length eye piece. And the two lenses are mounted 200 mm apart. If the final image is 225mm from the eye piece. What is the magnification produced?

Given Data:

Focal length of objective lens = f_o = 12mm

Focal length of eye piece lens = f_e = 75mm

Length of microscope = L = 200mm

Distance of final image from eye piece = d = 225cm

To Find:

Magnification = M = ?

Solution:

For eye piece lens

$q_e = -225mm$

$p_e = ?$

Using Lens Formula:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$

$$\frac{1}{75} = \frac{1}{p_e} + \frac{1}{-225}$$

$$\frac{1}{75} + \frac{1}{225} = \frac{1}{p_e}$$

$$\frac{3+1}{225} = \frac{1}{p_e}$$

$$p_e = \frac{225}{4}$$

$$\boxed{p_e = 56.25mm}$$

$q_o = L - p_e$

$q_o = 200 - 56.25$

$q_o = 143.75mm$

For objective lens:

$q_o = 143.75mm$

$p_o = ?$

Using Lens Formula:

$$\frac{1}{f_o} = \frac{1}{p_o} + \frac{1}{q_o}$$

$$\frac{1}{12} - \frac{1}{p_o} = \frac{1}{143.75}$$

$$\frac{1}{12} - \frac{1}{143.75} = \frac{1}{p_o}$$

$$\frac{143.75 - 12}{12 \times 143.75} = \frac{1}{p_o}$$

$$p_o \frac{12 \times 143.75}{131.75}$$

$$\boxed{p_o = 13.09mm}$$

Magnification of the objective:

$$M_o = \frac{q_o}{p_o}$$

$$M_o = \frac{143.75}{13.09}$$

$M_o = 10.98$

Magnification of the eye piece:

$$M_e = 1 + \frac{d}{f_e}$$

$$M_e = 1 + \frac{225}{75}$$

$M_e = 1 + 3 = 4$

Magnification of the compound microscope:

$M = M_o \times M_e$

$M = 10.98 \times 4$

$M = 43.92$

$\boxed{M = 44}$ **Ans.**

(4) Self Test:

A compound microscope has lens of focal length 1.0cm and 3.0cm and two lenses are mounted 8.7cm apart. If a virtual image is formed 25cm from the eye piece calculate the magnification of the instrument.

Ans. [47.13]

Q.10. An astronomical telescope of angular magnification 1000 has an objective of 15m focal length. What is the focal length of the eye piece?

Given Data:

Angular magnification of telescope = M = 1000

Focal length of objective lens $= f_o = 15m$

To find:

Focal length of eye piece lens $= f_e = ?$

Solution:

The magnifical of astronomical telescope

$$M = \frac{f_o}{f_e}$$

$$1000 = \frac{15}{f_e}$$

$$f_e = \frac{15}{1000}$$

$$\boxed{f_e = 0.015m}$$ **Ans.**

(5) Self Test:

(i) **A simple astronomical telescope in normal adjustment has an objective of focal length 100cm and an eye piece of focal length 5.0cm. Calculate the angular magnification.**

Ans. [M = 20]

(ii) **An astronomical telescope of magnifying power 1000 has an objective of 12m focal length. What is the focal length of eyepiece? (2007 Supp.)**

Ans. [$f_e = 0.012m$]

Q.12. A compound microscope has an objective with a focal length of 10 mm and a tube 100 mm long. An image is produced 250 mm from the eye piece when the object is 12 mm from the objective. What is the angular magnification?

Given Data:

Focal length of objective lens $= f_o = 100mm$

Length of microscope $= L = 100mm$

Distance of final image from eye piece $= d = 250mm$

Distance of object from objective $= p_o = 12mm$

$q_e = -250mm$

To Find:

Angular magnification = M = ?

Solution:

For Objective:

$q_o = ?$

Using lens formula:

$$\frac{1}{f_o} = \frac{1}{p_o} + \frac{1}{q_o}$$

$$\frac{1}{10} = \frac{1}{12} + \frac{1}{q_o}$$

$$\frac{1}{10} - \frac{1}{12} = \frac{1}{q_o}$$

$$\frac{6-5}{60} = \frac{1}{q_o}$$

$$\frac{1}{60} = \frac{1}{q_o}$$

$$q_o = 60\text{mm}$$

$$L = q_o + p_e$$

$$p_e = L - q_o$$

$$p_e = 100 - 60$$

$$p_e = 40\text{mm}$$

For eye piece:

$q_e = -250\text{mm}$

$f_e = ?$

Using Lens Formula:

$$\frac{1}{f_e} = \frac{1}{p_e} + \frac{1}{q_e}$$

$$\frac{1}{f_e} - \frac{1}{40} = \frac{1}{-250}$$

$$\frac{1}{f_e} = \frac{1}{40} - \frac{1}{250}$$

$$\frac{1}{f_o} = \frac{250-40}{40 \times 250}$$

$$\frac{1}{f_e} = \frac{210}{10000}$$

$$\frac{1}{f_e} = \frac{10000}{210}$$

$$f_e = 47.61\text{ mm}$$

Magnification of objective:

$$M_o = \frac{q_o}{p_o}$$

$$M_o = \frac{60}{12}$$

$$\boxed{M_o = 5}$$

Magnification of eye piece:

$$M_e = 1 + \frac{d}{f_e}$$

$$M_e = 1 + \frac{250}{47.61}$$

$$\boxed{M_e = 6.25}$$

Magnification of microscope:

$$M = M_o \times M_e$$

$$M = 5 \times 6.25$$

$$\boxed{M = 31.25}$$ **Ans.**

(6) Self Test:

(i) **A compound microscope has an objective of focal length 1cm, and an eye piece of focal length 5cm. The object located 1.05cm from the objective and the final image is 25cm from the eye piece. Find the separation of the lenses and magnifying power of the microscope.**

Ans. [L = 25.16cm, M = 120]

(ii) **A microscope has an objective lens of 10mm focal length, and an eye piece of 25.0mm focal length. What is the distance between the lenses and its magnification if the object is in sharp focus when it is 10.5mm from the objective?** **[2008]**

Ans. [L = 232.72mm, M = 220]

Q.13. A converging lens of 4 diopters is combined with a diverging lens of -2 diopters. Find the power and focal length of the combination. **[2009]**

Given Data:

Power of converging lens = P_1 = 4 diopters
Power of diverging lens = P_2 = –2 diopters

To Find:

Power of combination = P = ?
Focal length of combination = f = ?

Solution:

Power of combination is given by.

$$P = P_1 + P_2$$

$$P = 4 - 2$$

$$\boxed{P = 2 \text{ diopters}}$$ **Ans.**

$$f = \frac{1}{P}$$

$$f = \frac{1}{2}$$

$$\boxed{f = 0.5 \text{ m}}$$

$$\boxed{f = 50 \text{ cm}}$$ **Ans.**

Self test (7):

Two converging lenses of power 2.5 diopters and 2 diopters are placed in contact. What is the power of combination and what is the focal length of combination?

Ans. [P = 4.5 diopter, f = 0.222m]

Q.14. A convex lens forms image of an object on a fixed screen 20 cm from the lens. On moving the lens 60 cm towards the object, the image is again formed on the screen. What is the focal length of the lens?

Given Data:

In first case:

Object distance = $p_1 = x$

I mage distance = $q_1 = 20cm$

In second case:

Object distance = $p_2 = x - 60$

Image distance = $q_2 = 80cm$

To find:

Focal length of lens = $f = ?$

First case:

Using lens formula:

$$\frac{1}{f} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{20}$$

$$\frac{1}{f} = \frac{20 + x}{20x}$$

$$\mathbf{f = \frac{20x}{20 + x}} \longrightarrow \mathbf{(1)}$$

Second case:

Using lens formula:

$$\frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$$

$$\frac{1}{f} = \frac{1}{x - 60} + \frac{1}{80}$$

$$\frac{1}{f} = \frac{80 + x - 60}{80(x - 60)}$$

$$\frac{1}{f} = \frac{(20 + x)}{80(x - 60)}$$

$$\mathbf{f = \frac{80(x - 60)}{(20 + x)}} \longrightarrow \mathbf{(2)}$$

Comparing eq.(1) and eq.(2)

$$\frac{20x}{\cancel{(20 + x)}} = \frac{80(x - 60)}{\cancel{(20 + x)}}$$

$$\cancel{20}x = \overset{4}{\cancel{80}}(x - 60)$$

$$x = 4x - 240$$

$$x - 4x = -240$$

$$-3x = -240$$

$$x = \frac{\cancel{-240}}{\cancel{-3}}$$

$$\boxed{\mathbf{x = 80cm}}$$

From eq. (1):

$$f = \frac{20 \times 60}{20 + 80}$$

$$f = \frac{\cancel{20} \times \cancel{60}}{\cancel{100}}$$

$$\boxed{f = 12cm}$$ **Ans.**

Q.15. Two converging lenses are 25 cm apart. Focal length of each is 10 cm. An object is placed in front of one lens at 50 cm. Find the distance between the object and the final image?

Given Data:

Focal length of each lens $= f_1 = f_2 = 10cm$
Distance between the lenses $= L = 25cm$
Object distance from first lens $= p_1 = 50$ cm

To Find:

Distance between object and final image = d = ?

$$d = p_1 + q_1 + p_2 + q_2$$

For first lens:

$q_1 = ?$

Using lens formula:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{10} = \frac{1}{50} + \frac{1}{q_1}$$

$$\frac{1}{10} - \frac{1}{50} = \frac{1}{q_1}$$

$$\frac{5 - 1}{50} = \frac{1}{q_1}$$

$$\frac{4}{50} = \frac{1}{q_1}$$

$$q_1 = \frac{50}{4}$$

$$\boxed{q_1 = 12.5cm}$$

$$L = q_1 + p_2$$

$$p_2 = L - q_1$$

$$p_2 = 25 - 12.5$$

$$p_2 = 12.5 \text{ cm}$$

For second lens:

$q_2 = ?$

Using lens formula:

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$$

$$\frac{1}{10} = \frac{1}{12.5} + \frac{1}{q_2}$$

$$\frac{1}{10}-\frac{1}{12.5}=\frac{1}{q_2}$$

$$\frac{12.5-10}{10\times 12.5}=\frac{1}{q_2}$$

$$\frac{1}{q_2}=\frac{2.5}{125}$$

$$q_2=\frac{125}{2.5}$$

$\boxed{q_2 = 50cm}$

Distance between object and final image = $d = p_1 + q_1 + p_2 + q_2$

$d = 50 + 12.5 + 12.5 + 56$

$\boxed{d = 125cm}$ **Ans.**

EXTRA PROBLEMS:

Q.1. If a magnifying glass produced an image of magnification 6, what is the power of the lens? What is the best position of the object, if the watch maker holds the same lens close to his eye to see the damaged spring of watch?
(1993, 2001, 2004, 2006, 2010, 2013)

Given Data:

Magnification = M = 6
Image distance = d = 25cm

To Find:

Power of Lens = P = ?
Object distance = p = ?

Solution:

Using relation:

$$M = 1+\frac{d}{f}$$

$$6 = 1+\frac{25}{f}$$

$$6-1=\frac{25}{f}$$

$$5=\frac{25}{f}$$

$$f=\frac{25}{5}$$

$\boxed{f = 5cm}$

Power of lens:

$$P=\frac{100}{f_{(cm)}}$$

$$P=\frac{100}{5}$$

$\boxed{P = 20 \text{ diopters}}$ **Ans.**

Using lens formula:

$$\frac{1}{f}=\frac{1}{p}=\frac{1}{q}$$

$q = -25cm$

$$\frac{1}{5}=\frac{1}{p}=\frac{1}{-25}$$

$$\frac{1}{5}+\frac{1}{25}=\frac{1}{p}$$

$$\frac{5+1}{25}=\frac{1}{p}$$

$$\frac{6}{25}=\frac{1}{p}$$

$$p=\frac{25}{6}$$

$\boxed{p = 4.16cm}$ **Ans.**

(8) SELF TEST:

(i) **The focal length of a simple magnifier is 12.5cm. What is the magnifying power of the final image formed 25cm infront of the eye?**

Ans. [M = 3]

(ii) **A magnifying glass gives a five time enlarged image at a distance of 25cm from the lens. Find the focal length of the lens.**

Ans. [f = 6.25cm]

(iii) **A converging lens of focal length 12.5cm is used as a magnifying glass. If the near point of the observer is 25cm and the lens is held close to the eye calculate.**

i. The distance of the object from the lens.

ii. Angular magnification.

Ans. [M = 3, p = 8.33cm]

Q.2. The convex lenses of power 2 diopters and 10 diopters are used as an objective and an eye piece of a telescope. Find the magnifying power and the length of the telescope when focused for infinity.

Given Data:

Power of objective $= P_o = 2$ diopters

Power of eye piece $= P_e = 10$ diopters

To find:

Magnifying power $= M = ?$

Length of telescope $= L = ?$

Solution:

$$P_o = \frac{1}{f_o}$$

$$f_o = \frac{1}{P_o}$$

$$f_o = \frac{1}{2}$$

$$f_o = 0.5m$$

$$f_o = 50cm$$

$$P_e = \frac{1}{f_e}$$

$$f_e = \frac{1}{P_e}$$

$$f_e = \frac{1}{10}$$

$$f_e = 0.1m$$

$$\boxed{f_e = 10cm}$$

Magnifying power of telescope:

$$M = \frac{f_o}{f_e}$$

$$M = \frac{50}{10}$$

$$\boxed{M = 5}$$ **Ans.**

Length of telescope:

$$L = f_o + f_e$$

$$L = 50 + 10$$

$$\boxed{L = 60cm}$$ **Ans.**

(9) Self Test:

i) **A telescope has objective and eye piece lenses of power 2 dioters and 20 diopters respectively. What is the magnifying power and length of the telescope if the telescope is in normal adjustment? (2009)**

Ans. [M = 10, L = 0.55cm]

ii) **An astronomical telescope has the magnification equal to 100, the power of its objective 0.2 diopters. What is the focal length of the eye piece and the length of the telescope? (2004 Supplementary)**

Ans. [f_e = 0.005m, L = 0.505m]

Q.3. The magnifying power of a telescope is 9 and the length of the telescope is 100cm. Determine the focal length of the objective and the eye piece.

Give Data:

Magnifying power of telescope = M = 9

Length of telescope = L = 100cm

To find:

Focal length of objective = f_o = ?

Focal length of eye piece = f_e = ?

Solution:

Magnifying power of telescope:

$$M = \frac{f_o}{f_e}$$

$$9 = \frac{f_o}{f_e}$$

$$\boxed{9f_e = f_o} \longrightarrow (1)$$

Length of telescope:

$$L = f_o + f_e$$

$$100 = f_o + f_e$$

$$100 = 9f_e + f_e$$

$$100 = 10\, f_e$$

$$\boxed{f_e = 10\text{cm}}$$ Ans.

from (1)

$$f_o = 9 \times 10$$

$$\boxed{f_o = 90\text{cm}}$$ Ans.

(10) SELF TEST:

An astronomical telescope has a length 105cm and magnification is 6 Determine the power of objective and eye-piece. (2012)

Ans. [f_o = 90cm, f_e = 15cm]