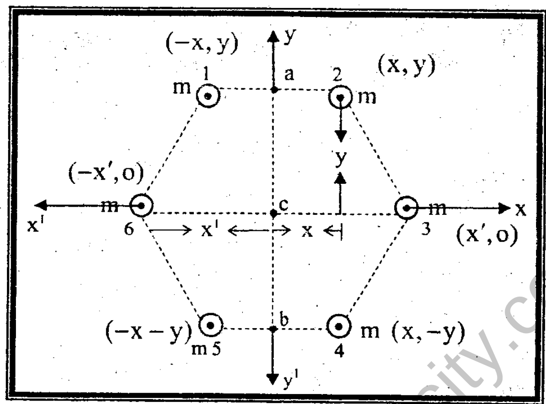
Q:(1) Locate the centre of mass of a system of particles each of mass 'm' arranged to correspond in position to the six corner of a regular (Planar) hexagon.

Solution:-

Taking X and Y – axis, such that origin O lies at the centre of the hexagon. We have for the co–ordinates (X_C) and (Y_C) of the centre of mass. Is



$$x_c = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \text{-----------} + m_6x_6}{m_1 + m_2 + m_3 + \text{-----------} + m_6}$$

$$x_c = \frac{m(-x) + m(x) + m(x') + m(x) + m(-x) + m(-x')}{m + m + m + m + m + m}$$

$$x_c = \frac{-\cancel{m}x + \cancel{m}x + \cancel{m}x + \cancel{m}x' - \cancel{m}x - \cancel{m}x'}{6m} = \frac{0}{6m}$$

$$\boxed{x_c = 0}$$

$$y_c = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \text{-----------} + m_6y_6}{m_1 + m_2 + m_3 + \text{-----------} + m_6}$$

$$y_c = \frac{m(y) + m(y) + m(0) + m(-y) + m(-y) + m(0)}{m + m + m + m + m + m}$$

$$y_c = \frac{my + my + 0 - my - my + 0}{6m}$$

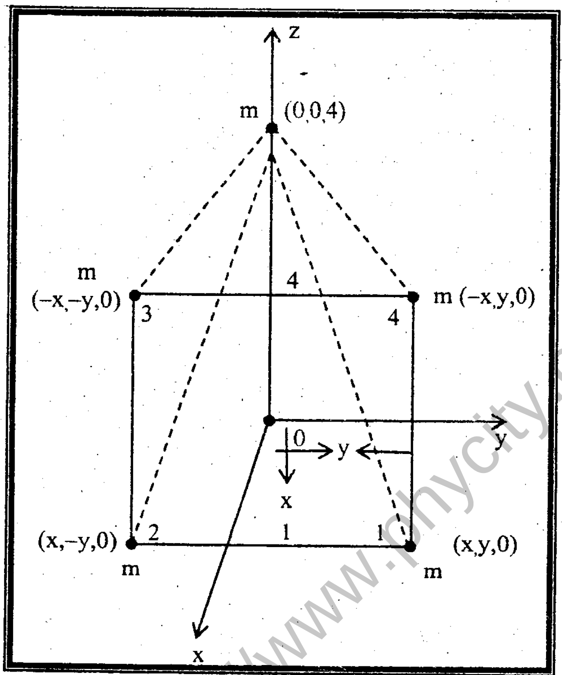
$$y_c = \frac{0}{6m}$$

$$y_c = 0$$

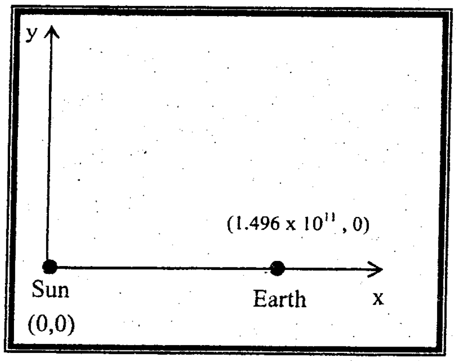
Thus the coordinates of centre of mass = (x_c , y_c) = (0 , 0)

Self test:(1)

Q:(2) Find the position of centre of mass of five equal-mass particles located at the five corners of a square-based right pyramid with sides of 'l' length 'h' and altitude



Q:(3) The mass of the sun is 329.390 times the earth's mass and the mean distance from the centre of the sun to the centre of the earth is 1.496×10^8km. Treating the earth and sun as particles with each mass concentrated at the respective geometric centre, how far from the centre of the sun is the C.M (centre of mass) of the earth-sun system? Compare this distance with the mean radius of the sun (6.9960×10^5km)



$m_s = 329390\, m_E$

$x_E = 1.496 \times 10^8 \text{ km} = 1.496 \times 10^{11}\text{m}$

$r_s = 6.9960 \times 10^5 \text{ km} = 6.9960 \times 10^8\text{m}$

The X co-ordinate of centre of mass

$$x_C = \frac{m_s \times x_s + m_E \times x_E}{m_s + m_E}$$

$$x_C = \frac{m_s \times 0 + m_E \times 1.496 \times 10^{11}}{329320\, m_E + m_E}$$

$$x_C = 0 + \frac{1.496 \times 10^{11} m_E}{329320\, m_E}$$

$$x_C = 4.54 \times 10^5 \text{m}$$

$$x_C = 4.54 \times 10^2 \text{km}$$

$$\frac{x_C}{r_s} = \frac{4.54 \times 10^5}{6.99960 \times 10^8} = 6.49 \times 10^{-4}$$

Q:(4) A particle with mass 4 kg moves along the x-axis with a velocity v = 15 t m/ sec. When t = 0 is the instant that the particle is at the origin.

(a) At t = 2sec. what is the angular momentum of the particle about a point P located on the + y-axis, 6m from the origin? (b) What torque about P acts on the particle?

Given Data:

Mass of particle = m = 4kg

Velocity of particle = v = 15t m/s

Point P located on y-axis = y = 6m

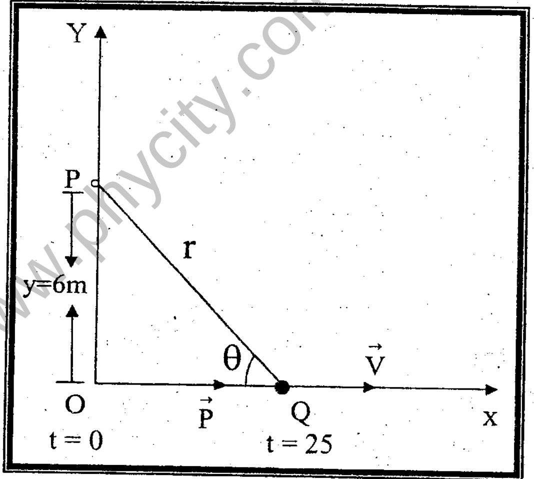
(a) Time = t = 2sec.

To Find:

(a) Angular momentum = L = ?

(b) Torque = τ = ?

Solution:-



(a) The velocity of particle after 2sec.

V=15x2 = 30 m/s

Angular momentum of the particle

$L = mv\,r\,\sin\theta$ $\left[\sin\theta = \dfrac{OP}{PQ} \quad \sin\theta = \dfrac{6}{r} \right]$

$L = 4 \times 30 \times r \times \dfrac{6}{r}$

$$\boxed{L = 720\ kg\ m^2 / S}$$

(b) We know the torque acting on a Particle is equal to rate of change of its angular momentum.

$$\tau = \frac{L}{t}$$

$$\tau = \frac{720}{2}$$

$$\boxed{\tau = 360\ N.m}$$

Self test:(2)

A particle of mass 400gm rotates in a circular orbit of radius 20cm with constant angular speed of 1.5 revolution per second. Calculate the magnitude of angular momentum of particle with respect to centre of the orbit. **(2001)**

Q:(5) A particle of mass 'm' is located at the vector position $\vec{r}$ and has a linear momentum vector $\vec{P}$. The vectors r and P are non zero. If the particle moves only in the xy–plane, prove that

$L_x = L_y = 0$ and $L_z \neq 0$

Given Data:

Particle moves only in the xy – plane

$\vec{Y} = x\hat{i} + y\hat{j}$

$\vec{P} = Px\hat{i} + Py\hat{j}$

Prove that:

$Lx = 0$
$Ly = 0$
$Lz \neq 0$

Solution:-

The angular momentum is given by.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ x & y & 0 \\ p_x & p_Y & 0 \end{vmatrix}$$

$$\vec{L} = \hat{\imath}\begin{vmatrix} y & 0 \\ p_y & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} x & 0 \\ p_x & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} x & y \\ p_x & p_y \end{vmatrix}$$

$$Lx\hat{\imath} + Ly\hat{j} + Lz\hat{k} = \hat{i}(0) - \hat{j}(0) + \hat{k}(xp_y - yp_x)$$

Comparing the coefficient of $\hat{\imath}, \hat{j}$ and $\hat{k}$ we get

$Lx = 0$

$Ly = 0$

$Lz = xp_y - yp_x \neq 0$

Q:(6) A light rigid rod 1 m in length rotates in the xy-plane about a pivot through the rod's centre. Two particles of mass 2kg and 3kg are connected to its ends. Determined the angular momentum of the system about the origin at the instant the speed of each particle is 5m/sec.

Given Data:

Mass of particle A= 2kg

Mass of particle B= 3kg

Velocity of each particle = 5 m/s

Length of rod = L = 1m

Distance of each particle from the pivot = r = 0:5m

To Find:

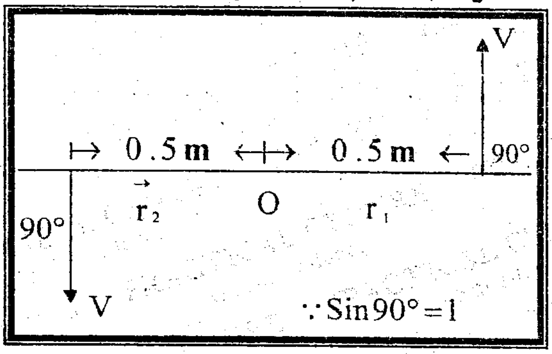
Angular momentum of the system = L = ?

Solution:

The total angular momentum of the system is

$L = L_1 + L_2$

$L = m_1 v\, r_2 \text{ Sin } \theta + m_2 v\, r_1 \text{ Sin } \theta$



$L = 2 \times 5 \times 0.5 \times \text{Sin } 90° + 3 \times 5 \times 0.5 \text{ Sin } 90°$

$L = 5 + 7.5$

$\boxed{L = 12.5 \text{ Js}}$

Q:(7) A uniform beam of mass 'M' supports two masses m_1 and m_2. If the knife edge of the support is under the beam's centre of gravity and m_1 is at a distance 'd' from the centre, determine the position of m_2 such that the system is balanced.

Given Data:

Mass of beam = M

Mass of body A = m_1

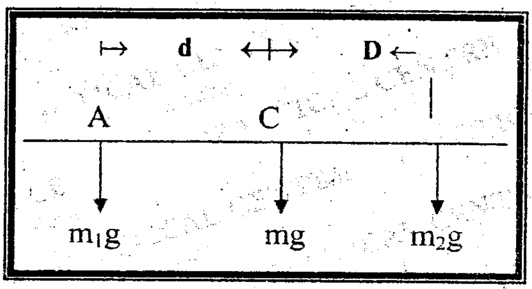
Mass of body B = m_2

Distance of body A from the centre = d

To Find:

Distance of body B from the centre = D = ?

Solution:



Since system is balance, therefore
Apply second condition of equilibrium.

$\Sigma \tau = 0$ about 'C'

$m_1 g \times d + (-m_2 g \times D) + Mg\,(0) = 0$

$m_1 g d - m_2 g D = 0$

$m_1 g d = m_2 g D$

$\boxed{D = \frac{m_1}{m_2} d}$

Q:(8) A uniform ladder of length ℓ and weight W=50N rests against a smooth vertical wall. If the coefficient of friction between the ladder and the ground is 0.40, find the minimum angle θ_{min} such that the ladder may not slip.

(2011, 2010)

Given Data:

Length of ladder = L

Weight of Ladder = W = 50N

Coefficient of friction = μ = 0.4

To Find:

The minimum angle made by ladder with horizontal = θ_{min} = ?

Solution:

In the figure

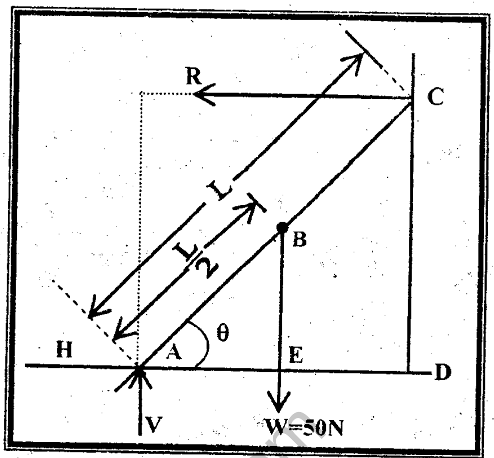
R = Reaction of the wall

H and V = Horizontal and vertical Components of reaction of ground

$\mu = \frac{H}{V}$

$0.4 = \frac{H}{V}$

$H = 0.4V \rightarrow (1)$



Apply first condition of equilibrium

$\Sigma Fx = 0$
$H + (-R) = 0$
$H - R = 0$
$H = R$

$\Sigma Fy = 0$
$V + (-W) = 0$
$V - W = 0$
$V = W = 50N$

From eq.1 $H = 0.4 \times 50 = 20N$; $R = 20N$

Now apply second condition of equilibrium.

$\Sigma\tau = 0$ about A

$R \times \overline{DC} - W \times \overline{AE} = 0$

Consider Δ ADC

$\sin\theta = \frac{\overline{DC}}{\overline{AC}}$

$\sin\theta = \frac{\overline{DC}}{L}$

$\overline{DC} = L\sin\theta$

Consider Δ AEB

$\cos\theta = \frac{\overline{AE}}{\overline{AB}}$

$\cos\theta = \frac{\overline{AE}}{L/2}$

$\overline{AE} = L/2 \cos\theta$

$20 \times L\sin\theta - 50 \times \frac{L}{2}\cos\theta = 0$

$20L\sin\theta = 25L\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{25}{20}$

$\tan\theta = 1.25$

$\theta = \tan^{-1}(1.25)$

$\boxed{\theta = 51.3^\circ}$

Self test:(3)

A uniform ladder of length 'ℓ' and weight 60N rest against a smooth vertical wall. If the Coefficient of friction between the ladder and ground is 0.40, Find the minimum angle such that the ladder may not slip. **(2003 P.M)**

Q:(9) A ladder with a uniform density and a mass 'm' rests against a frictionless vertical wall at an angle of 60°. The lower end rests on a flat surface where the coefficient of friction (static) is 0.40. A student with a mass (M=2m) attempts to climb the ladder. What fraction of the length 'L' of the ladder will the student have reached when the ladder begins to slip?

Given Data:

Mass of Ladder = m

Weight of Ladder = mg

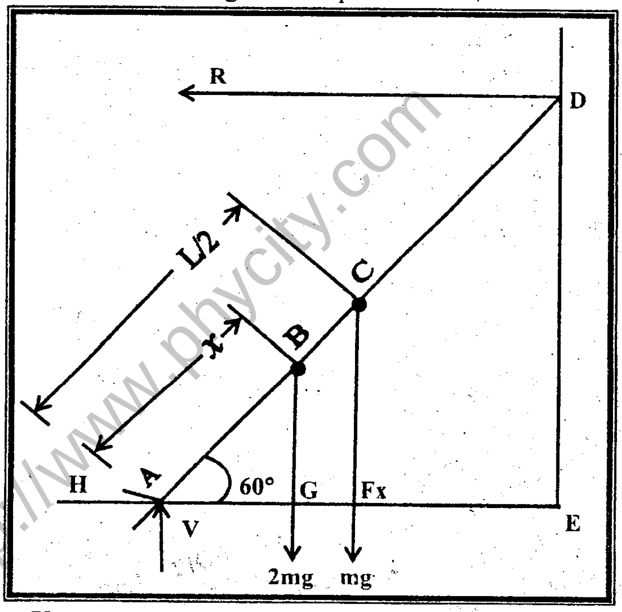
Length of Ladder = L

Mass of student = 2m

Weight of Student = 2mg

$\theta = 60$

$\mu = 0.40$



To Find:

Fraction of length reached $= \dfrac{X}{L} = ?$

Solution:

In figure

R = Reaction of the wall

H and V = Horizontal and vertical components of reaction of ground.

$$\mu = \frac{H}{V}$$

$$H = \mu V$$

$$H = 0.40V \rightarrow (1)$$

Apply first condition of equilibrium

$\Sigma Fx = 0$	$\Sigma Fy = 0$
$H + (-R) = 0$	$V + (-2mg) + (-mg) = 0$
$H - R = 0$	$V - 2mg - mg = 0$
$H = R$	$V - 3mg = 0$
	$V = 3mg$

From eq.(1)

$$H = 0.4 \times 3mg$$

$$H = 1.2mg$$

$$R = 1.2mg \qquad \because H = R$$

Now apply second condition of equilibrium.

$\sum \tau = 0$ about 'A'

R X E D – 2mg x AG – mg x AF = 0 →

Consider Δ AED

$\sin 60° = \frac{ED}{AD}$

$0.866 = \frac{ED}{L}$

$ED = 0.866\ L$

Consider Δ AGB

$\cos 60° = \frac{AG}{AB}$

$0.5 = \frac{AG}{X}$

$0.5 = \frac{AG}{X}$

$AG = 0.5X$

Consider Δ AFC

$\cos 60° = \frac{AF}{AC}$

$0.5 = \frac{AF}{L/2}$

$AF = 0.25\ L$

$1.2\ mg \times 0.866L - 2mg \times 0.5x - mgx\ 0.25L = 0$

$1.0392\ mg\ L - mg\ x - 0.25mg\ L = 0$

$0.789\ \cancel{m}\cancel{g}\ L = \cancel{m}\cancel{g}\ x$

$\boxed{\frac{X}{L} = 0.789}$

Self test:(4)

(i) A 15 metre ladder weighing 350N, rests against a smooth wall at a point 12m above the ground. The centre of gravity of the ladder is one third the way up. A body 420N climbs half way up the Ladder, calculate the reaction of the wall and the ground. **(2002 P.M)**

(ii) A ladder of length 'ℓ' and weight 200N rest against a smooth vertical wall at an angle 50° the centre of gravity of the ladder is 0.4L from the base. How large a force of friction must exist at the base of the Ladder if it is not slip? What is the necessary coefficient of static friction. **(2003 P.E)**

Q:(10) A particle of mass 0.3kg moves in the xy-plane. At the instant its coordinates are (2,4)m. its velocity is (3i + 4j) m/sec. At this instant determine the angular momentum of the particle relative to the origin.

Given Data: **(2012)**

Mass of particle = m = 0.3 kg

Position of particle = (2,4) m

Velocity of particle = $\vec{V} = \left(3\hat{i} + 4\hat{j}\right)$ m/s

To Find:

Angular momentum $= \vec{L} = ?$

Solution:

$$\text{Position Vector} = \vec{r} = X\hat{i} + y\hat{j}$$

$$\vec{r} = \left(2\hat{i} + 4\hat{j}\right)m$$

The angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = \vec{r} \times m\vec{V} \qquad \because \vec{P} = m\vec{V}$$

$$\vec{L} = m\,\vec{r} \times \vec{V}$$

$$\vec{L} = 0.3\left[\left(2\hat{i} + 4\hat{j}\right) X \left(3\hat{i} + 4\hat{j}\right)\right]$$

$$\vec{L} = 0.3\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$\vec{L} = 0.3\left[\hat{i}\begin{vmatrix} 4 & 0 \\ 4 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix}\right]$$

$$\vec{L} = 0.3\left[\hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(8-12)\right]$$

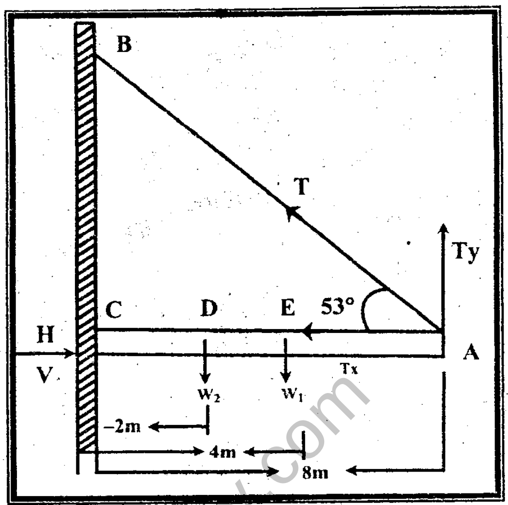
$$\vec{L} = 0.3\left[-4\hat{k}\right] \qquad \boxed{\vec{L} = -1.2\hat{k}\,JS}$$

Self test:(5)

A particle of mass 0.2kg moves is xy–plane at an instant the coordinates are (1,2) m, its velocity is (2i + 2j) m/s. At this instant determine the angular momentum of the particle with respect to the origin. **(2005 Failures)**

Q:(11) A uniform horizontal beam of length 8 m and weighing 200N is pivoted at the wall with its far end supported by a cable that makes an angle of 53° with the horizontal. If a person weighing 600N stands 2m from the wall, find the tension and the reaction force at the pivot.

Given Data:



Length of beam = L = 8m

Weight of beam = W_1 = 200N

Weight of person = W_2 = 600N

Angle of cable with beam = 53°

Distance of person from wall = 2m

Centre of beam = 4m

To Find:

Tension in cable = T = ?

Horizontal reaction = H = ?

Vertical reaction = V = ?

Solution:

Apply first condition of equilibrium

$\Sigma F_X = 0$	$\Sigma F_y = 0$
$H + (-Tx) = 0$	$V + Ty + (-W_1) + (-W_2) = 0$
$H - T\,Cos53° = 0$	$V + T\,Sin53° - 200 - 600 = 0$
$H - T x O.602 = 0$	$V + TxO.799 - 800 = 0$
$H - 0.602T = 6$	$V + O.799T = 800 \rightarrow 2$ (2)
$H = 0.602T \rightarrow (1)$	

Apply second condition of equilibrium

$\Sigma\tau = 0$ about 'C'

$Ty \times AC - W_1 \times EC - W_2 \times DC = 0$

$TSin53° x8 - 200x4 - 600x2 = 0$

$T(0.799)8 - 800 - 1200 = 0$

$6.388T - 2000 = 0$

$6.388T = 2000$

$$T = \frac{2000}{6.388}$$

$$\boxed{T = 313.05N}$$

Putting the value of T in Eq. (1) and (2) we get

$$H = 0.602T$$

$$H = 0.602 \times 313.05$$

$$\boxed{H = 188.4N}$$

$$V + 0.799T = 800$$

$$V + 0.799 \times 313.05 = 800$$

$$V = 800 - 250$$

$$\boxed{V = 550N}$$

There for Resultant reaction R is

$$R = \sqrt{H^2 + V^2}$$

$$R = \sqrt{(188.4)^2 + (550)^2}$$

$$R = \sqrt{337994.5}$$

$$\boxed{R = 581.3N}$$

Self test: (6)

A uniform horizontal rod weighing 98N is pivoted at a vertical wall. Its for end is supported by a string whose other end is tied to a point on the wall above the pivot. A boy of mass 20kg stands at the centre of the rod. Find only the vertical component of the tension in the string and only vertical component of the reaction at the pivot. Length of the horizontal rod is 4 metre. **(2000)**

Self test: (7)

A 12 metre ladder weighting 400N rest against a vertical wall at a point 10 metre above the grownd. The centre of gravity of the ladder is on half the way up. A man weighting 200N climbs three forth up the ladder. Assuming that the wall is smooth, find the reaction of the ground and the wall. **(2009, 1992)**

Self test: (8)

A ladder rests against a smooth vertical wall at an angle of 60° with the ground and its centre of gravity is at rd$\frac{1}{3}$ of its length from the base. Determine (a) the frictional form which prevents the ladder from slipping (b) The coefficient of static friction.

(1996)