Numerical:

Q1. Calculate the work done by a force $\vec{F}$ specified by $\vec{F}=3\hat{i}+4\hat{j}+5\hat{k}$ in displacing a body from position A to position B along a straight path the position A and B are respectively given by: 1994, 1999, 2009 F

$$\vec{r}_A = 2\hat{i}+5\hat{j}-2\hat{k}$$

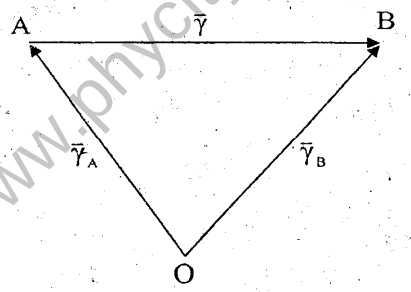
$$\vec{r}_B = 7\hat{i}+3\hat{j}-5\hat{k}$$

GIVEN DATA:

$$\vec{F}=3\hat{i}+4\hat{j}+5\hat{k}$$

$$\vec{r}_A = 2\hat{i}+5\hat{j}-2\hat{k}$$

$$\vec{r}_B = 7\hat{i}+3\hat{j}-5\hat{k}$$



TO FIND:

Work done = ?

SOLUTION:

Displacement from A to B

$$\vec{r}=\vec{r}_B-\vec{r}_A$$

$$\vec{r}=7\hat{i}+3\hat{j}-5\hat{k}-(2\hat{i}+5\hat{j}-2\hat{k})$$

$$\vec{r}=7\hat{i}+3\hat{j}-5\hat{k}-2\hat{i}-5\hat{j}+2\hat{k}$$

$$\vec{r}=5\hat{i}-2\hat{j}-3\hat{k}$$

We know

$$\text{Work done} = \vec{F}\bullet\vec{r}$$

$$= (3\hat{i}+4\hat{j}+5\hat{k})\bullet(5\hat{i}-2\hat{j}-3\hat{k})$$

Work done = 3 x 5 + 4(-2) + 5 (-3)

$\because \quad \vec{A}\bullet\vec{B} = AxBx + Ay\,By + Az\,Bz$

Work done = ~~15~~ –8 –~~15~~

Work done = –8 unit Ans.

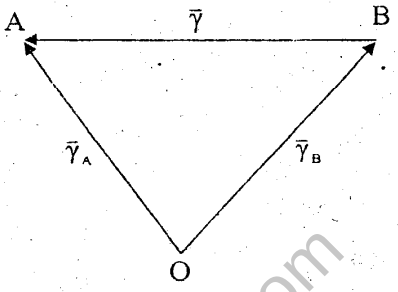
SELF TEST (1):

Calculate the work done by a force $\vec{F}$ specified by $\vec{F} = 2\hat{i} + 2\hat{j} - \hat{k}$ in displacing a body from position B to position A along a straight path. The position B and A are respectively given as.

$$\hat{r}_B = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{r}_A = 4\hat{i} + 6\hat{j} + \hat{k}$$

19 unit Ans.



Q2. A 2000kg car travelling at 20m/s comes to rest on a level ground in a distance 100m. How large is the average frictional force tending to stop it?

GIVEN DATA:

Mass of car = m = 2000kg

Initial velocity of car = v_i = 20m/s

Final velocity of car = v_f = 0

Distance covered by car = S = 100m

TO FIND:

Frictional force = f = ?

SOLUTION:

Using work – energy relation

Work done = Final K.E – Initial K.E

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i$$

$$\because \quad W = F \times S$$

$$f \times 100 = \frac{1}{2}(2000)(0)^2 - \frac{1}{2}(2000)(20)^2$$

$$f \times 100 = 0 - \frac{1}{2} \times 2000 \times 400$$

$$100f = -400000$$

$$f = -\frac{400000}{100}$$

f = – 4000 N Ans.

SELF TEST (2):

Q3. A 100kg man is in car travelling at 20m/s.

a) Find his kinetic energy

b) The car strikes a concrete wall and comes to rest after the front of the car has collapsed 1m. The man is wearing a seat belt and harness. What is the average force exerted by the belt and harness during the crash?

Q4. When an object is thrown upward, it rises to a height 'h' How high is the object, in terms of 'h' when it has lost one third of its original kinetic energy.

SOLUTION:

let at height of h', object has lost one third of its original K.E

According to law of conservation of energy

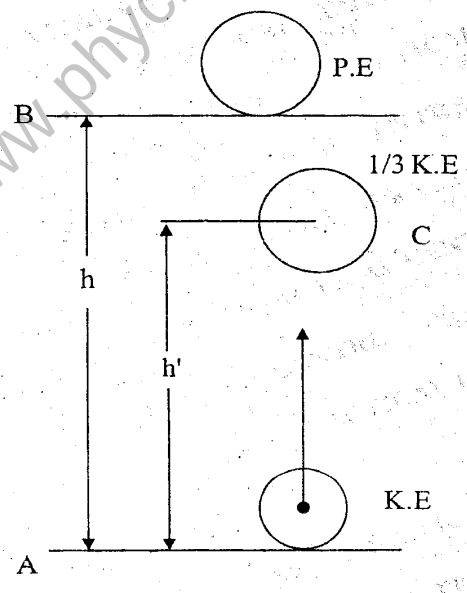
Loss of K.E = Gain of P.E

$$\frac{1}{3}K.E = P.E$$

$$\frac{1}{3} \times \frac{1}{2} mv^2 = mgh'$$

$$\frac{1}{6}v^2 = gh'$$

$$h' = \frac{1}{6g}v^2 \longrightarrow (i)$$



Using the equation of motion

$2as = v_f^2 - v_i^2$

$a = -g \qquad S = h$

$v_f = 0 \qquad v_i = v$

$2(-g)h = (0)^2 - (v)^2$

$+2gh = +v^2$

$v^2 = 2gh$, put in $\longrightarrow$ (i)

$$h' = \frac{1}{6} \times 2gh$$

$$h' = \frac{1}{3}h$$

SELF TEST (3):

When an object is thrown upward, it rises to height 'h' How high is the object, in term of 'h' when it has lost one fourth of its original kinetic energy.

$\left(\text{Ans: } \frac{1}{4}h\right)$

Q5. A pump is needed to lift water through a height 2.5m at the rate of 500g/min. What must the minimum horse power of the pum be?

(2011, 2010)

GIVEN DATA:

Height = h = 2.5m

Rate of lifting water = 500g/min

TO FIND:

P_{hp} = ?

SOLUTION:

Mass of water lifted per second $= \frac{500}{60}$

$= 0.00833kg$

P.E acquired by water per second = mgh

$= 0.00833 \times 9.8 \times 2.3$

$= 0.2042J/sec$

P = 0.2042 J/S

P = 0.2042 watt

1hp = 746 watt

$P = \frac{0.2042}{746}$ hp $\quad \because$ 1hp = 746 watt

P = 0.000274hp

$\boxed{P = 2.74 \times 10^{-4} \text{ hp}}$ Ans.

Q6. A horse pulls a cart horizontally with a force of 40lb. at an angle of 30° above the horizontal and moves along at a speed of 6.0 mile/h.

(a) How much work does the horse do in 10 minutes.

(b) What is the power output of the horse.

GIVEN DATA:

$\theta = 30^\circ$

$V = 6 \text{ mile/hour} = \dfrac{6 \times 1760 \times 3}{60 \times 60} = 8.8 \text{ft/s}$ $\because$ [1 mile = 1760 yard, 1 yard = 3 feet]

$F = 40\text{lb}$

$t = 10 \text{ minutes} = 10 \times 60 = 600\text{sec}$

TO FIND:

Work done = ?

Power = ?

SOLUTION:

$S = vt = 8.8 \times 600 = 5280\text{ft}$

$\text{Work} = FS \cos\theta = 40 \times 600 \text{ Cos } 30^\circ$ $\because \text{Cos}30^\circ = 0.866$

$= 40 \times 5280 \text{ Cos } 30^\circ$

$= 40 \times 5280 \times 0.866$

$= 1.8 \times 10^5 \text{ ft-lb}$

$\text{Power} = Fv \cos\theta$

$= 40 \times 8.8 \text{ Cos } 30^\circ$ $\because \text{Cos}30^\circ = 0.866$

$= 40 \times 8.8 \times 0.866$

304.8 ft –lb/sec

$P_{hp} = \dfrac{304.8}{550} = 0.55 \text{ hp}$

$P_{hp} = 0.55$ hp Ans.

Q7. A body of mass 'm' accelerates uniformly from rest to speed v_f in time t_f. Show that the work done on the body as a function of time 't' in term of v_f and t_f is.

$$\frac{1}{2} \quad m \quad \frac{v_f^2}{t_f^2} t^2$$

SOLUTION:

Work done $= F \times S \longrightarrow$ (1)

According to Newton's second law of motion $F = ma$

Work done $= ma \times S \longrightarrow$ (2)

We find 'a' in term of v_f and t_f from the equation

$$v_f = v_i + at$$

$$v_i = 0 \quad t = t_f$$

$$v_f = 0 + a\, t_f$$

$$a = \frac{v_f}{t_f}$$

Also we find S from the equation

$$S = vit + \frac{1}{2}at^2$$

$$v_i = 0$$

$$S = \frac{1}{2}at^2$$

$$\text{Work done} = m \times \frac{v_f}{t_f} \times \frac{1}{2} at^2$$

$$\text{Work done} = \frac{1}{2} m \frac{v_f}{t_f} \times \frac{v_f}{t_f} t^2$$

$$\boxed{\textbf{Work done} = \frac{1}{2} m \frac{v^2_f}{t^2_f} t^2}$$

Q.8 A rocket of mass 0.2kg is launched from rest. It reaches a point "p" lying at a height 30.0m above the surface of the earth from the starting point. In the process + 425J of work is done on the rocket by the burning chemical propellant. Ignoring air resistance and the amount of mass lost due to the burning propellant. Find the speed "V_f" of the rocket at the point "P".

GIVEN DATA:

m = 0.2kg

h = 30.0 m

work = + 425J

TO FIND:

$V_f = ?$

SOLUTION:

Gain in K.E + Gain in P.E = work

$$\frac{1}{2}mv_f^2 + mgh = 425$$

$$\frac{1}{2}x0.2v_f^2 + 0.2 \times 9.8 \times 30 = 425$$

$$0.1\ v_f^2 + 58.8 = 425$$

$$0.1\ v_f^2 = 425 - 58.8$$

$$0.1\ v_f^2 = 366.2$$

$$V_f^2 = \frac{366.2}{0.1}$$

$$V_f^2 = 3662$$

$\boxed{V_f = 60.5\ m/s}$ **Ans.**

Self Test# 4:

A horse pulls a cart horizontally with a force of 40N at an angle of 25 above the horizontal and moves along at a speed of 15m/s. How much work will horse do in 10 minutes? What is power out put of horse? Give your answer is horse power.

(2013)

EXTRA PROBLEMS

Q.1. Neutron travels a distance of 12m in time interval of 3.6×10^{-6} Sec. Assuming the speed to be constant find the kinetic energy $m_n = 1.7 \times 10^{-27}$ kg. **(2005 Failures)**

Given Data:

$S = 12m$

$t = 3.6 \times 10^{-6}$ S

$m_n = 1.7 \times 10^{-27}$kg

To Find:

$$V = \frac{S}{t}$$

$$V = \frac{12}{3.6 \times 10^{-6}}$$

$$V = 3.3 \times 10^{6} \text{ m/s}$$

$$K.E. = \frac{1}{2}mv^2$$

$$K.E. = \frac{1}{2} \times 1.7 \times 10^{-27} (3.3 \times 10^{6})$$

$$K.E. = 9.44 \times 10^{-15} \text{ Jule}$$

Q.2. A ball of mass 100gm in thrown up in air vertically and reaches a height of 9.8m. Calculate the velocity with which it is thrown and its initial energy (Neglect air friction and take $g = 10$ m/sec^2) **(2002)**

Given Data:

$m = 100gm = \frac{100}{1000} = 0.1kg$

$h = 9.8m$

$g = 10 \text{ m/s}^2$

To Find:

$V = ?$

$K.E = ?$

Less of K.E. = Gain of P.E.

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$v^2 = 2 \times 10 \times 0.1$$

$$v^2 = 2$$

$$\boxed{v = 1.414 \text{ m/s}^2}$$

$$K.E = \frac{1}{2}mv^2$$

$$K.E. = \frac{1}{2} \times 0.01 \times (1.414)^2$$

$$\boxed{K.E. = 0.1 \text{ Jule}}$$

Q.3. The jet engine of an airline develops of forward force 200KN on the aeroplane when the aeroplane is flying 250 m/s. calculate the horse power which is being develop when the force and the velocity are parallel.

Given Data:

$F = 200\ KN = 200 \times 10^3\ N$

$v = 250\ m/s$

$\theta = 0°$

To Find

$P_{hp} = ?$

Power = $FV\ Cos\theta$

$= 200 \times 10^3 \times 250\ Cos\theta°$ $\quad \because Cos\theta° = 1$

$$\boxed{P = 50 \times 10^6 \text{ watt}}$$

$1hp = 746\ J$

$$P_{hp} = \frac{50 \times 10^6}{746}$$

$$\boxed{P_{hp} = 6702412 \text{ hp}}$$