NUMERICAL PROBLEMS

Q.1 Find the area of a rectangular plate having length (21.3 ± 0.2) cm and width (9.80 ± 0.10) cm:

SOLUTION:

Area of rectangular plate = Length x width.

$A = (21.3 \pm 2) \times (9.80 \pm 0.10)$

$A = (21.3 \times 9.80) \pm (21.3 \times 0.10) \pm (0.20 \times 9.80) \pm (0.2 \times 0.10).$

$= 208.76 \pm 2.13 \pm 1.96$

$= 208.78 \pm 4.09$

$= (208.8 \pm 4.1)\ cm^2.$

$A = (209 + 4)\ cm^2$ Ans.

Q.2 Calculate (a) the circumference of a circle of radius 3.5cm and (b) area of a circle of radius 4.65cm:

SOLUTION:

(a) Circumference of circle

$S = 2\pi r$

$S = 2 \times 3.141 \times 3.5$

$S = 21.99cm$ Ans.

(b) Area of circle

$A = \pi r^2$

$= 3.141 \times (4.65)^2$

$= 3.141 \times 21.6225$

$A = 67.89cm^2$ Ans.

Q.4 Find the dimension of 'C' in the equation $S = Ct^3$:

SOLUTION:

The dimension of 'S' is L

The dimension of t is T

Then The dimension of $C = \frac{s}{t^3} = \frac{L}{T^3}$

$= LT^{-3}$ Ans.

Q.5 Estimate the number of liters of gasoline used by cars in Pakistan each year (consider cars in Pakistan = 500000):

DATA:

No. of cars = 500000 = 5×10^5

Average distance/year = 16000km

Gasoline consumption = 6km/litres

No. of litre of gasoline used/year = ?

SOLUTION:

Total distance covered by all cars/year = $5 \times 10^5 \times 16000$

$= 8 \times 10^9$ km

Gasoline consumption = 6km/litres

Gasoline consumption = 1/6 litres

Gasoline consumption = 8×10^9km = $\frac{1}{6} \times 8 \times 10^9$

Gasoline consumption = 1033×10^9 litres.

DIMENSIONS

Q.1. Deduce the dimension of:

(a) Velocity **(b)** Acceleration **(c)** Force

(a) Velocity $= \dfrac{\text{Displacement}}{\text{Time}}$

Dimension of displacement $= L$

Dimension of time $= T$

Dimensions of velocity $= \dfrac{L}{T}$

Dimensions of velocity $= LT^{-1}$

(b) Acceleration $= \dfrac{\text{Change in velocity}}{\text{Time}}$

Dimensions of velocity $= \dfrac{L}{T}$

Dimension of time $= T$

Dimensions of acceleration $= \dfrac{\frac{L}{T}}{T}$

Dimensions of acceleration $= \dfrac{L}{T^2}$

Dimensions of acceleration $= LT^{-2}$

c) Force = Mass x Acceleration

Dimension of mass $= M$

Dimensions of acceleration $= \dfrac{L}{T^2}$

Dimensions of force $= M\dfrac{L}{T^2}$

Dimensions of force $= MLT^{-2}$

SELF TEST:(1)

Q.2. Deduce the dimension of:

(a) Momentum ($p = mv$) **(b)** Work ($W = F\,d$) **(c)** Power ($P = F\,v$)

(d) Weight ($w = mg$) **(e)** Angular Momentum ($L = mvr$)

(f) Torque ($\tau = Fxd$) **(g)** Area **(h)** Volume

(i) Density

Q.3. What are the dimensions of gravitational constant 'G' in the formula:

$$F = \frac{G\,m_1\,m_2}{r^2}$$

SOLUTION:

As $F = \dfrac{G\,m_1\,m_2}{r^2}$

Therefore 'G' $= \dfrac{F\,r^2}{m_1\,m_2}$

Dimensions of 'F' $= M\dfrac{L}{T^2}$

Dimension of 'r' = L
Dimension of 'm_1' = M
Dimension of 'm_2' = M
Dimensions of 'G' = $\frac{M\,L/T^2\,(L)^2}{M\,M}$
Dimensions of 'G' = $\frac{M\,L^3}{M^2\,T^2}$
Dimensions of 'G' = $\frac{L^3}{M\,T^2}$

Dimensions of 'G' = $M^{-1}\,L^3\,T^{-2}$

SELF TEST: (2)

Q.4. What are the dimensions of spring constant "k" in the formula:
F = kx, where F = force & x = displacement

Q.5. What are the dimensions of angular velocity "ω" in the formula:
V = rω, <u>where</u>
V = Linear velocity
r = Radius

Q.6. What are the dimensions of angular acceleration "α" in the formula:
a = r α <u>where</u>
a = linear acceleration
r = radius.

Q.7. Show that the expression $S = V_i t + \frac{1}{2} at^2$ is dimensionally correct:

SOLUTION:

$S = V_i t + \frac{1}{2} at^2$
L.H.S. = S
Dimension of S = L
Dimension of L.H.S = L
R.H.S. = $V_i t + \frac{1}{2} at^2$
Dimensions of $V_i = \frac{L}{T}$
Dimension of t = T
Dimensions of a = $\frac{L}{T^2}$
Dimension of $\frac{1}{2}$ = <u>nil</u>
Dimensions of R.H.S. = $\frac{L}{\cancel{T}} \times \cancel{T} + \frac{L}{\cancel{T^2}} \times \cancel{T^2}$
Dimension of R.H.S. = L + L
Dimension of R.H.S. = 2L
Dimension of R.H.S. = L
Dimension of L.H.S = Dimension of R.H.S.
The equation is dimensionally correct.

Q.8. Show that the following equations are dimensionally correct:

(i) $T = 2\pi\sqrt{\frac{m}{k}}$ **(ii)** $H = \frac{V_0^2 \sin^2\theta}{2g}$

L. H. S. = T

Dimension of L.H.S = T

R. H. S. = $2\pi\sqrt{\frac{m}{k}}$

2π is a number having no dimensions.

Dimension of m = M

Dimensions of k = $\frac{M}{T^2}$

Dimensions of R.H.S. = $\sqrt{\frac{M}{\frac{M}{T^2}}}$

Dimensions of R.H.S. = $\sqrt{\frac{MT^2}{M}}$

Dimensions of R.H.S. = $\sqrt{T^2}$

Dimension of R.H.S. = T

Dimension of L.H.S. =Dimension of R.H.S.

The equation is dimensionally correct.

(ii) $H = \frac{V_0^2 \sin^2\theta}{2g}$

L.H.S. = H *(Height)*

Dimension of H = L

Dimension of L.H.S. = L

R. H. S. = $\frac{V_0^2 \sin^2\theta}{2g}$

$\sin^2\theta$ is a number having no dimens

Dimensions of $V_0 = \frac{L}{T}$

Dimensions of g = $\frac{L}{T^2}$

Dimensions of R.H.S = $\frac{\left(\frac{L}{T}\right)^2}{\frac{L}{T^2}}$

Dimensions of R.H.S = $\frac{\frac{L^2}{T^2}}{\frac{L}{T^2}}$

Dimension of R.H.S = L

Dimension of L.H.S = Dimension of R.H.S.

The equation is dimensionally correct.

SELF TEST:(3)

Q.9. Show that the expression $V_f = Vi + at$ is dimensionally correct.

Q.10. Show that the expression $T = 2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.

Q.11. Show that the following equations are dimensionally correct.

(i) $V = \lambda f$ (ii) $T = \frac{2V_0 \sin\theta}{g}$ iii) $R = \frac{V_0^2 \sin 2\theta}{g}$

Where **V_0 = Initail Velocity g = Acceleration due to gravity**