2010, 1985

PROBLEMS:

Q.1 An object is connected to one end of a horizontal spring whose other end is fixed. The object is pulled to the right (in the positive x-direction) by an externally applied force of magnitude 20N causing the spring to stretch through a displacement of 1 cm.

(a) Determine the value of force constant if, the mass of the object is 4kg.

(b) Determine the period of oscillation when the applied force is suddenly removed.

Given Data:

Force applied = F = 20N

Mass of the object = m = 4kg

Spring Stretch = x = 1cm

$= \frac{1}{100} = 0.01m$

To Find

Force constant = k = ?

Period of oscillation = T = ?

Solution:

Stretching Force

$$F = kx$$

$$k = \frac{F}{x}$$

$$k = \frac{20}{0.01}$$

$$\boxed{k = 2000\ N/m}$$ **Ans.**

The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2(3.14)\sqrt{\frac{4}{2000}}$$

$$T = 6.284 \times \frac{2}{44.72}$$

$$\boxed{T = 0.281S}$$ **Ans.**

Self Test (1).

A body of mass 2Kg attached to a spring is displaced through 0.04m from its equilibrium position and then released. If the spring constant is 200 N/m.

i) Find the period and frequency of vibration.

ii) The maximum velocity. $\left[V_{max} = x_0\sqrt{\frac{k}{m}}\right]$ **Ans.** $[V_{max} = 0.4m]$

iii) The maximum acceleration. $\left[a_{max} = -\frac{k}{m}x_0\right]$ **Ans.** $[a_{max} = -4m/s^2]$

Self test (2)

i) A body of mass 0.025 attached to a spring is displaced through 0.1m to the right of equilibrium position. If the spring constant is 0.4 N/m and its velocity at the end of this displacement be 0.4 m/s. calculate. *(2004)*

(a) Time Period (b) Total Energy (c) Amplitude

[Ans. T = 1.575 T.E = 0.004J x_o = 0.141m]

ii) A body of mass 32gm attached to an elastic spring is performing SHM. Its velocity is 0.4 m/s. When the displacement is 8cm toward right. If the spring constant 0.4 N/m, calculate (i) total energy (ii) the amplitude of its motion. *(2008)*

[Ans. T.E = 0.00384J x_o = 0.138m]

Q.2 A body hanging from a spring is set into motion and the period of oscillation is found to be 0.50 S. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest? *(2013)*

Given Data:

Time period = T = 0.50 S

To Find:

Extension in the spring = x = ?

Solution:

The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} \longrightarrow (1)$$

If the hanging body has mass, then stretching force on the spring is

$$F = mg \longrightarrow (2)$$

If x is the extension of the length of the spring due to weight of body, then

$$F = kx \longrightarrow (3)$$

Comparing e.g. (2) and e.g. (3)

$$mg = kx$$

$$\frac{m}{k} = \frac{x}{g}$$

Putting the value of $\frac{m}{k}$ in e.g. (1)

$$T = 2\pi\sqrt{\frac{x}{g}}$$

Squaring both sides.

$$T^2 = 4\pi^2\frac{x}{g}$$

$$x = \frac{T^2 g}{4\pi^2}$$

$$x = \frac{(0.50)^2(9.8)}{4(3.14)^2}$$

$$x = \frac{0.25 \times 9.8}{4 \times 9.872}$$

$$x = 0.062\text{m}$$

$$\boxed{x = 6.2\text{ cm}}$$ Ans.

Self test (3).

i) **A body hanging from a spring is set into motion and the period of oscillation is to be 0.8 sec. After the body has come to rest it is removed. How much shooters will the spring be when it comes to rest?** *(2002P.E)*

Ans. [x = 0.159m]

(ii) **When a mass m is hung on a vertical spring it stretches through 6cm. Determine its period of vibration if it is pulled down a little and released.** *(1996)*

Ans. [T = 0.491 S]

(iii) **A mass of 0.5 kg is suspended from a spring is stretched 0.089m. Calculate the period of oscillation of the mass when it is given a small displacement.**

Ans. [T = 0.598 Sec]

Example 8.4

A simple pendulum completes one oscillation in 2S. Calculate its length when $g = 9.8\ m/s^2$. **(2007 Failure)**

Given Data:

Time period = T = 2S

g = $9.8\ m/s^2$

To find:

Length of pendulum = l = ?

Solution:

The time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Squaring both sides.

$$T^2 = 4\pi\, l/g$$

$$L = \frac{gT^2}{4\pi^2}$$

$$L = \frac{9.8(2)^2}{4(3.14)^2}$$

$$L = \frac{9.8 \times 4}{4 \times 9.859}$$

$$\boxed{L = 0.994m}$$ **Ans.**

Self Test (4)

Calculate the length of a second pendulum at a place where $g = 10m/s^2$ **(1997)**

$$\boxed{L = 1.01m}$$

EXTRA PROBLEM:

Q.1 A mass at the end of a spring oscillates with simple harmonic motion with a period of 0.40 S, find the acceleration when displacement is 4.0cm.

(2003 P.M)

Given Data:

Time period = T = 0.40S

Displacement = x = 4cm = $\frac{4}{100}$ = 0.04m

To Find:

Acceleration = a = ?

Solution:

The acceleration of simple Harmonic motion is given

$$a = -\frac{k}{m}x \longrightarrow (1)$$

The time period of simple harmonic motion is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Squaring both sides.

$$T^2 = 4\pi^2\frac{m}{k}$$

$$\frac{k}{m} = \frac{4\pi^2}{T^2}$$

Putting the value of $\frac{k}{m}$ in equation. (1)

$$a = -\left(\frac{4\pi^2}{T^2}\right)x$$

$$a = -\frac{4 \times (3.14)^2}{(0.40)^2} \times (0.04)$$

$$a = -\frac{4 \times 9.859 \times 0.04}{0.16}$$

$$\boxed{a = -9.859\ m/s^2}$$ **Ans.**

Self test (5)

A mass at the end of a spring describes simple harmonic motion will a period of 0.50 S. Find the acceleration when the displacement is 5cm.

Ans. [$a = -7.88\ m/s^2$]

Q.2 A simple pendulum completes 4 vibration is 8 S on the surface of earth. Find out its time period on the surface of moon where acceleration due gravity is one-sixth that on earth. 2010, 2009, 2007, *1993, 2005 Supp, 2004 Supp.*

Solution:

4 vibration take time = 8 s

1 vibration take time $= \frac{8}{4} = 2s$

Time period on the surface of earth 2s.

Time period on the surface of earth is

$$T = 2\pi\sqrt{\frac{L}{g_e}} \longrightarrow (1)$$

Acceleration due to gravity on the surface of moon

$$g_m = \frac{1}{6}g_e$$

Therefore time period on the surface of moon

$$T' = 2\pi\sqrt{\frac{L}{g_m}}$$

$$T' = 2\pi\sqrt{\frac{L}{1/6g_e}}$$

$$T' = 2\pi\sqrt{6\frac{L}{g_e}}$$

$$T' = \sqrt{6}\ 2\pi\sqrt{\frac{L}{g_e}}$$

From equation (1)

$$T' = \sqrt{6}\ T$$

$$T' = \sqrt{6} \times 2$$

$$T' = 2.449 \times 2$$

$$\boxed{T' = 4.898\ S}$$ **Ans.**

Self Test (6)

A simple pendulum complete 6 vibration in 12 s on the surface of earth. Find the time period on the surface of Jupiter where acceleration due to gravity is 2.63 times that on earth.

Ans. [T = 1.233 S]

Q.3 Calculate length of second pendulum on the surface of moon where the acceleration due to gravity is 0.167 time that on the earth surface.

(1995)

Given Data:

$g_m = 0.167\, g_e$

$T = 2S$

To Find:

Length of pendulum = ?

Solution:

Time period of simple pendulum on the surface of moon is:

$$T = 2\pi \sqrt{\frac{L}{g_m}}$$

Squaring both sides

$$T^2 = 4\pi^2 L/g_m$$

$$L = \frac{g_m T^2}{4\pi^2}$$

$$L = \frac{0.167 g_e (2)^2}{4(3.14)^2}$$

$$L = \frac{0.167 \times 9.8 \times 4}{4 \times 9.869}$$

$$\boxed{L = 0.165\text{m}}$$ **Ans.**

Self Test (7)

(i) Find the length of second pendulum on planet Jupiter where value of 'g' is 2.63 times the value of 'g' on the surface of earth. ***(2000, 2006)***

Ans. [L = 2.61m]

(ii) What will be the length of second pendulum on the surface of moon having acceleration due to gravity equal to $1/6^{th}$ of acceleration due to gravity on earth? ***(2007, 2004 Supp.)***

Ans. [L = 0.165m]

Q.4 Determine the value of acceleration due to gravity at a place where time period of simple pendulum becomes twice its time period at Karachi. What will be the length of a second's pendulum at this place?

(2006 Failures)

Solution:

Acceleration due to gravity at Karachi.

$$g = \frac{4\pi^2 L}{T^2} \longrightarrow \quad (1)$$

Acceleration due to gravity at a place where time period of a simple pendulum becomes twice its time period at Karachi.

$$g' = \frac{4\pi^2 L}{T'^2}$$

$$T' = 2T$$

$$g' = \frac{4\pi^2 L}{(2T)^2}$$

$$g' = \frac{4\pi^2 L}{4T^2}$$

$$g' = \frac{1}{4}\left[\frac{4\pi^2 L}{T^2}\right]$$

From equation (1)

$$g' = \frac{1}{4} g$$

$$g' = \frac{1}{4} \times 9.8$$

$$\boxed{g' = 2.45 \text{ m/s}^2}$$

$$L = \frac{g' T^2}{4\pi^2}$$

$$L = \frac{2.45 \times 4}{4 \times 9.859}$$

$$\boxed{L = 0.248 \text{ m}}$$

Self Test (8)

Compute the acceleration due to gravity on the surface of the moon where a simple pendulum 1.5m long makes 100 vibration in 605 seconds. ***(2002 P.E)***
Ans. [$g = 1.617 m/s^2$]

Q3. A pipe has a length of 2.46m.

(a) Determine the frequencies of the fundamental mode and the first two overtones if the pipe is open at both ends. Take V = 344 m/s as the speed of sound in air.

(b) What are the frequencies determine in (a) if the pipe is closed at one end?

(c) For the case of open pipe. How many harmonics are present in the normal human being hearing range (20 to 2000 Hz)?

Solution:

Length of pipe	=	L	=	2.46m
Speed of sound of air	=	V	=	344 m/s

Required:

$\upsilon_1 = ?$ $\upsilon_2 = ?$ $\upsilon_3 = ?$

(a) Solution:

$$\upsilon_1 = \frac{\upsilon}{2L}$$

$$= \frac{344}{2(2.46)}$$

$$\boxed{\upsilon_1 = 70\ \text{Hz}}$$

As $\upsilon_N = N\upsilon_1$

$$\upsilon_2 = 2\upsilon_1$$

$$\upsilon_2 = 2 \times 70$$

$$\boxed{\upsilon_2 = 140\ \text{Hz}}$$

$$\upsilon_3 = 3\upsilon_1$$

$$= 3 \times 70$$

$$\boxed{\upsilon_3 = 210\ \text{Hz}}$$

(b) $(\upsilon_1)_{closed} = ?$ $(\upsilon_2)_{closed} = ?$ $(\upsilon_3)_{closed} = ?$

As $(\upsilon_1)_{closed} = \frac{\upsilon}{4L}$

$$(\upsilon_1)_{closed} = \frac{344}{4(2.46)}$$

$$\boxed{(\upsilon_1)_{closed} = 35\ \text{Hz}}$$

And $(\upsilon_N)_{closed} = (2N - 1)(\upsilon_1)_{closed}$

So $(\upsilon_2)_{closed} = (2 \times 2 - 1) \times 35$

$$\boxed{(\upsilon_2)_{closed} = 105\ \text{Hz}}$$

$(\upsilon_3)_{closed} = (2 \times 3 - 1)(\upsilon_1)_{closed}$

$= 5 \times 35$

$$\boxed{(\upsilon_3)_{closed} = 175\ \text{Hz}}$$

(c) $\upsilon_1 = 70\text{Hz}$

$\upsilon_N = 2000° - 20$

$= 19980\text{Hz}$

$N = ?$

As $\upsilon_N = N\upsilon_1$

$$N = \frac{\upsilon_N}{\upsilon_1} = 285 \text{ harmonics}$$

Thus there are 285 harmonics.

Q7. The frequency of the second harmonic of an pipe (open at both ends) is equal to the frequency of the second harmonic of a closed pipe (open at one end).

(a) Find the ratio of the length of the closed pipe to the length of the open pipe.

(b) If the fundamental frequency of the open pipe is 256 Hz, what is the length of pipe? (Use v = 340 m/s)

Solution:

Let the length of open pipe = L_1

And the length of closed pipe = L_2

Given:

$(\upsilon_2)_{open} = (\upsilon_2)_{closed}$

Required:

(a) $\frac{L_2}{L_1} = ?$

(b) If $(\upsilon_1)_{open} = 256\text{Hz}$

$V = 340\text{m/s}$

Then $L_1 = ?$ $L_2 = ?$

(a) As $(\upsilon_2)_{open} = (\upsilon_2)_{closed}$

$2(\upsilon_1)_{open} = (2 \times 2 - 1)(\upsilon_1)_{closed}$

$$2\frac{\upsilon}{2L_1} = 3.\frac{\upsilon}{4L_2}$$

$$\boxed{\frac{L_2}{L_1} = \frac{3}{4}}$$

(b) As $(\upsilon_1)_{open} = \frac{\upsilon}{2L_1}$

$$L_1 = \frac{\upsilon}{2(\upsilon_1)_{open}}$$

$$\boxed{L_1 = \frac{340}{2 \times 256} = 0.66\text{m}}$$

And $\frac{L_2}{L_1} = \frac{3}{7} \Rightarrow L_2 = \frac{3}{4} \times L_1$

$$\boxed{L_2 = 0.498\text{m}}$$

Q4. **A standing wave is established in 120 cm long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz. Determine the wave length and the fundamental frequency.** *(1999)*

Given Data:

Length of string = L = 120 cm = $\frac{120}{100}$ = 1.20 m

Frequency = υ_4 = 120 Hz

No. of Loops = n = 4

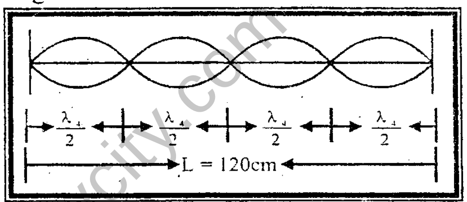
To Find:

Wave length = λ_4 = ?

Fundamental frequency = υ_1 = ?

Solution:

As we know that the string vibrates in four segments i.e.



$$L = \frac{\lambda_4}{2} + \frac{\lambda_4}{2} + \frac{\lambda_4}{2} + \frac{\lambda_4}{2}$$

$$L = \frac{\lambda_4 + \lambda_4 + \lambda_4 + \lambda_4}{2}$$

$$L = \frac{4\lambda_4}{2}$$

$$L = 2\lambda_4 \qquad OR \qquad \lambda_4 = \frac{L}{2}$$

$$\lambda_4 = \frac{1.20}{2}$$

$$\boxed{\lambda_4 = 0.60\ \text{m}}$$

The relation between fundamental frequency υ_1 and nth υ_n is

$$\upsilon_N = N\upsilon_1$$

$$\upsilon_4 = 4\upsilon_1$$

$$\upsilon_1 = \frac{\upsilon_4}{4}$$

$$\upsilon_1 = \frac{120}{4}$$

$$\boxed{v_1 = 30.0\ \text{Hz.}}$$

Self Test:

(i) **A standing wave is established in a 2.4 m long string fixed at both ends. The string vibrates in four segments when driven at 200 Hz. Determine**
(i) The wave length. (ii) The fundamental frequency.
Ans. [$\lambda_4 = 1.2$m $\upsilon_1 = 50$Hz]

(ii) **A standing wave is established in a string of length 80 cm, fixed at both ends. The string vibrates in five segments when driven at 120 Hz, calculate its wave length and fundamental frequency.** *(2002 Supplementary)*
Ans. [$\lambda_5 = 0.32$m $\upsilon_1 = 24$Hz]

(iii) **A standing wave established in a 110 cm long fixed at both ends. The string vibrates in four segments when driven at 110 Hz. Determine the wave length and fundamental frequency.** *(2004 Failure)*
Ans. [$\lambda_4 = 0.55$m $\upsilon_1 = 27.5$Hz]

(iv) **A standing wave is established in a 2.4m long sting fixed at both ends. The string vibrates in four segments when driven at 200 Hz. Determine the velocity of the wave.** *(2007 Supplementary)*
Ans. [V = 240m/s]

EXTRA PROBLEMS:

Q.1 The frequency of vibration produced in a string is 150 Hz when it vibrates is 3 loops. The length of the string is 120 cm. Find the velocity wave. Also find the frequency when it vibrates is 5 loops.

Given Data:

Frequency = ν_3 = 150 Hz.

No. of loops = n = 3

Length of string = L = 120 = $\frac{120}{100}$ = 1.20m

To Find:

Speed of wave = V = ?

ν_5 = ?

Solution:

The relation between fundamental frequency and nth ν_n is

$$\nu_n = n\nu_1$$

$$\nu_3 = 3\nu_1$$

$$\nu_1 = \frac{\nu_3}{3} = \frac{150}{3} = 50 \text{ Hz}$$

$$\nu_5 = 5\nu_1$$

$$\nu_5 = 5 \times 50$$

$$\boxed{\nu_5 = 250 \text{ Hz.}}$$

$$\nu_1 = \frac{V}{2L}$$

$$V = 2L\nu_1$$

$$V = 2 \times 1.2 \times 50$$

$$\boxed{V = 120 \text{ m/s}}$$

Self Test (2)

(i) In a sonometer a wire of length 1 m when pluck at the centre vibrates with a frequency 250Hz, calculate the wave length and speed of wave in the wire.

(2009, 2005)

Ans. [V = 500m/s, λ_1 = 2m]

(ii) The frequency of a string is 125 Hz. When it is vibration in 5 segments. Calculate the frequency when the string vibrates in three and four segments. What is the speed of the wave is it if the length of the string is 0.8m.

Ans. [υ_3= 75Hz, υ_4 = 100Hz, V = 40m/s]

Q.2 **A 2 meter long string with mass 0.004kg, when one end is fixed, is stretched horizontally by passing over a pully attached with 1 kg mass to its ends, find the speed and the frequency of fifth and sixth harmonic.**
(2002, 2003, 2005 Supp., 2010)

Given Data:

Length of string = L = 2m
Mmass of string = m = 0.004 Kg
M = 1 Kg

To Find:

Speed of wave = V = ?
Frequency of fifth harmonic = ν_5 = ?
Frequency of sixth harmonic = ν_6 = ?

Solution:

Linear density = $\mu = \frac{m}{l} = \frac{0.004}{2}$
$\mu = 0.002$ kg/m

Tension in the string = $T = Mg = 1 \times 9.8$
$T = 9.8$ N

Speed of transverse wave in stretched string

$$V = \sqrt{T/\mu}$$
$$V = \sqrt{\frac{9.8}{0.002}}$$
$$V = \sqrt{4900}$$
$$\boxed{V = 70 \text{ m/s}}$$

Fundamental frequency

$$\nu_1 = \frac{V}{2L}$$
$$\nu_1 = \frac{70}{2 \times 2}$$
$$\nu_1 = \frac{70}{4}$$
$$\boxed{\nu_1 = 17.5 \text{ Hz}}$$
$$\nu_5 = 5\nu_1$$
$$\nu_5 = 5 \times 17.5$$
$$\boxed{\nu_5 = 87.5 \text{ Hz}}$$
$$\nu_6 = 6\nu_1$$
$$\nu_6 = 6 \times 17.5$$
$$\boxed{\nu_6 = 105 \text{ Hz}}$$

Self Test (3)

A 50cm wire of linear density 2.5×10^{-3} kg/m is stretched between two fixed supports by applying the tension of 25N, calculate. *(2006 Failure)*

(i) **The speed of the wave on the wire**
(ii) **The fundamental frequency of the vibration of wire.**

Ans. [$V = 100$m/s, $\upsilon_1 = 100$Hz]

Self Test (4)

A string 1m long and of mass of 0.004kg is stretehed with a force. Calculate the force if the speed of wave in the string is 140m/sec. *(2012)*

Ans. [F = 78.4N]

NUMERICAL:

Q.5 Calculate the speed of sound in air at atmospheric pressure $p = 1.01 \times 10^5$ N/m^2, taking $\Upsilon = 1.40$ and $\rho = 1.2$ kg/m^3.

Given Data:

$P = 1.01 \times 10^5$ N/m^2

$\gamma = 1.4$

$\rho = 1.2$ Kg Kg/m^3

To Find:

$V = ?$

Using the relation

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

$$V = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.2}}$$

$$V = \sqrt{11.78 \times 10^4} = 3.43 \times 10^2$$

$$\boxed{V = 343 \text{ m/s}}$$ **Ans.**

Self test (1):

Find the speed of sound in air at S.T.P what will be the speed of sound at 37 c°
(Given: $\rho = 1.29$ Kg/m', $\gamma = 1.42$, $P = 1.01 \times 10^5$ N/m^2) **(2002 P.M)**

Ans. [$V_{STP} = 333.43$m/s $V_{37} = 355.30$m/s]

Q.6 A Sound wave propagating in air has a frequency of 4000Hz. Calculate the percent fractional change in wave length when wave front initially in a region where T = 27°C enter a region where the air temperature decreases to 10°C.

Given Data:

Frequency of sound wave = υ = 4000 Hz

Initial temperature = T_1 = 27C° + 273 = 300K

Final temperature = T_2 = 10C° + 273 = 283K

To Find:

$$\frac{\Delta\lambda}{\lambda_1} \% = ?$$

Using the relation for speed of sound wave at temperature T

$$V = 332\sqrt{\frac{T}{273}}$$

$$V_1 = 332\sqrt{\frac{T_1}{273}}$$

$$V_1 = 332\sqrt{\frac{300}{273}}$$

$$V_1 = 332 \times 1.048$$

$$\boxed{V_1 = 348.02 \text{ m/s}}$$

$$V_2 = 332\sqrt{\frac{T_2}{273}}$$

$$V_2 = 332\sqrt{\frac{283}{273}}$$

$$V_2 = 332 \times 1.018$$

$$\boxed{V_2 = 338.02\ m/s}$$

Wave length of sound at T

$$\lambda = \frac{V}{\upsilon}$$

Wave length of sound wave at T_1 is

$$\lambda_1 = \frac{V_1}{\upsilon}$$

$$\lambda_1 = \frac{348.02}{4000} = 0.0870\ m$$

Wave length of sound wave at T_2 is

$$\lambda_2 = \frac{V_2}{\upsilon}$$

$$= \frac{338.02}{4000}$$

$$\lambda_2 = 0.0845m$$

Change in wave length $= \Delta\lambda = \lambda_1 - \lambda_2$

$$\Delta\lambda = 0.870 - 0.845$$

$$\Delta\lambda = 0.0025\ m$$

$$\frac{\Delta\lambda}{\lambda_1}\% = \frac{\Delta\lambda}{\lambda_1} \times 100$$

$$= \frac{0.0025}{0.087} \times 100$$

$$= 2.9\ \%$$

Self test (2):

A sound wave of frequency 500Hz in air enters from a region of 25°C to a regior of 5°C. Calculate the percentage fractional charge in wavelcngth. **(2013)**

Q.8 A 256 Hz tuning fork produces four beats per second when sounded with another fork of unknown frequency. What are two possible values for the unknown frequency?

Given Data:

Known frequency = υ_1 = 256Hz

No. of beats per second = υ_b = 4 beats/s

To Find:

Unknown frequency = υ_2 = ?

Solution:

$\upsilon_1 - \upsilon_2 = \pm \upsilon_b$

$256 - \upsilon_2 = \pm 4$

$256 - \upsilon_2 = 4$ $\qquad$ $256 - \upsilon_2 = -4$

$256 - 4 = \lambda_2$ $\qquad$ $256 + 4 = \upsilon_2$

$\boxed{\upsilon_2 = 252 \text{ Hz}}$ $\qquad$ $\boxed{\upsilon_2 = 260 \text{ Hz}}$

$\boxed{\upsilon_2 = 252 \text{ Hz or } 260 \text{ Hz}}$

Self test (2):

A 250 Hz tuning fork produced two beat/s when sounded with on at fork of unknown frequency what are two possible values of frequency. **(2005 Failures)**

Ans. **[υ_2 = 248 Hz, υ_2 = 252 Hz]**

Q.9 An ambulance travels down a highway at a speed of 75mi/h. Its siren emits sound at a frequency of 400Hz. What is the frequency heard by a person in a car traveling at 55mi/h in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance.

Given Data:

Speed of ambulance = V_s = 75 mil/h

Speed of car = V_0 = 55 mil/h

Frequency of sound = υ = 400Hz

To Find:

Frequency of sound heard by the listener = υ' = ?

$$\upsilon' = \left(\frac{750+55}{750-75}\right) 400$$ [Velocity of sound = V = 750 mil/h]

$$\upsilon' = \frac{805}{675} \times 400$$

$\boxed{\upsilon' = 477 \text{ Hz}}$ **Ans.**

When both source and listener moves away from each other, then the frequency of sound as heard by the listener is given by the relation.

$$\upsilon' = \left(\frac{V - V_0}{V + V_s}\right) \lambda$$

$$\upsilon' = \frac{750-55}{750+75} \times 400$$

$$\upsilon' = \frac{695}{825} \times 400$$

$\boxed{\upsilon' = 337 \text{ Hz}}$ **Ans.**

Q.10 A car has siren sounding a 2 kHz tone. What frequency will be detected as stationary observer as the car approaches him at 80 km/h?

Speed = 1200 km/h. (1998)

Given Data:

The frequency of the siren = υ = 2kHz = 2000 Hz

The speed of the source = V_S = 80km/h

The speed of the sound = V = 1200 Km/h

To Find:

The frequency of the sound heard by the listener = υ' = ?

Solution:

When source moves towards stationary listener, then the frequency of sound heard by listener is given by the relation.

$$\upsilon' = \left(\frac{V}{V - V_s}\right)\upsilon$$

$$\upsilon' = \frac{1200}{1200-80} \times 2000$$

$$\upsilon' = \frac{1200}{1120} \times 2000$$

$$\upsilon' = 2143\ \text{Hz}$$

$$\boxed{\upsilon' = 2.143\text{K Hz}}$$ Ans.

Self Test (4):

i) A car has been sounding a 4 kHz tone. What frequency will be detected by a stationary listener as the car approaches him at 50 Km/h?

The speed of sound = 1200 km/h. *(1999)*

Ans. [υ' = 4173.9 Hz]

Self Test (5):

An ambulance has a siren producing sound waves of frequency 2000 Hz and moving towards a listener standing on a bus stop. If the frequency heard by him is 2150 Hz. Find the speed of the ambulance (Speed of sound in air 340m/s)

(2001)

Ans. [υ_s= 23.72 m/s]

Self Test (6):

A Car has a siren sending 2.5 KHz tone what frequency will be detected by a stationary listener when the car approaches him at 72 km/h (velocity of sound = 1200 km/h)

Ans. [υ' = 2659.57 Hz]

EXTRA PROBLEMS:

Q.1 A source of sound and listener are moving toward each other with velocity which are 0.5 times and 0.2 times the speed of sound respectively. If the source is emitting 2KHz. Tone. Calculate the frequency heard by the listener.

(1995, 2003 P.E)

Given Data:

The speed of source	=	V_S	=	0.5V
The speed of listener	=	V_0	=	0.2V
Frequency of sound	=	υ	=	2 KHz

To Find:

Frequency of sound heard by the listener = υ^1 = ?

Solution:

When both the source and the listener moving towards each other, then the frequency of sound as heard by the listener is given by the relation.

$$\upsilon' = \left(\frac{V+V_0}{V-V_s}\right)\upsilon$$

$$\upsilon' = \left(\frac{V+0.2V}{V-0.5V}\right)2000$$

$$\upsilon' = \frac{V(1+0.2)}{V(1-0.5)} \times 2000$$

$$\upsilon' = \frac{1.2}{0.5} \times 2000$$

$$\upsilon' = 4800 \text{ Hz}$$

$$\boxed{\upsilon' = 4.8 \text{ KHz}} \quad \textbf{Ans.}$$

Self Test (7):

Two cars are moving straight to each other from opposite direction with the same speed. The horn of one is blowing with the frequency of 3000 Hz and is heard by people in the car with the frequency of 3400Hz. Find the speed of the cars if the speed of sound in are in 340m/s. **(2003 P.M)**

Ans. **[v′ = 20.75 m/s]**

Q.2 **Calculate the intensity level of the faintest audible sound when Io = 10^{-12} watt/m²** **(1993)**

Intercity level in decibel is.

$$B = 10\log\left(\frac{I}{I_0}\right)$$

$$B = 10\log\frac{10^{-12}}{10^{-12}}$$

$$B = 10\log 1 \qquad \log 1 = 0$$

$$B = 10 \times 0$$

$$\boxed{B = 0}$$

Self Test: (7)

Calculate the intercity level of the whisper when $Io = 10^{-12}$ watt/m^2 and intercity of whisper $= I = 10^{-10}$ watt/m^2

Ans. [$\beta = 20$ dB]

Q.3 Find the velocity of sound in a gas is which two waves of wave length 0.80m and 0.81m produced 5 beats per second. (2012)

Given Data:

Wave length of first wave $= \lambda_1 = 0.80\text{m}$

Wave length of second wave $= \lambda_2 = 0.81\text{m}$

No. of beats per second $= \upsilon_b = 5 \text{ beats/s}$

To Find:

Speed of sound $= V = ?$

Solution:

$$\upsilon_1 - \upsilon_2 = \upsilon_b$$

$$\upsilon_1 = \frac{V}{\lambda_1} \quad \upsilon_2 = \frac{V}{\lambda_2}$$

$$\frac{\upsilon}{\lambda_1} - \frac{\upsilon}{\lambda_1} = \upsilon_b$$

$$\frac{V}{0.80} - \frac{V}{0.81} = 5$$

$$\frac{0.81V - 0.8V}{0.80 \times 0.81} = 5$$

$$\frac{0.01V}{0.80 \times 0.81} = 5$$

$$V = \frac{5 X 0.80 X 0.81}{0.01}$$

$$\boxed{V = 324 \text{ m/s}}$$ **Ans.**

Q.4 Find the speed of sound in air at OC° Given $\gamma = 1.4$ for air, molecular mass of air M = 0.0288kg/mal. R = 8.314 J/mol. K. (2000)

Given Data:

$T = OC° + 273 = 273\text{K}$

$\gamma = 1.4$

$M = 0.0288 \text{ Kg/mol.}$

$R = 8.314 \text{J/mol K}$

To Find

$V = ?$

Solution:

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$V = \sqrt{\frac{1.4 X 8.314 X 273}{0.0288}}$$

$$V = \sqrt{\frac{3177.61}{0.0288}}$$

$$V = \sqrt{110333.7}$$

$$\boxed{V = 332.16 \text{ m/s}}$$ **Ans.**

Q.5 **A Car has its siren sounding 2KHz tone. If the frequency heared by a stationary observer is 2, 143Hz. Find the speed at which it approaches stationary observers (Speed of sound = 340 m/s).**

GIVEN DATA:

υ = 2 KHz = 2000 Hz

υ' = 2143Hz

V = 340 m/sec

TO FIND:

V_S = ?

SOLUTION:

$$\upsilon' = (\frac{V}{V - V_s})\upsilon$$

$$V - V_s = \frac{\upsilon}{\upsilon'}V$$

$$V_s = V - \frac{\upsilon}{\upsilon'}V$$

$$= (1 - \frac{\upsilon}{\upsilon'})V$$

$$= 1 - \frac{2000}{2143})340$$

$$= (1 - 0.3933271)\,340$$

$$= (0.06673)\,340$$

$$\boxed{V_s = 22.688 \text{ m/sec}}$$

SELF TEST (8):

A stationary source is producing 2000Hz tone. An observer in a car observe it as 1750Hz. Find out speed of Car (Speed of sound in air is 345 m/sec).

Ans. [V_s = 49.28 m/s]

SELF TEST (9):

Calculate the speed of sound at an altitude of 4000m above the car this surface where temperature is about 40°C (Speed of Sound = 322m/s atoi)

Ans. [V_T = 355.49 m/s]