PROBLEMS:

Q.1 A 10kg mass is at a distance of 1m from a 100kg mass. Find the gravitational force of attraction when.

(a) 10kg mass exerts force on the 100 kg mass.

(b) 100kg mass exerts force on the 10kg mass.

Give Data:

Mass of body A = m_A = 10kg
Mass of body B = m_B = 100kg
Distance between them = r = 1m

Constant:

Gravitational constant = G = 6.673×10^{-11} Nm^2/kg^2

To Find:

Gravitational Force = ?

Solution:

By Newton's Law of gravitation force extered by body A on the body B.

$$F_{AB} = G \frac{m_A m_B}{6}$$

$$F_{AB} = \frac{6.673 \times 10^{-11} \times 10 \times 100}{1}$$

$$F_{AB} = 6.673 \times 10^{-11} \times 10^3$$

$$\boxed{F_{AB} = 6.673 \times 10^{-8} N}$$ **Ans.**

The force exerted by body B on the body A

$$F_{BA} = \frac{G\, m_B m_A}{r}$$

$$F_{BA} = \frac{6.673 \times 10^{-11} \times 100 \times 10}{1}$$

$$\boxed{F_{BA} = 6.673 \times 10^{-8} N}$$ **Ans.**

This shows that both the bodies will attract each oth with equal and opposite force

Self Test:

Q.5 The mass of the planet Jupiter is 1.9×10^{27} kg and that of the sun is 2.0×10^{30}kg. If the average distance between them is 7.8×10^{11}m. Find the gravitational force of the sun on Jupiter. **(2003 P.M)**

Ans. $\boxed{41.68 \times 10^{22} N}$

Q.7 Compute the gravitational attraction between two college student of mass 80 and 50kg, respectively 2m apart from each other. Is this force worth worrying about?

Ans. $\boxed{6.673 \times 10^{-8} N}$

No, the force is not worth worrying about because the mutual gravitational force of attraction is negligible as compared to attraction of earth.

Q.8 Determine the gravitational attraction between the proton and electron in hydrogen atom assuming that the electron describes a circular orbit with a radius of 0.53×10^{-10}m.

Ans. $\boxed{3.60 \times 10^{-47}\text{N}}$

Q.2 Compute the gravitational acceleration at the surface of the planet Jupiter which has a diameter as 11times as compared with that of earth and a mass equal to 318 times that of earth. **(2002 P.E, 2003 Failures)**

Given Data:

Diameter of Jupiter = $D_J = 11\ D_e$

Radius of Jupiter = $R_J = 11\ R_e$

Mass of Jupiter = $M_J = 318\ M_e$

To Find:

Gravitational acceleration on Jupiter = $g_J = ?$

Solution:

Gravitation acceleration on the surface of earth

$$g_e = \frac{G M_e}{R_e^2} \longrightarrow (1)$$

Gravitation acceleration on the surface of Jupiter.

$$g_J = \frac{G M_J}{R_J^2} \longrightarrow (2)$$

$$g_J = \frac{G(318\, M_e)}{(11\, R_e)^2}$$

$$g_J = \frac{318\, G M_e}{121\, R_e^2}$$

$$g_J = \frac{318}{121}\left[\frac{G M_e}{R_e^2}\right]$$

From equation (1).

$$\boxed{g_J = 2.63\ g_e}$$

$\because\ g_e = 9.8\ \text{m/s}^2$

$g_J = 2.63 \times 9.8$

$\boxed{g_J = 25.774\ \text{m/s}^2}$ **Ans.**

Self Test (2):

(i) The mass moon of is approximately one eightieth $\left(\frac{1}{80}\right)$ of the mass of earth and its radius is approximately one fourth $\left(\frac{1}{4}\right)$ that of the earth. Determine the acceleration due to gravity at the surface of the moon.

Ans. $\boxed{g_m = \frac{1}{4}g}$

Q.4 The radius of the moon is 27% of the earth radius and its mass is 1.2% of the earth's mass. Find the acceleration due to gravity on the surface of the moon. How much will a 424N body weight there? **(1997, 2002 P.M, 2013 Failure)**

Hint:

Mass of moon = $M_m = 1.2\%\ M_e = \frac{1.2}{100} M_e$

$= 0.012\ M_e$

Radius of moon = $R_m = 27\%\ R_e = \frac{27}{100} R_e$

$= 0.27\ R_e$

Gravitantional acceleration on mass = $g_m = ?$

Ans. ($g_m = 1.61^3 m/s^2$ $W_m = 69.78N$)

(iii) Suppose a new planet "x" is discovered which has a diameter 10 times bigger as compared to that of earth and a mass equal to 300 times that of earth. Compute the gravitational acceleration at the surface of the planet "x" **(2008)**

Ans. $\boxed{g_x = 29.4\ m/s^2}$

Q.5 What is the value gravitational acceleration at a distance of
(i) earth radius above the earth's surface.
(ii) twice earth radius above the earth surface.

Give Data:

(i) $h = R_e$ (ii) $h = 2R_e$

To Find:

$g_h = ?$

Solution:

Gravitational acceleration on the surface of earth

$$g_e = \frac{G M_e}{R_e^2}$$

Gravitational acceleration at height "h" above the earth surface.

$$g_h = \frac{G M_e}{(R_e + h)^2} \longrightarrow (2)$$

(i) $h = R_e$

$$g_h = \frac{G M_e}{(R_e + R_e)^2}$$

$$g_h = \frac{G M_e}{(2R_e)^2}$$

$$g_h = \frac{G M_e}{4R_e^2} = \frac{1}{4}\frac{GM_e}{R_e^2}$$

From equation (1)

$$\boxed{g_h = \frac{1}{4} g_e}$$

(ii) $h = 2R_e$

$$g_h = \frac{GM_e}{(R_e + 2R_e)^2}$$

$$g_h = \frac{GM_e}{(3R_e)^2}$$

$$g_h = \frac{GM_e}{9R_e^2} = \frac{1}{9}\frac{GM_e}{R_e^2}$$

From equation (1)

$$\boxed{g_h = \frac{1}{9} g_e}$$

Self Test: (3)

What is the value of gravitational acceleration at a distance of half earth radius above the earth's surface.

Ans. $(\frac{4}{9}g)$

Q.6 At what distance from the centre of earth does the gravitational acceleration have one half the value that it has on the earth's surface.

(2005, 2003P.E, 2005, 2004 Failures 2005 Supp.)

Solution:

Gravitational acceleration on the surface of earth.

$$g_e = \frac{GM_e}{R_e^2} \longrightarrow (1)$$

Gravitational acceleration at height 'h' above the earth surface.

$$g_h = \frac{G M_e}{(R_e + h)^2}$$

$$(R_e + h) = R$$

$$g_h = \frac{G M_e}{R^2} \longrightarrow (2)$$

$$g_h = \frac{1}{2} g_e$$

$$(1/2)g_e = \frac{G M_e}{R^2}$$

$$R^2 = 2\frac{G M_e}{g_e}$$

And $g_h = \frac{1}{2}g_e$; $R_E - d = ?$

$$g_h = \left(1 - \frac{d}{R_E}\right)g$$

$$\frac{1}{2}g = \left(1 - \frac{d}{R_E}\right)g$$

$$\frac{-1}{2} = -\frac{d}{R_E}$$

$$d = \frac{1}{2}R_E$$

From equation (1)

$$R^2 = 2\frac{GM_e}{\frac{GM_e}{R_e^2}}$$

$$R^2 = 2\frac{GM_e R_e^2}{GM_e}$$

$$R^2 = 2R_e^2$$

$$R = \sqrt{2R_e^2}$$

$$\boxed{R = 1.41\ R_e}$$ **Ans.**

Let

Distance form centre of earth $= x\ R_E - d$

$$x = R_E - d = R_E - \frac{1}{2}R_E$$

$$\boxed{x = \frac{1}{2}R_E}$$

Self Test (4)

(i) At what distance from the centre of earth does the gravitational acceleration have one third, the value that it has on the earth's surface. **(1993)**

Ans. $\boxed{1.73\ R_e}$

(ii) At what distance from the centre of earth does the gravitational acceleration have one fourth, the value that it has on the earth's surface. **(2004)**

Ans. $\boxed{2\ R_e}$

Q.9 A woman with a mass of 45kg is standing on a scale in an elevator. The elevator accelerates upward with a constant acceleration of 1.2 m/s^2. What is the woman weight as measured her in the elevator?

Given Data:

Mass of woman = m = 45kg

Acceleration of elevator = a=1.2m/s^2

To Find:

Apparent weight of woman in elevator = F_w =?

Solution:

When elevator is moving up.

$Fw = ma + mg$

$Fw = 45 \times 1.2 + 45 \times 9.8$

$Fw = 54 + 4W$

$\boxed{Fw = 495W}$ Ans.

Self Test: (5)

A man weighing 60kg is standing on the floor of an elevator calculate the force exerted by the man on the floor of the elevator when it (i) Is at rest (ii) moving up with an acceleration of 2m/s^2 (iii) moves down with an acceleration of 2m/s^2.

Ans. **(i)** 588N **(ii)** 808N **(iii)** 468 N

EXTRA PROBLEMS:

Q.1 Compute the density of Jupiter which has diameter 11 times as compared that of earth and mass equal to 318 times that of the earth ($R_e = 6.4 \times 10^6$m $M_e = 6 \times 10^{24}$ kg).

Give Data:

Diameter of Jupiter = $D_J = 11D_e$

Radius of Jupiter = $R_J = 11R_e = 11 \times 6.4 \times 10^6$m
$= 70.4 \times 10^6$m

Mass of Jupiter = $M_J = 318\ M_e = 318 \times 6 \times 10^{24}$kg
$= 1908 \times 10^{24}$kg

To Find:

Density of Jupiter = $\rho_j = ?$

Solution:

Volume of Jupiter = $\frac{4}{3}\ \pi\ R_J^3$

$= \frac{4}{3}\ (3.14)\ (70.4 \times 10^6)^3$

$= 1.461 \times 10^{24}\text{m}^3$

Density of Jupiter $= \rho_j = \frac{M_J}{V_J}$

$= \frac{1908 \times 10^{24}}{1.461 \times 10^{24}}$

$\boxed{\rho_j = 1.305 \times 10^3\ \text{kg/m}^3}$ **Ans.**

Self Test: (6)

The radius of the moon is 27% of earth's radius and its mass is 1.2% of the mass of the earth. Find the density of the moon.

Ans. **$(3.33 \times 10^3\ \text{kg/m}^3)$**

Q.2 Find how deep from the surface of the earth a point where acceleration due to gravity is half the value on the earth surface. **(1995)**

Solution:

Gravitational acceleration at depth x from the earth surface.

$$g_x = g\left(1 - \frac{x}{R_e}\right)$$

$$\frac{g_x}{g} = 1 - \frac{x}{R_e}$$

$$g_x = \frac{1}{2}\ g$$

$$2g_x = g$$

$$\frac{g_x}{2g_x} = -\frac{x}{R_e}$$

$$\frac{1}{2} = 1 - \frac{x}{R_e}$$

$$\frac{x}{R_e} = 1 - \frac{1}{2}$$

$$\frac{x}{R_e} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{2} R_e}$$

Self Test: (7)

At what depth from the surface of the earth is the value of acceleration due to gravity one fourth the value at the earth's surface. **(2004)**

Ans. $(\frac{3}{4}R_e)$.

Q.3 A satellite of mass "m" moves in a circular orbit around the earth with constant speed "v" at height of 290km above the earth's surface. Find the orbital speed of the satellite (Re = 6. 38 x 10^6m Me =5.98 x 10^{24}kg G = 6.673 x 10^{-11} Nm^2/kg^2. **(2008 Failures)**

Give Data:

h = 290km = 290 x 10^{-3}m
Re = 6.38 x 10_6m
Me = 5.98 x 10^{24}kg
G = 6.673 x 10^{-11} Nm^2/kg^2

To Find:

Orbital speed of satellite = v = ?

Solution:

Distance from center of earth to satellite: r = R + h
r = 6.38 x 10^6m + 290 x 10^3 = 6.670 x 10^6m
Centripetal force

$$F_c = ma_c \longrightarrow (1)$$

Centripetal acceleration of the satellite is given by

$$a_c = \frac{v^2}{r} \longrightarrow (2)$$

$$F_c = \frac{mv^2}{r} \longrightarrow (2)$$

Gravitational force between the earth and satellite is given by

$$F_c = \frac{GmMe}{r^2} \longrightarrow (3)$$

Comparing eq. 2 and eq. 3

$$\frac{mv^2}{r} = \frac{GmMe}{r^2}$$

$$v^2 = \frac{GMe}{r}$$

$$v = \sqrt{\frac{GMe}{r}}$$

$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{6.670 \times 10^6}}$$

$$\boxed{V = 7.733 \times 10^3 \text{ m/s}}$$ **Ans.**