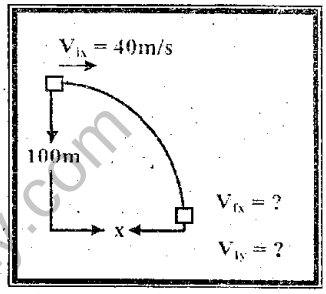
PROBLEMS

Q.1. A rescue helicopter drops a package of emergency ration to a stranded party on the ground. If the helicopter is traveling horizontally at 40m/s at a height of 100m above the ground.

(i) Where does the package strike the ground relative to the point at which it was released?

(ii) What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Given Data:



Initial horizontal velocity $= V_{ix} = 40m/s$
Initial vertical velocity $= V_{iy} = 0$
Vertical height $= y = -100m$
Horizontal Acceleration $= a_x = 0$
Vertical Acceleration $= a_y = -g = -9.8m/s$

To Find:

(a) Horizontal distance = x = ?
(b) Final horizontal velocity = V_{fx} = ?
(c) Final vertical velocity = V_{fy} = ?

Solution:

Using the relation:

$$y = V_{iy}t + \frac{1}{2}a_y t^2$$

$$-100 = 0 \times t + \frac{1}{2}(-9.8)t^2$$

$$-100 = -4.9t^2$$

$$t^2 = \frac{-100}{-4.9} = \frac{1000}{49} = 20.40\text{sec}$$

$$\boxed{t = 4.5\text{sec}}$$

(a) Using the relation

$$x = V_{ix}t^2 + \frac{1}{2}a_x t^2$$

$$x = 40 \times 4.5 + \frac{1}{2} \times 0 \times (4.5)^2$$

$$x = 180 + 0$$

$$\boxed{x = 180m}$$ Ans.

(b) Since in projectile horizontal velocity does not change, always remain same. But the vertical velocity is change, therefore,

$V_{ix} = V_{fx} = 40m/s$

OR Using the relation

$V_{fx} = V_{ix} + a_x t$

$V_{fx} = 40 + 0 \times 4.5$

$\boxed{V_{fx} = 40m/s}$ **Ans.**

(c) Using the relation

$V_{fy} = V_{iy} + a_y t$

$V_{fy} = 0 + (-9.8) \times 4.5$

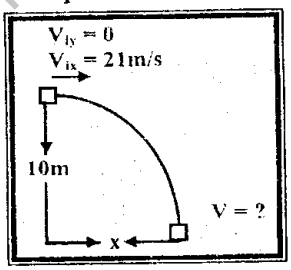
$\boxed{V_{fy} = -44m/s}$ **Ans.**

Self Test (1)

Q.4(i) A ball is thrown in horizontal direction from a height of 10m with velocity of 21m/s.

(a) How far will it hit the ground from its initial position on the ground?

(b) And with what velocity.



$$\left[V = \sqrt{V_{fx}^2 + V_{fy}^2}\right]$$

Ans. [30m, 25.2 m/s]

(ii) A bomber dropped a bomb at a height of 490m with the speed 120m/s along the horizontal, calculate the distance where the bomb will strike the ground relative to the point at which it was released. **(2002 supp.)**

Ans. [1200m]

Q.2. A long jumper leaves the ground at an angle of 20° to the horizontal and a speed of 11m/s.

(a) How far does the jump.

(b) What is the maximum height reached? Assume the motion of long jumper is that of projectile.

Given Data:

Initial velocity of jumper = $V_o = 11m/s$

Angle of jumper to the horizontal = $\theta = 20^\circ$

To Find:

Range = R = ?

Height = H = ?

Solution:

We know the range of projectile is:

$$R = \frac{V_o^2 \text{Sin}2\theta}{g}$$

$$R = \frac{(11)^2 \text{Sin}(2 \times 20^\circ)}{9.8}$$

$$R = \frac{121 \text{Sin}40^\circ}{9.8}$$

$$R = \frac{121 \times 0.643}{9.8} \qquad \because \quad \text{Sin}40^\circ = 0.643$$

R = 7.939m Ans.

We know height of projectile is:

$$H = \frac{V_o^2 \text{Sin}^2\theta}{2g}$$

$$H = \frac{(11)^2 (\text{Sin}20)^2}{2 \times 9.8}$$

$$H = \frac{121(0.342)^2}{19.6}$$

H = 0.722m Ans.

Self Test (2)

i. An artillery cannon is pointed upwards at an angle 35° with respect to the horizontal and fires a projectile with an initial velocity of 200m/s. If the air resistance is negligible find:

(i) The maximum height of the projectile.

(ii) Range of projectile. *(1998)*

Ans. [R = 3835.4m H = 669.38m]

ii. A machine gun is pointed upward at an angle of 30° with respect to the horizontal and fires a projectile with a velocity of 200m/s. Calculate the range of the projectile and the height of the projectile. *(2002 P.E), (2006)*

Ans. [R = 3534.19m, H = 510.20m]

iii. A projectile is launched at an angle of 40° at 10m/s. calculate its range and the maximum height attained by it. *(2006 Failures)*

Ans. [R = 10.04m, H = 2.108m]

iv. A cricket ball is thrown at a speed of 20m/s in a direction 30° to the horizontal. Calculate the maximum height of the ball and horizontal range. *(2008 Failures)*

Ans. [R = 35.34m, H = 5.10m]

Q.5. A rocket is launched at an angle of 53° to the horizontal with an initial speed of 100m/s. It moves along it initial line of motion with an acceleration of 30m/s^2 for 3sec. At this time the engine fails and rocket proceed to moves as free body.

Find:

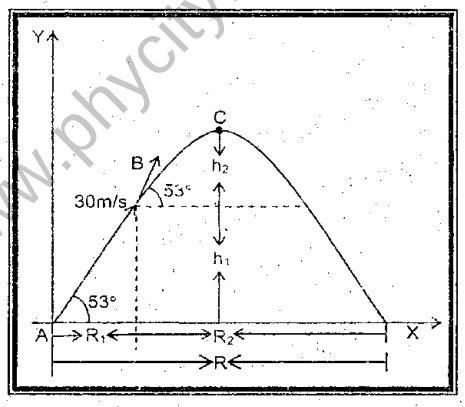
(a) The maximum altitude reached of the rocket.

(b) Its total time of flight and

(c) Its horizontal range.

Ans.

The rocket is moving for 3sec with acceleration 30m/s, so it is not a projectile for 3sec i.e. from A to B.



Consider motion from A to B

$V_i = 100m/s$

$a = 30m/s$

$t = 3sec$

$v_f = ?$

$v_f = v_i + at$

$v_f = 100 + 30 \times 3$

$\boxed{v_f = 190m/s}$

at the point B $v_f = v_o = 190m/s$

$$S = v_i t + \frac{1}{2} at^2$$

$$S = 100 \times 3 + \frac{1}{2} \times 30 \times (3)^2$$

$S = 300 + 15 \times 9$

$S = 300 + 135$

$\boxed{S = 435m}$

Resolving this distance into horizontal and vertical components.

$R_1 = S\ Cos\theta = 435\ Cos53°$

$R_1 = 435 \times 0.6018$

$\boxed{R_1 = 261.79m}$

$h_1 = S\ Sin\theta = 435\ Sin\ 53°$

$= 435 \times 0.7986$

$\boxed{h_1 = 347.40m}$

Consider motion form B to C:

$V_o = 190 m/s$

$\theta = 53^\circ$

$t_2 = ?$

$h_2 = ?$

We know that the time to reach maximum height is

$$t = \frac{V_o \, Sin\theta}{2}$$

$$t_2 = \frac{190 \times Sin}{9.8}$$

$$t_2 = \frac{190 \times 0.7986}{9.8}$$

$$\boxed{t_2 = 15.48 sec}$$

We know that maximum height is

$$H = \frac{V_o^2 \, Sin^2\theta}{2g}$$

$$h_2 = \frac{(190)^2 \, (Sin53^\circ)^2}{2 \times 9.8}$$

$$h_2 = \frac{(190)^2 \, (0.7936)^2}{19.6}$$

$$\boxed{h_2 = 1174.75 m}$$

Total Height $= h_1 + h_2$

$= 347.40 + 1174.75$

$$\boxed{H = 1522.15 m}$$ **Ans.**

Consider motion from C to E

$v_{iy} = 0$

$a_y = -9.8 \, m/s^2$

$H = y = -1522.15 m$

$t_3 = ?$

$$y = v_{iy} \, t + \frac{1}{2} a_y \, t_3$$

$$-1522.15 = 0 \times t_3 + \frac{1}{2}(-9.8) \, t^2_3$$

$$-1522.15 = 0 - 4.9 t_3$$

$$t^2_3 = -\frac{1522.15}{-4.9}$$

$$t^2_3 = 310.46$$

$$\boxed{t_3 = 17.62 sec}$$

Total time $= t_1 + t_2 + t_3$

$= 3 + 15.48 + 17.62$

$$\boxed{t = 36.1 sec}$$ **Ans.**

Time from B to E = $t_2 + t_3$

$= 15.48 + 17.62$

$t = 33.1$sec

There fore horizontal range from F to E

$R_2 = v_x \times t$

$R_2 = V_o \cos\theta\, t$ $\qquad v_x = V_o \cos\theta$

$R_2 = 190 \cos 53^\circ \times 33.1$

$R_2 = 190 \times 0.6018 \times 33.1$

$R_2 = 3784.81$m

Total Rang = $R_1 + R_2$

$= 261.79 + 3784.81$

$\boxed{R = 4046.6\text{m}}$ **Ans.**

Self Test (3)

Q.3 A stone is thrown upward from the top of a building at an angle of 30° to the horizontal and with an initial speed of 20m/s. If the height of the building is 45m.

(i) Calculate the total time the stone in flight.

(ii) What is the speed of the stone just before it strikes the ground.

(iii) Where does the stone strike the ground.

Ans. [4.22 sec, 35.82m/s, 73.0m]

Q.6. A diver leaps from tower with an initial horizontal velocity component of 7m/s and upward velocity component of 3m/s. Find the component of her position and velocity after one second. *(2012)*

Given Data:

Initial Horizontal velocity = $V_{ix} = 7$m/s

Initial Vertical velocity = $V_{iy} = 3$m/s

Time = t = 1sec.

Vertical Acceleration = $a_y = -9.8\text{m/s}^2$

To Find:

(a) V_{fx} =?

(b) V_{fy} =?

(c) x =?

(d) y =?

Solution:

(a) Since in projectile motion horizontal velocity does not change, always remain same but the vertical velocity is change, therefore

$V_{ix} = V_{fx} = 7$m/s

(b) Using the relation

$V_{fy} = V_{iy} + a_y t$

$V_{fy} = 3 + (-9.8) \times 1$

$V_{fy} = 3 - 9.8$

$\boxed{V_{fy} = -6.8\text{m/s}}$ **Ans.**

(c) Using the relation

$$x = V_{ix}t + \frac{1}{2}a_x t^2$$

$$x = 7x1 + \frac{1}{2}x0x1 \qquad \because \quad a_x = 0$$

$$\boxed{x = 7m} \text{ Ans.}$$

$$y = V_{iy}t + \frac{1}{2}a_y t^2$$

$$y = 3x1 + \frac{1}{2}(-9.8)1^2$$

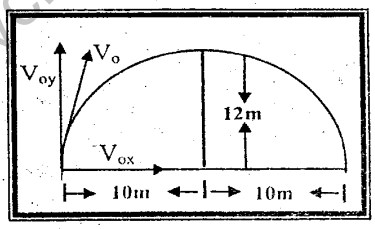
$$y = 3 - 4.9$$

$$\boxed{y = -1.9m} \text{ Ans.}$$

Q.7. A boy standing 10m from a building. Can just barely reach the roof 12m above him when he thrown a ball at the optimum angle with respect to the ground. Find the initial velocity component of the ball.

Given Data:

Horizontal distance = x = 10m
Vertical height = H = 12m



To Find:

(a) Initial Horizontal Velocity = V_{ox} = ?
(b) Initial Vertical velocity = V_{oy} = ?

Solution:

(a) We know that vertical height of projectile is:

$$H = \frac{V_o^2 Sin^2\theta}{2g}$$

$$12 = \frac{V_o^2 Sin^2\theta}{2g} \quad \Rightarrow \quad 12 = \frac{(V_o Sin\theta)^2}{2g}$$

$$V_o Sin\theta = V_{oy}$$

$$12x2g = V_{oy}^2$$

$$V_{oy}^2 = 12 \times 2 \times 9.8$$

$$V_{oy}^2 = 235.2$$

$$\boxed{V_{oy} = 15.33m/s} \text{ Ans.}$$

We know that horizontal range of projectile is:

$$R = \frac{V_o^2 Sin2\theta}{g}$$

$$10 + 10 = \frac{V_o^2 Sin2\theta}{g} \quad \Rightarrow \quad 20 = \frac{V_o^2 Sin2\theta}{g}$$

$$20 = \frac{V_o^2 x 2Sin\theta Cos\theta}{g}$$

$$g \times 20 = 2(V_o \sin\theta)(V_o \cos\theta)$$
$$20 \times 9.8 = 2\, V_{oy}\, V_{ox}$$
$$V_{ox} = \frac{20 \times 9.8}{2V_{oy}}$$
$$V_{ox} = \frac{20 \times 9.8}{2 \times 15.33}$$

$$\boxed{V_{ox} = 6.39 \text{m/s}} \text{ Ans.}$$

Q.8. A marter shell is fired at ground level target 500m distance with an initial velocity of 90m/s. What is its launch angle? Calculate the maximum and minimum time of flight to hit target.

Given Data:

Range = $R = 500$m
Initial velocity = $V_o = 90$m/s

To Find:

(a) Launch angle = $\theta = ?$
(b) Minimum time of flight = $T_{min} = ?$
(c) Maximum time of flight = $T_{max} = ?$

Solution:

(a) We know that range of projectile is:

$$R = \frac{V_o^2 \sin 2\theta}{g}$$
$$\sin 2\theta = \frac{R \times g}{V_o^2}$$
$$\sin 2\theta = \frac{500 \times 9.8}{(90)^2}$$
$$\sin 2\theta = 0.6049$$
$$2\theta = \sin^{-1}(0.6049)$$
$$2\theta = 37.2^\circ$$
$$\theta = \frac{37.2^\circ}{2}$$

$$\boxed{\theta = 18.6^\circ} \text{ Ans.}$$

If the shell is fired with angle $(90 - 18.6) = 71.4^\circ$ then range will also 500m

(b) For minimum time of flight

$$\theta = 18.6^\circ$$
$$T_{min} = \frac{2V_o \sin\theta}{g}$$
$$T_{min} = \frac{2 \times 90 \sin(18.6^\circ)}{9.8}$$

$$\boxed{T_{min} = 5.8\text{sec}} \text{ Ans.}$$

(c) For maximum time of flight

$$\theta = 71.4^\circ$$

$$T = \frac{2V_o \sin\theta}{g}$$

$$T_{max} = \frac{2 \times 90 \sin(71.4^\circ)}{9.8}$$

$$T_{max} = \frac{180 \times 0.947}{9.8}$$

$$\boxed{T_{max} = 17.40 \text{sec}}$$ **Ans.**

Self Test: (4)

(i) A morter shell is fired at a ground level target 490m away with an initial velocity 98m/s. Find the two possible value of the launch angle. Calculate the minimum and maximum time to hit the target. ***(1995)***

Ans. [$\theta = 15^\circ, \theta' = 75^\circ, T_{min} = 5.176$ sec $T_{max} = 19.3$ sec.]

(ii) Two possible angle to hit a target by a marter shell fired with initial velocity 98m/s are 15° and 75°. Calculate the range of projectile and the minimum time required to hit the target. ***(2004)***

Ans. [R = 490m $T_{min} = 5.176$ sec.]

(iii) A rocket is fired at a ground level target 600m away with initial velocity 85m/s. Find the two possible value of the launch angle. Calculate the minimum time to hit the target. ***(2008)***

Ans. [$\theta = 27.23^\circ, \theta' = 62.76^\circ$ $T_{min} = 7.93$ sec.]

(iv) A marter shell is fixed at target 800m away with velocity of 100m/s. Calculate the maximum possible value of launch angle ***(2009)***

Ans.

Q.9. What is the takeoff speed of a locust if launch angle is 55° and its range is 0.8m. ***(2004 Failures) (2006 Supply) (2005)***

Given Data:

Launch angle $= \theta = 55^\circ$

Range $= R = 0.8$m

To Find:

Takeoff speed $= V_o = ?$

We know that range of projectile is:

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

$$V_o^2 = \frac{R \times g}{\sin 2\theta}$$

$$V_o^2 = \frac{0.8 \times 9.8}{\sin(2 \times 55)}$$

$$V_o^2 = \frac{0.8 \times 9.8}{\sin 110}$$

$$V_o^2 = \frac{0.8 \times 9.8}{0.9396}$$

$$V_o^2 = 8.34$$

$$\boxed{V_o = 2.88 \text{m/s}}$$ **Ans.**

Self Test: (5)

(i) What is the take off speed of a locust if launch angle is 30° and its range is 0.8m. *(2004 Supp.)*

Ans. [3m/s]

(ii) What is take of speed of a locust if its launch angle is 50° and its.

(iii) An artillery with maximum range 1020m is to be fired for a target 600m. What should be:

(a) Angle for maximum and minimum time.
(b) What will be the time of flight in both cases?
(c) What will be maximum height in both cases?

Ans. [18°, 72°, 6.3s, 19.4s, 48.9m, 461.4m]

Q.10. A car is traveling on a flat circular track of radius 200m at 20m/s and has centripetal acceleration $a_c = 4.5m/s^2$

(a) If the mass of car is 1000kg What frictional force is required to provide the acceleration?

(b) If the co-efficient of static friction μ_s is 0.8 what is the maximum speed at which the car circle the tracks.

Given Data:

Radius of track = r = 200m
Speed of car = V = 20m/s
(a) man of car = m = 1000kg
(b) Co-efficient of friction = $\mu = 0.8$

To Find:

(a) Frictional force = f = ?
(b) Maximum speed of car = v_{max} = ?

Solution:

(a) Here the frictional force is equal to the centripetal force

$$f = F_c$$
$$f = ma_c$$
$$f = 1000 \times 4.5$$
$$\boxed{f = 4500N}$$

(b) $F_c = \mu R$
$R = w = mg$
$F_c = \mu mg = 0.8 \times 1000 \times 9.8 = 7840N$

Using the relation

$$F_c = \frac{mV^2_{max}}{r}$$
$$7840 = \frac{1000V^2_{max}}{200}$$
$$V^2_{max} = \frac{7840 \times 200}{1000}$$
$$V^2_{max} = 0.1568$$
$$\boxed{V_{max} = 39.6m/s} \text{ Ans.}$$

Self Test: (6)

Find out angle of projection for which maximum height of projectile is $\frac{1}{3}$rd of it's horizontal range.

Given Data:

$$H = \frac{1}{3}R$$

To Find:

$$\theta = ?$$

Solution:

$$H = \frac{1}{3}R$$

$$\frac{V_o^2 \, Sin^2\theta}{g} = \frac{1}{3} \, \frac{V_o^2 . 2Sin\theta \, Cos\theta}{g}$$

$$Sin\theta = \frac{2}{3}Cos\theta$$

$$\frac{Sin\theta}{Cos\theta} = \frac{2}{3}$$

$$Tan\theta = \frac{2}{3}$$

$$\theta = Tan^{-1}\frac{2}{3}$$

Q.11. The turn table of a record player rotates initially at a rate of 33 rev./min and takes 20sec. to come to rest.

(a) What is the angular acceleration of the turn table?

(b) How many rotation does the turn table make before coming to rest?

(c) If the radius of the turn table is 0.14m, what is the initial linear speed bug ridging on the rim?

(d) What is the magnitude of the tangential acceleration of the bug at time?

(2005 Failures)

Given Data:

Initial angular speed = $\omega_i = 33$ rev./min

$$\omega_i = \frac{33 \times 2\pi}{60}$$

$$\omega_i = 3.45 \text{ rad./s}$$

Final angular speed $\omega_f = 0$

Time $= t = 20\text{sec.}$

Radius $= r = 0.14\text{m}$

To Find:

(a) Angular acceleration $= \alpha = ?$

(b) No. of rotation $= ?$

(c) Initial linerat speed $= V_i = ?$

Solution:

(a) Using the relation

$$\omega_f = \omega_i + \alpha t$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{0 - 3.45}{20}$$

$$\boxed{\alpha = -0.173 \text{ rad./s}^2.}$$

(b) $\theta = \omega_i t + \alpha t^2 \frac{1}{2}$

$\theta = 3.45 \times 2 + \frac{1}{2} \times (-0.173)(20)^2$

$\theta = 69 - 34.6$

$\boxed{\theta = 34.4 \text{ rad.}}$

No. of rotation $= \frac{\theta}{2\pi} = \frac{34.4}{2(3.14)}$

No. of rotation = 5.5 rotation

(c) Using the relation

$V_i = r\omega_i$

$V_i = 0.14 \times 3.45$

$\boxed{V_i = 0.484 \text{m/s}^2}$

(d) $a = r\alpha$

$a = 0.14 \times -0.173$

$\boxed{a = -0.0242 \text{m/s}^2}$

Q.12. Tarzan swing on a vine of length 4m in a vertical circle under the influence of gravity. When the vine makes an angle $\theta = 20^\circ$ with the vertical. Tarzan has speed of 5m/s^2. Find. ***(2013)***

(a) His centripetal acceleration at this instant.
(b) His tangential acceleration and
(c) The resultant acceleration.

Given Data:

Length of vine = L = 4m
Angle = $\theta = 20^\circ$
Linear speed = $V = 5\text{m/s}^2$

To Find:

(a) $a_c = ?$
(b) $a_t = ?$
(c) $a = ?$

Solution:

(a) Using the relation

$a_c = \frac{V^2}{r}$

$a_c = \frac{(5)^2}{4}$

$\boxed{a_c = 6.25 \text{m/s}^2}$

(b) $a_t = g \sin\theta$

$a_t = 9.8 \sin 20^\circ$

$\boxed{a_t = 3.35 \text{m/s}^2}$

(c) $a = \sqrt{a_c^2 + a_t^2}$

$a = \sqrt{(6.25)^2 + (3.35)^2}$

$\boxed{a = 7.09 \text{m/s}^2}$

EXTRA PROBLEMS:

Q.1 Calculate the centripetal acceleration and centripetal force on a man of mass 80kg. when resting on the ground at the Equator. Given that the radius of earth is 6.4×10^6 m and the earth completes one rotation is a day.

(2011, 2006, 2007 Failures, 2003 Failures)

Given Data:

Mass of man = m = 80kg
Radius of earth = Re = 6.4×10^6m
Earth completes one Rotation = 1 day = $1 \times 24 \times 60 \times 60$ = 86400S

To Find:

Centripetal acceleration = a_c = ?
Centripetal force = F_c =?

Solution:

$$a_c = \frac{4\pi^2 Re}{T^2}$$

$$a_c = \frac{4(3.14)^2 (6.4\times10^6)}{(86400)^2}$$

$$a_c = \frac{4\times9.87\times6.4\times10^6}{7464960000}$$

$$\boxed{a_c = 0.0338 m/s^2}$$ Ans.

$$F_c = \frac{4\pi^2 m\, Re}{T^2}$$

$$F_c = \frac{4(3.14)^2\, 80\times6.4\times10^6}{(86400)^2}$$

$$F_c = \frac{4\times9.87\times80\times6.4\times10^6}{7464960000}$$

$$\boxed{F_c = 2.707N}$$ Ans.

Q.2.

Given data:

T_{max} = 69.6N
m = 2kg
r = 1m

To find:

v = ?

Solution:

When body is moving in a vertical circle

$$T_{max} = F_c + Mg$$

$$T_{max} = \frac{Mv^2}{r} + Mg$$

$$69.6 = \frac{2v^2}{1} + 2 \times 9.8$$

$$69.6 = 2v^2 = 169.6$$

$$69.6 - 19.6 = 2v^2$$

$$2v^2 = 50$$

$$v^2 = 25$$

$$\boxed{v = 5m/sec}$$