CHAPTER 4
WORK AND ENERGY.

4.1 WORK DONE BY A CONSTANT FORCE.

- **WORK.** The work done on a body by a constant force is defined as;

Def — "The product of the magnitudes of the displacement and the component of the force in the direction of the displacement."

Mathematically;

$$\text{Work} = W = \vec{F}\cdot\vec{d} = Fd\cos\theta = d(F\cos\theta) \quad \text{—(1)}$$

i.e; the scalar or dot product of force and displacement is called work. Where the quantity $(F\cos\theta)$ is the component of the force $\vec{F}$ in the direction of the displacement $\vec{d}$ (fig b)

Work is a scalar quantity.

Explanation

Consider, an object which is being pulled by a constant force $\vec{F}$ at an angle 'θ' to the direction of motion. Work is said to be done when this force causes a displacement in the body.

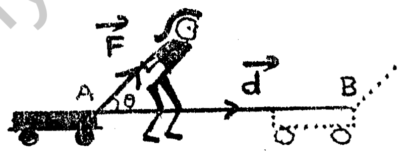

fig (a)

The work is also equal to the product of the magnitude of the force and the component of the displacement along the direction of the force (fig c). Hence;

$$W = \vec{F}\cdot\vec{d} = F(d\cos\theta) \quad \text{—(2)}$$

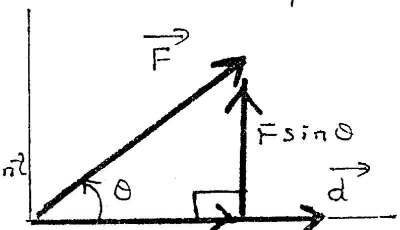

fig (b)

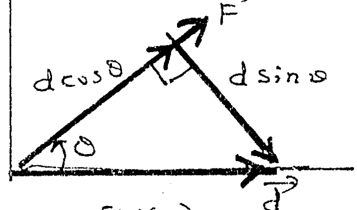

fig (c)

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UNIT. The SI unit of work is Nm or **joule (J)**, named after James Prescott Joule (1818–1889).

Def — "Work done will be equal to 1J, when a force of 1N acting on a body of mass 1kg displaces it through a distance of 1m along the direction of force."

Dimension Work $= N \times m = kg\, m\, s^{-2} \times m = kg\, m^2 s^{-2}$

Dim. of work $= [W] = [ML^2T^{-2}]$

EXPLAINATION OF WORK DONE BY GRAPH.

When a **constant force** acts through a distance 'd', the event can be plotted on a simple graph by taking distance along x-axis and the force along y-axis. As the force does not vary, the graph will be a horizontal straight line. If the constant force $\vec{F}$ (newton) and the displacement $\vec{d}$ (metre) are in the **same direction** then the work done is Fd (joule). Hence the area under a force-displacement curve can be taken to represent the work done by the force.

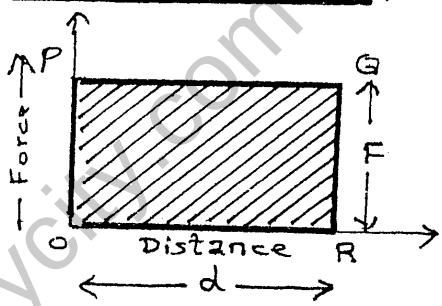


fig (d)

In case the force $\vec{F}$ is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and d.

SPECIAL CASES:

1 — **Positive Work**. If the angle between force and displacement is less than 90° i.e; $\theta < 90°$, then the work is said to be positive.

$$W = \vec{F}.\vec{d} = Fd\cos\theta$$

$$W = Fd \text{ (max value) for } \cos 0° = 1$$

i.e; Work is positive when force and displacement

(P.T.O)

are in the same direction or the force has some component in the direction of displacement.

2— Negative work. If the angle between force and displacement is greater than 90° i.e; $\theta > 90°$, then the work is said to be negative.

e.g;

(i) If a bucket of water is lifted against gravity very slowly, the angle between gravity and displacement is 180°. Hence;

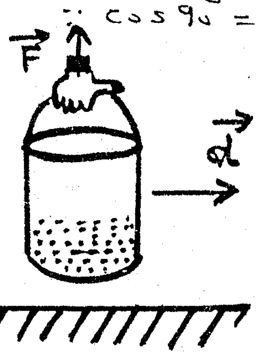
$$W = \vec{F}.\vec{d} = Fd\cos 180° = -Fd.$$

(ii) The body moves against the force of friction on a horizontal plane.

3— Zero work. If the angle between force and displacement is 90°, then no work is done.

$$W = \vec{F}.\vec{d} = Fd\cos 90° = 0 \qquad \because \cos 90° = 0$$

e.g; The supporting force exert to balance the weight of the bucket is vertical while the displacement is horizontal. So the supporting force does not work on the bucket.



4.2 WORK DONE BY A VARIABLE FORCE:

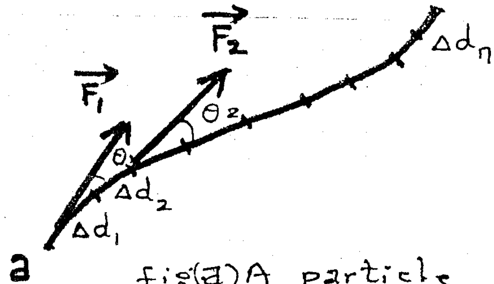
In many cases the force does not remain constant during the process of doing work.

For example, ① As rocket moves away from the Earth, work is done against the force of gravity which varies as the inverse square of the distance from Earth's centre.

② The force exerted by a spring increases with the amount of stretch.

EXPLANATION.

Consider a particle in XY-plane moving from point 'a' to 'b'. The path has been divided into 'n' short intervals of displacements $\vec{\Delta d_1}, \vec{\Delta d_2}, \ldots \ldots, \vec{\Delta d_n}$ and $\vec{F_1}, \vec{F_2}, \ldots, \vec{F_n}$ are the forces acting during these intervals. During each small interval, the force is supposed to be approx. constant. So the workdone for the first interval can be written as;



fig(a) A particle acted upon by a variable force, moves along the path shown from point 'a' to 'b'.

$$\Delta W_1 = \vec{F_1} \cdot \vec{\Delta d_1} = F_1 \cos\theta_1 \Delta d_1$$

Similarly, work done in the second interval is;

$$\Delta W_2 = \vec{F_2} \cdot \vec{\Delta d_2} = F_2 \cos\theta_2 \Delta d_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\Delta W_n = \vec{F_n} \cdot \vec{\Delta d_n} = F_n \cos\theta_n \Delta d_n$$

The total work done in moving the object can be calculated by adding all these terms. Therefore,

$$W_{Total} = \Delta W_1 + \Delta W_2 + \ldots\ldots\ldots + \Delta W_n$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + \ldots\ldots\ldots + F_n \cos\theta_n \Delta d_n$$

or $$W_{Total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \qquad (1)$$

GRAPHICAL EXPLANATION

We can examine this graphically by plotting $F\cos\theta$ verses 'd' as shown in fig(b). The displacement 'd' has been subdivided into the same 'n' intervals. The value of $F\cos\theta$ at the begining of each interval is indicated by the horizontal lines.

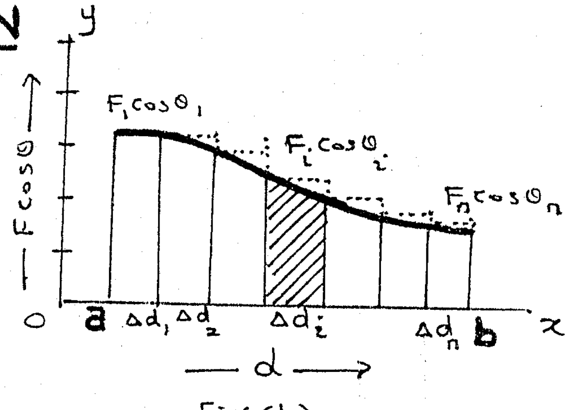


fig (b)

Now the ith shaded rectangles has an area $F_i \cos\theta_i \Delta d_i$ which is the work done during the ith interval. So eq. (1) gives work done which is approx equal to the sum of areas of all the rectangles as shown in fig (b).

To get a more accurate result, we divide the total displacement into a very large number of equal intervals such that each Δd approaches zero. In this case the work done is;

$$W_{Total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad \text{——— (2)}$$

Thus, the work done by a **variable force** in moving a particle between two points is equal to the area under the $F\cos\theta$ verses 'd' curve between the two points 'a' and 'b' as shown in fig (c)

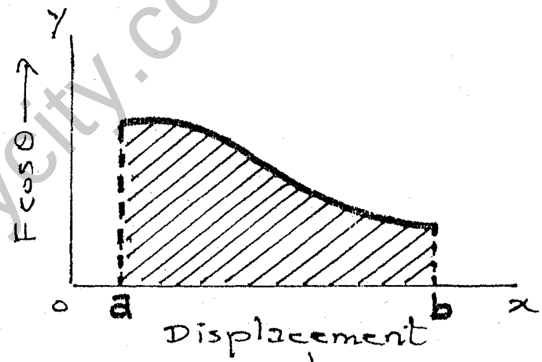


fig (c)

EXAMPLE 4.1

A force 'F' acting on an object varies with distance 'x' as shown in fig. Calculate the work done by the force as the object moves from $x = 0$ to $x = 6\,m$.

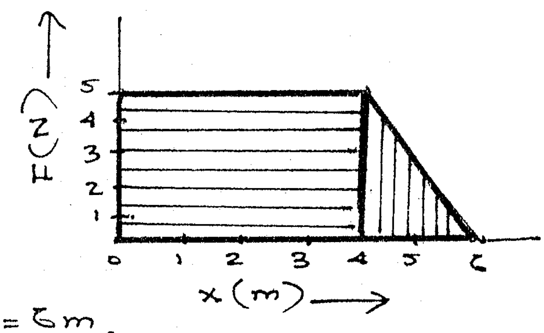


Fig.

Sol:

Work done represented by the area of rectangle

$$= 4\,m \times 5\,N = 20\,Nm = 20\,J \quad \text{——— (1)}$$

Work done represented by the area of triangle

$$= \frac{1}{2} \times 2\,m \times 5\,N = 5\,J \quad \text{——— (2)}$$

$\because$ Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$.

$\therefore$ Total work done $= 20\,J + 5\,J = \boxed{25\,J}$ ——— (3)

#4.3 WORK DONE IN GRAVITATIONAL FIELD.

• GRAVITATIONAL FIELD.

Def—— "The space around the Earth in which the gravitational force acts on a body is called the gravitational field."

When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive, if against, the work is negative.

WORK DONE IN GRAVITATIONAL FIELD.

Let us consider an object of mass 'm' being displaced with constant velocity from point 'A' to 'B' along various paths in the presence of a gravitational force (fig a). In this case the gravitational force is equal to the **weight 'mg'** of the object.

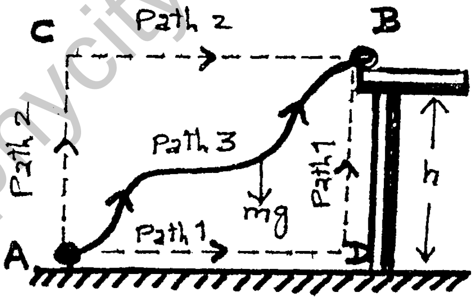


fig. (a)

1—— Work done along the path ADB.

The work done by the gravitational force along the path ADB can be split into two parts:

(a) The work done along AD is zero, because the weight mg is perpendicular to this path.

(b) The work done along DB is $(-mgh)$ because the direction of mg is opposite to that of the displacement i.e; $\theta = 180^\circ$.

Hence, the work done in displacing a body from 'A' to 'B' through path 1 is;

$$W_{ADB} = 0 + (-mgh) = -mgh \qquad (1)$$

2 — Work done along the path ACB.

If we consider the path ACB, it can also be split up into two parts;

(a) The work done along the path AC is also $(-mgh)$

(b) The work done along the path CB is zero because angle between weight and path is 90°.

Thus, the work done in displacing the object from 'A' to 'B' by path 2 is;

$$W_{ACB} = -mgh + 0 = -mgh \quad \text{(2)}$$

3 — Work done along the curved path AB.

Let us now consider path 3 i.e; curved one. Imagine the curved path, to be broken down into a series of horizontal and vertical steps as shown in fig (b). There is no work done along the horizontal steps, because mg is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. Thus

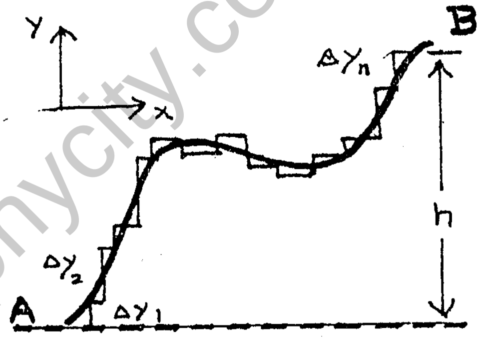


fig (b) A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during the y displacement.

$$W_{AB} = -mg\,(\Delta y_1 + \Delta y_2 + \ldots\ldots\ldots + \Delta y_n)$$

As $\Delta y_1 + \Delta y_2 + \ldots\ldots + \Delta y_n = h$

Hence, $W_{AB} = -mgh \quad \text{(3)}$

CONCLUTION.

"Work done in the Earth's gravitational field is independent of the path followed."

• CONSERVATIVE FIELD :

Def — "The field in which the work done is independent of the path followed or work done in a closed path is zero is called a conservative field."

For example ;

1 — Gravitational field
2 — Electric field
3 — Magnetic field etc.

CLOSED PATH.

Def — "A path in which a body after passing through several points reaches the starting point is called a closed path or loop."

Conservative Forces ① Gravitational force ② Elastic spring force ③ Electric force.

• NON CONSERVATIVE FIELD :

Def — "The field in which the work done in moving a body between two points depends upon the path followed between the two paths."

The **frictional force** is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

Non Conservative Forces.

① Air resistance ② Tension in a string
③ Normal force ④ Propulsion force of a rocket
⑤ Propulsion force of a motor.

4.4 POWER:

Def — "Rate of doing work is called power."

Mathematically; If work ΔW is done in a time interval Δt, then the average power during the interval Δt is:

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \text{———} (1)$$

INSTANTANEOUS POWR.

If the rate of doing work is not uniform and work is expressed as a function of time, then the instantaneous power at any instant is defined as;

Def — "Ratio of the work done to the time interval, when both are extremely small is called instantaneous power."

$$P_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \quad \text{———} (2)$$

When the work is done at uniform rate, the average power and instantaneous power are equal.

UNITS.

1— The SI unit of power is **Watt (W)** after James Watt.

Def — "If one joule of work is done in one second than power will be one watt."

$$1 \text{ watt} = 1 \text{ J } s^{-1} \quad \text{———} (3)$$

1 Kilowatt $= 1 KW = 1000 W = 10^3 W$ (Bigger unit of power)
1 Mega watt $= 1 MW = 10^6 W$ "
1 Giga watt $= 1 GW = 10^9 W$ "

2— In British Engineering system, the unit of power is **horse power (hp)**;

Def — "when work is done at the rate of 550 ft-lb in one second is called horse power."

$$1 hp = 550 \frac{ft\text{-}lb}{s} = 746 \text{ Watts} \quad \text{———} (4)$$

UNITS OF WORK IN TERMS OF POWER.

In electrical measurements, the unit of work is expressed as watt-second. However, a commerical unit of electrical energy is a **Kilowatt-hour (KWh)**. It is also called as Board of trade unit (BOTU).

Def — "one kilowatt hour is the work done in one hour by an agency whose power is one kilowatt."

$$\therefore \quad 1\,KWh = 1000\ W \times 3600\ S = 3.6\times10^6\ WS$$
$$= 3.6\times10^6\ J = 3.6\ MJ \quad \text{——— (5)}$$

DIMENSIONS OF POWER.

$$Power = \frac{Work}{Time} = \frac{1J}{1S} = \frac{ML^2T^{-2}}{T}$$

$$\text{Dim. of Power} = [P] = [ML^2T^{-3}] \quad \text{——— (6)}$$

RELATION BETWEEN FORCE, POWER AND VELOCITY

It is sometime, convenient to express power in terms of constant force $\vec{F}$ acting on an object moving at constant velocity $\vec{V}$. For example; when the propeller of a motor boat causes the water to exert a constant force $\vec{F}$ on the boat, it moves with a constant velocity $\vec{V}$.

As $$P_{in} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \quad \text{——— (7)}$$

we know $$\Delta W = \vec{F}\cdot\Delta\vec{d} \quad \text{——— (8)}$$

Putting values, we have

$$P_{in} = \lim_{\Delta t \to 0} \vec{F}\cdot\left(\frac{\Delta\vec{d}}{\Delta t}\right)$$

Hence, $$P_{in} = \lim_{\Delta t \to 0} \vec{F}\cdot\vec{V} \quad \text{——— (9)}$$

or $$\boxed{P = \vec{F}\cdot\vec{V}} \quad \text{——— (10)}$$

i.e; Power is the dot product of force and velocity.

EXAMPLE 4.2

A 70 kg man runs up a long flight of stairs in 4 S. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Sol. $$Power = \frac{mgh}{t} = \frac{70\,kg \times 9.8\,ms^{-2} \times 4.5\,m}{4\,S}$$

$$P = 7.7\times10^2\ kg\,m^2s^{-3} = \boxed{7.7\times10^2\ W}$$

4.5 ENERGY:

Def — "Capacity to do work is called energy."

MECHANICAL ENERGY.

Def — "Energy possessed by a body due to its state of rest or motion is called Mechanical energy."

TYPES. There are two types of mechanical energy.

1 — Kinetic energy (K.E)
2 — Potential energy (P.E)

① K.E.

Def — "Energy possessed by a body due to its motion is called K.E."

Formula; $K.E = \frac{1}{2} \times m v^2$ —— (1)

or $K.E = \frac{1}{2} \times m(\vec{v}.\vec{v})$ —— (2)

② P.E.

Def — "Energy possessed by a body due to its position in a force field e.g gravitational field or because of its constrained state is called P.E."

Formula; The P.E of a body due to gravitational field near the surface of the Earth at a height 'h' can be expressed as;

$P.E = mgh$ —— (3)

This is called gravitational P.E, which is always determined relative to some arbitrary position which is assigned the value of zero P.E. The Earth's surface or a point at infinity from the Earth can be chosen as zero reference level of gravitational P.E. In present case, this reference level is the surface of the Earth as position of zero P.E.

UNITS. The SI unit of energy is joule (J).

DIMENSION. [Dim. of energy] = $[ML^2T^{-2}]$

ELASTIC P.E.

Def — "The energy possessed by the spring due to its compressed or stretched state is called elastic P.E."

• WORK ENERGY PRINCIPLE:

Statement — "Work done on the body is equal to the change in its K.E."

EXPLANATION.

Whenever work is done on a body, it increases its energy. Suppose a body of mass 'm' is moving with velocity v_i. A force 'F' acting through a distance 'd' increases the velocity to v_f, then from 3rd equation of motion:

$$2ad = v_f^2 - v_i^2$$

or $$d = \frac{v_f^2 - v_i^2}{2a} \quad \text{--- (1)}$$

From second law of motion

$$F = ma \quad \text{--- (2)}$$

Multiplying eq. (1) and (2), we get

$$Fd = \frac{1}{2} \times m\left(v_f^2 - v_i^2\right)$$

or $$Fd = \frac{1}{2} \times m v_f^2 - \frac{1}{2} m v_i^2$$

$$Fd = (K.E)_f - (K.E)_i \quad \text{--- (3)}$$

As Fd = work done on the body;

$\therefore$ $$W = (K.E)_f - (K.E)_i$$

$$\boxed{W = \text{change in K.E of the body.}}$$

This is called work-energy principle.

N.B If work is positive, the final K.E is greater than the inital K.E and K.E increases. If the work is negative, the K.E decreases. When work is zero, the K.E remains constant.

If a body is raised up from Earth's surface, the work done increases the gravitational P.E. **Similarly**, if a spring is compressed, the work done on it equals the increase in its elastic P.E.

ABSOLUTE POTENTIAL ENERGY :

Def —— "The work done in moving a body from Earth's surface to a point far away from the Earth where the value of 'g' is negligible is called Absolute P.E."

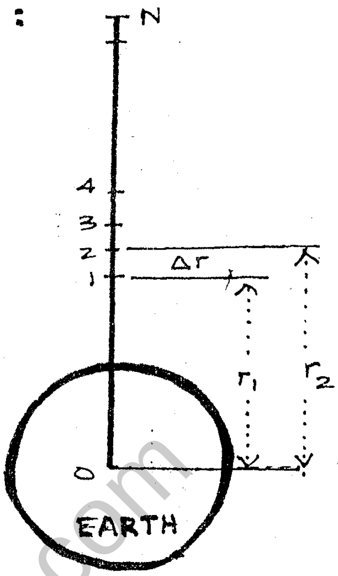


fig.

DETERMINATION OF A.P.E .

The relation for the calculation of the workdone by the gravitational force or P.E=mgh, is true only near the surface of the Earth where the gravitational force is constant. **But** if the body is displaced through a large distance in space from, let, point 1 to N (fig) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance. So, we divide the distance between 1 and N into small steps each of length Δr such that the value of the force remains constant for each small step. Hence, the total workdone can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre 'O' of the Earth, then the workdone during the first step i.e. displacing a body from point 1 to point 2 can be calculated as: The distance between the centre of the step and the centre of the Earth will be;

$$r = \frac{r_1 + r_2}{2} \quad \text{———} (1)$$

If $\quad r_2 - r_1 = \Delta r \quad$ ——— (2)

then $\quad r_2 = \Delta r + r_1 \quad$ ——— (3)

Putting the value of r_2 in eq (1), we have

$$r = \frac{r_1 + r_1 + \Delta r}{2} = \frac{2r_1 + \Delta r}{2}$$

$$r = r_1 + \frac{\Delta r}{2} \quad \text{——— (4)}$$

Squaring on both sides, we have;

$$r^2 = r_1^2 + \frac{\Delta r^2}{4} + r_1 \Delta r \quad \text{——— (5)}$$

As $(\Delta r)^2 << r_1^2$, So this term can be neglected as r_1^2

Hence $$r^2 = r_1^2 + r_1 \Delta r \quad \text{——— (6)}$$

Putting the value of Δr, we have

$$r^2 = r_1^2 + r_1 (r_2 - r_1)$$

$$= \cancel{r_1^2} + r_1 r_2 - \cancel{r_1^2}$$

$$r^2 = r_1 r_2 \quad \text{——— (7)}$$

The gravitational force F at the centre of the step is;

$$F = \frac{GMm}{r^2} \quad \text{——— (8)}$$

where M = mass of Earth
G = gravitational constt.
m = mass of an object.

becomes;

$$F = \frac{GMm}{r_1 r_2} \quad \text{——— (9)}$$

As this force is assumed to be constant during the interval Δr, so the work done is

$$W_{1 \to 2} = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos 180° = -\,GMm \times \frac{\Delta r}{r_1 r_2}$$

$\because \cos 180° = -1$

The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force.

Putting the value of Δr, we get

$$W_{1 \to 2} = -GMm \times \frac{(r_2 - r_1)}{r_1 r_2}$$

$$= -GMm \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Similarly;

$$W_{2 \to 3} = -GMm \times \left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$\vdots \qquad \qquad \vdots$$

$$W_{(N-1) \to N} = -GMm \times \left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\to 2} + W_{2\to 3} + \cdots\cdots + W_{(N-1)\to N}$$

$$= -GMm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\cdots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

On simplification, we get

$$W_{total} = -GMm\left[\frac{1}{r_1}-\frac{1}{r_N}\right] \quad \text{———} (10)$$

If the point N is situated at an infinite distance from the Earth, so

$$r_N = \infty \quad , \text{then } \frac{1}{r_N} = \frac{1}{\infty} = 0$$

Hence,

$$W_{total} = \frac{-GMm}{r_1}$$

Therefore, the general expression for the gravitational P.E of a body situated at distance 'r' from the Earth is

$$U = -\frac{GMm}{r} \quad \text{———} (11)$$

This is known as the absolute value of gravitational P.E of a body at a distance 'r' from the centre of Earth.

Now, when **'r' increases**, the gravitational force does negative work and U increases (i.e; becomes less negative). When **'r' decreases**, the body falls towards the Earth, the work is positive and P.E decreases (i.e; becomes more negative). U is zero when **($r=\infty$)**, the mass 'm' is infinitely away from the Earth, so it is quite a different choice for making $U=0$ at some arbitrary position. Therefore, the choice of zero point is arbitrary and the only difference of P.E from one point to another is significant, so negative values of U should not be too alarming.

Now the absolute P.E on the surface of the Earth is found by putting $r = R$ (Radius of the Earth)

$$\boxed{A.P.E = U_g = -\frac{GMm}{R}}$$

The negative sign shows that the Earth's gravitational field for mass 'm' is attractive. The above eq. gives the work or the energy required to take the body out of the Earth's gravitational field, where its P.E w.r.t Earth is zero.

ESCAPE VELOCITY:

When an object is projected upward it comes back to ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape out of the influence of gravity of the Earth.

Def — "The initial velocity of an object with which it goes out of the Earth's gravitational field is known as escape velocity."

EXPRESSION FOR ESCAPE VELOCITY.

The escape velocity corresponds to the initial K.E gained by the body, which carries it to an infinite distance from the surface of Earth is

$$\text{Intial K.E} = \frac{1}{2} m v_{esc}^2 \quad \text{———} (1)$$

The work done in lifting a body from Earth's surface to an infinite distance is equal to the absolute P.E of the body at Earth's surface is

$$A.P.E. = \frac{GMm}{R} \quad \text{———} (2)$$

The body will escape out of the gravitational field if the initial K.E of the body is equal to the absolute P.E. Then

$$\frac{1}{2}\cancel{m}V_{esc}^{2} = \frac{GM\cancel{m}}{R}$$

or $$V_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{——— (3)}$$

As $$g = \frac{GM}{R^2} \implies gR = \frac{GM}{R}$$

Hence, $$V_{esc} = \sqrt{2gR} \quad \text{——— (4)}$$

Putting values, we have

$$V_{esc} = \sqrt{2 \times 9.8\ ms^{-2} \times 6.4 \times 10^{6}\ m}$$

$$= 11 \times 10^{3}\ ms^{-1}$$

$$\boxed{V_{esc} = 11\ \ Km\ s^{-1}}$$

4.6 INTERCONVERSION OF P.E AND K.E :

Consider a body of mass 'm' at rest at a height 'h' above the surface of the Earth.

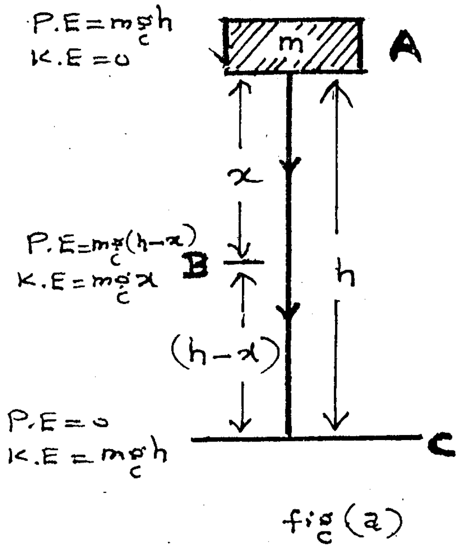


fig (a)

AT POINT 'A'.

$$(P.E)_P = mgh \quad \text{——— (1)}$$

and $$(K.E)_P = 0 \quad \text{——— (2)}$$

so $$(T.E)_P = \boxed{mgh} \quad \text{——— (3)}$$

When the body is dropped from a certain height, its K.E and P.E are interchange.

AT POINT 'B'.

The P.E and K.E at the position 'B' when the body has fallen a distance 'x', ignoring air friction is ;

$$(P.E)_B = mg(h - x) \quad \text{——— (4)}$$

and $$(K.E)_B = \frac{1}{2}mV_B^2 \quad \text{——— (5)}$$

Velocity V_B, at 'B' can be calculated from the rel. as;

$$v_f^2 = v_i^2 + 2gs$$

As $v_f = v_B$, $v_i = 0$, $S = x$, we have;

$$v_B^2 = 0 + 2gx = 2gx \quad \text{——— (6)}$$

$\therefore$

$$(K.E)_B = \frac{1}{\cancel{2}} \times m(\cancel{2}gx) = mgx \quad \text{——— (7)}$$

So

$$(T.E)_B = (P.E)_B + (K.E)_B$$

$$(T.E)_B = mg(h - x) + mgx$$

$$(T.E)_B = \boxed{mgh} \quad \text{——— (8)}$$

AT POINT 'C'

At position 'c', just before the body strikes the Earth;

$$(P.E)_c = 0 \quad \text{——— (9)}$$

$$(K.E)_c = \frac{1}{2} \times m v_c^2 \quad \text{——— (10)}$$

where v_c can be found out by the following rel;

$$v_f^2 = v_i^2 + 2gs$$

As $v_f = v_c$, $v_i = 0$, $S = h$

So

$$v_c^2 = 2gh \quad \text{——— (11)}$$

i.e;

$$(K.E)_c = \frac{1}{\cancel{2}} \times m \times \cancel{2}gh = mgh \quad \text{——— (12)}$$

$$(T.E)_c = (P.E)_c + (K.E)_c$$

$$= 0 + mgh$$

$$(T.E)_c = \boxed{mgh} \quad \text{——— (13)}$$

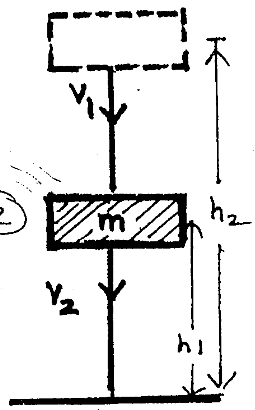


fig (b)

Thus at point 'c', K.E is equal to the original P.E of the body. Actually when a body falls, its velocity increases i.e; the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its K.E. On the other hand, as the body falls, it height decreases and hence, its P.E also decreases. So

$$\boxed{\text{Loss in P.E} = \text{Gain in K.E}}$$

or;

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2) \quad \text{——— (14)}$$

Where v_1 and v_2 are velocities of the body at the heights h_1 and h_2 respectively, when no friction is involved.

WHEN FRICTIONAL FORCE IS PRESENT.

If we assume that a frictional force "f" is present during the downward motion, then a part of P.E is used in doing work against friction equal to fh. The remaining $P.E = mgh - fh$ is converted in K.E;

Hence,

$$mgh - fh = \frac{1}{2}mv^2$$

or

$$mgh = \frac{1}{2}mv^2 + fh$$

Thus,

Loss in P.E = Gain in K.E + Work done against friction.

N.B After the body hitting the ground, the P.E and K.E are converted into sound and heat energies. This is called dissipation of energy.

4.7 LAW OF CONSERVATION OF ENERGY:

Statement — "Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant."

EXPLANATION.

The kinetic and potential energies are both different forms of the same basic quantity i.e; **mechanical energy**. Th s the total mechanical energy is the sum of its K.E and P.E. It is an experimental fact that one form of energy can be converted into another form. e.g; an electrical and chemical energy are more easily transferred into heat. Also the P.E of the falling object changes to K.E but on striking the ground, the K.E changes into heat and sound. If it seems in an energy transfer that some energy has disappeared, the lost energy is often converted into heat. Ultimately, all energy transfers result in heating of environment.

EXAMPLE 4.3: A brick of mass 2kg is dropped from a rest position 5m above the ground. What is its velocity at a height of 3m above the ground?

DATA. $m = 2\,kg$, initial height $= h_1 = 5m$,
final height $= h_2 = 3\,m$, $g = 9.8\,ms^{-2}$, $V_1 = 0$
V_2 = velocity of brick at height $h_2 = ?$

Sol. As Loss in P.E = Gain in K.E

So
$$\cancel{m}g(h_1 - h_2) = \frac{1}{2}\cancel{m}(V_2^2 - V_1^2)$$
$$9.8\,ms^{-2}(5m - 3m) = \frac{1}{2}(V_2^2 - 0)$$
$$V_2^2 = 2 \times 9.8 \times 2$$
$$V_2 = \sqrt{2 \times 9.8 \times 2} = \boxed{6.3\,ms^{-1}}$$

4.8 NON CONVENTIONAL ENERGY SOURCES:

Energy is being consumed at an enormous rate in the world today. The present annual consumption is about 6×10^{13} KWh. Scientists are always exploring the new sources of energy to fulfil the future needs. Some of these are as:

1 — ENERGY FROM TIDES.

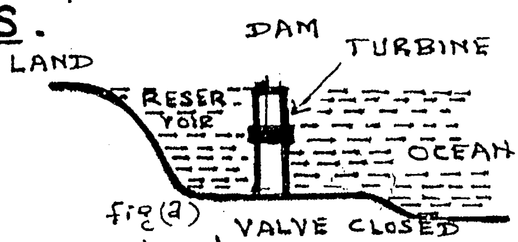


fig (a)
High Tide:
Water level equalized

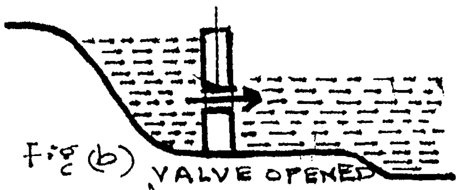


fig (b)
LOW Tide:
water is begining to flow out of basin to ocean, driving turbines.

Energy can be obtained from tides. Gravitational force of the moon give rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the rushing water also drives turbines and generates electricity as shown. Tidal power stations have been installed in France, Alaska, Argentina etc.

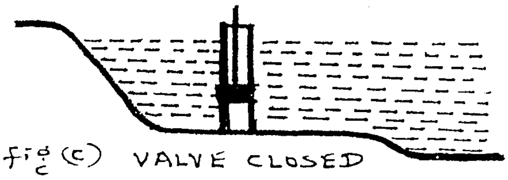


fig (c) VALVE CLOSED
Water level equalized.

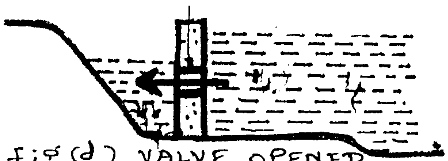
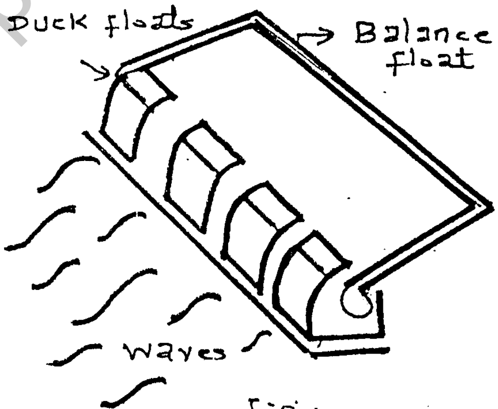


fig (d) VALVE OPENED
High tide:
Water is allowed to flow back into the basin, driving turbines.

fig: Tidal power plant. Turbines are located inside the dam.

2— ENERGY FROM WAVES.

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known as salter's duck. It consists of two parts:



fig

(i) — Duck float
(ii) — Balance float

The wave energy makes duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.

3— SOLAR ENERGY :

The Earth receives huge amount of energy directly from the sun each day. Energy from Sun is called solar energy. Every year the earth absorbs about 4×10^{7} KWh of solar energy. If we could utlize only 0.1% of the incident solar energy, it would be more than sufficient for the entire world's energy requirements. Solar energy at normal incidence outside the Earth's atmosphere is about $1.4\,KWm^{-2}$ which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of the solar energy reaching the Earth's surface is about $1\,KWm^{-2}$. This energy can be used directly to heat water using large solar reflectors and thermal absorbers or be converted to electricity.

In **One method** the flate plate collectors are used for heating water. A typical collector is shown in fig. It has a blanckened surface which absorbs energy directly from solar radiation. Cold water passes over the surface and is heated upto about 70°c.

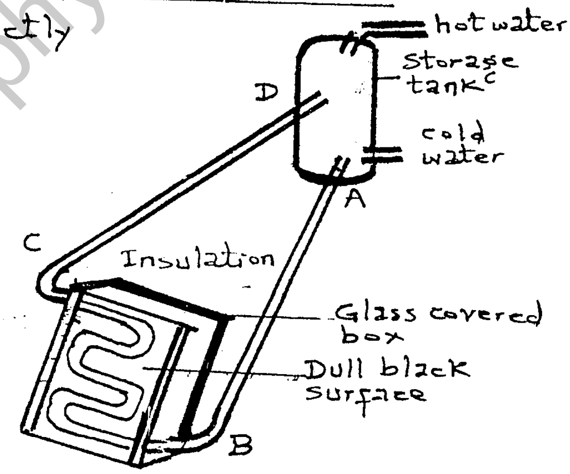


fig.

Much higher temp. can be achieved by concentrating solar radiation on to a small surface area by using huge reflectors (mirrors) or lenses to produce steam for running a turbine.

The **other method** is the direct conversion of sunlight into electricity through the use of semi conductor devices called solar cells also known as photo voltaic cells.

Def — "A solar cell is a device which is used to convert solar energy into electric energy."

Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single voltaic cell is very low. In order to get sufficient high voltage for practical use, a large no. of such cells are connected in series forming a solar cell panel.

For cloudy days or nights, electric energy can be stored during the sun light in Nickel cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

Solar cells, although, are expensive but last a long time and have low running cost.

USES OF SOLAR CELLS

(i) — Solar cells are used to power satellites having large solar panels which are kept facing the sun.

(ii) — Solar cells are used for remote ground based weather stations and rain forest communication systems.

(iii) — Solar calculators are also in use. etc.

4 — ENERGY FROM BIOMASS :

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or bio conversion refers to the use of this material as fuel or its conversion into fuels. There are many methods used for the conversion of biomass into fuels. But the most common are

(i) — Direct combustion
(ii) — Fermentation.

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste.

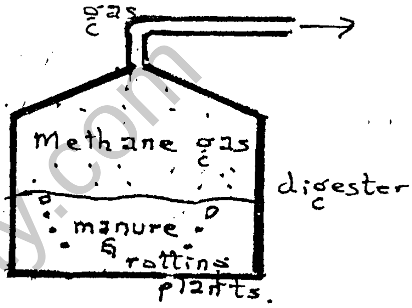


fig.

- Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by **fermentation** of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).
- The rotting of biomass in a closed tank called a digester produces Biogas which can be piped out to use for cooking and heating (fig)
- The waste material of the process is a good organic fertilizer. Thus, production of biogas provides us energy source and also solves the problem of organic waste disposal.

5— ENERGY FROM WASTE PRODUCTS :

Waste products like wood waste, crop residue, and particularly muncipal solid wast can be used to get energy by direct conversion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.

6 — GEOTHERMAL ENERGY :

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.

(i) Radioactive decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

(ii) Residual heat of the Earth

At some places hot igneous rocks, usually within 10 Km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temp. of these rocks is about 200°c or more.

(iii) Compression of material

The compression of material deep inside the Earth also causes generation of heat energy.

In some places water beneath the ground is in contact with hot rocks and is raised to high temp. and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators. Where water is not present and hot rocks are not very deep, the water is pumped down through them which returns as steam (Fig.). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a **geyser**. It is a hot spring that discharges steam and hot water, intermittently

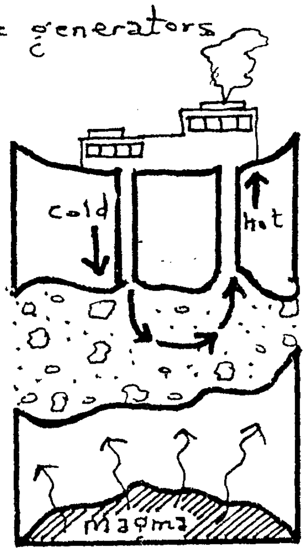


fig.

releasing an explosive coloumn into the air. Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.