MOTION AND FORCE

5/A

-:MOTION:-

Definition:- An object is said to be in the state of motion if it changes its position with respect to its surroundings. e.g. motion of car, Flying of birds etc.

-: DISPLACEMENT:-

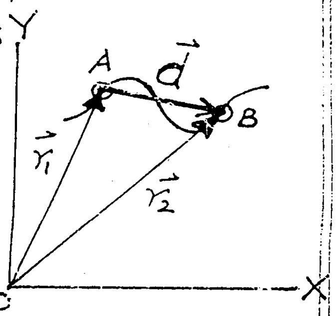
Def:- "The change in position of an object from initial position to the final position is called displacement"

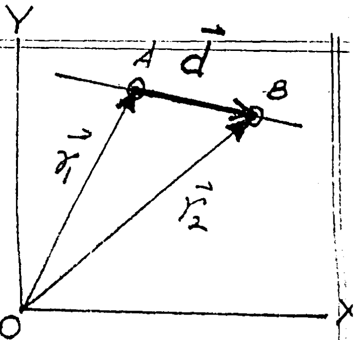
OR

"The shortest directed distance between the two points is called displacement".

It is a vector quantity which shows how far and in what direction the body has been moved or displaced from its initial position. The tail of the displacement vector lies at the initial position while its tip concides with the final position.

When an object moves along a curve, its displacement does not concide with its path of motion but when it moves along a straight path its displacement concides its path of motion.





Mathematically displacement is given as

$$\vec{d} = \vec{r_2} - \vec{r_1}$$

where $\vec{r_1}$ and $\vec{r_2}$ are the position vectors representing the position of an object with respect to the initial position A and final position B respectively. The straight line between the two points gives the magnitude of displacement.

-: VELOCITY :-

Def:- "The time rate of change of displacement is called velocity."

Let $\vec{d}$ be the displacement covered by an object in time t then its velocity is given as

$$\vec{V} = \frac{\vec{d}}{t}$$

Its direction is along the direction of $\vec{d}$

-: UNIT :- The S.I. unit of velocity is m/sec

-: DIMENSIONS :- The dimensions of $\vec{V}$ are $[LT^{-1}]$

-: AVERAGE VELOCITY :-

Def:- The ratio of total displacement to the total time is called average velocity. It is denoted by $<\vec{V}>$ or $\vec{V}_{av}$.

If $\vec{d}$ be the total displacement in time interval t then its average velocity is given as

$$\langle\vec{V}\rangle = \frac{\vec{d}}{t}$$

Average velocity does not tell about the motion between points A and B:∵

In particular case when the displacement of an object vanishes when it returns to its initial position, we use the term instantaneous velocity:-

-: INSTANTANEOUS VELOCITY:-

Def:- The limiting value of $\frac{\Delta \vec{d}}{\Delta t}$ as the time interval Δt approaches zero is called instantaneous velocity:-

OR

The velocity of an object at a particular instant of time is called instantaneous velocity and is denoted by V_{inst}.

Mathematically it is expressed as

$$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

where $\Delta\vec{d}$ represents the change in position of object.

When Δt is made very small, $\Delta\vec{d}$ also become very small and the final point B approaches to the initial point A. The direction of V_{inst} at A is along tangent at A.

If instantaneous velocity does not change, the body is said to be moving with uniform velocity.

-: ACCELERATION :-

-: **Def**:- The time rate of change of velocity of an object is called acceleration and is denoted by $\vec{a}$

If $\Delta\vec{v}$ be the change in velocity in time interval Δt then

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

The velocity can be changed either by changing its magnitude or direction or both.

-: **UNIT**:- The S.I. unit of acceleration is m/sec^2

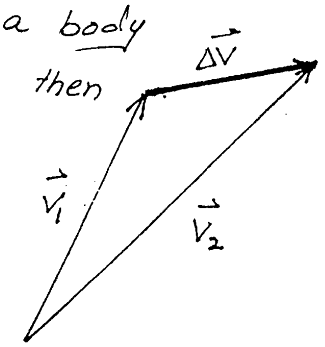
DIM. :- The dimensions of $\vec{a}$ are $[M^0LT^{-2}]$.

-: AVERAGE ACCELERATION :-

Def:- The ratio of total change in velocity in total time taken is called average acceleration:- and is denoted by $\langle\vec{a}\rangle$ or $\vec{a}_{av}$

Let $\vec{v}_1$ velocity of a body changes to $\vec{v}_2$ in time Δt then

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$



It direction is same as $\Delta\vec{v}$

-: INSTANTANEOUS ACCELERATION :-

Def:- The acceleration of a body at any particular instant of time is called instantaneous acceleration.

It is obtained by making Δt smaller and smaller such that it approaches to zero. Mathematically it is given as

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Acceleration is positive when velocity is increasing and is negative when it is decreasing.

-: UNIFORM ACCELERATION:-

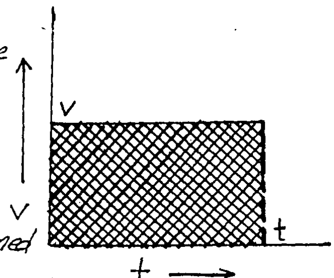
Def:- If the velocity of an object changes equally in equal intervals of time. its acceleration is said to be uniform.

VELOCITY TIME GRAPH

velocity time graph explain the change occurs in the velocity of an object with time.

If an object like car is moving on a road with constant velocity or zero acceleration. The graph is a horizontal line parallel to time axis.

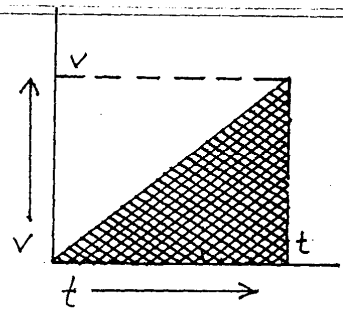
The distance covered by the object moving with uniform velocity can be given as $S = vt$ but this can also be determined by calculating the area of shaded rectangle under the curve and time axis. so



$$S = \text{Area of rectangle} = \text{Length} \times \text{width}$$
$$= v \times t$$

If the body is moving with increasing velocity. The graph is a straight line inclined to time axis.

The distance covered by the object under velocity time graph is equal to the area of shaded triangle. So

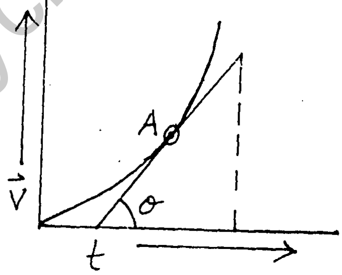


$S =$ Area of triangle

$= \frac{1}{2} \times$ base $\times$ Altitude $= \frac{1}{2} \times v \times t$

The distance can also be calculated by taking the product of mean velocity with time.

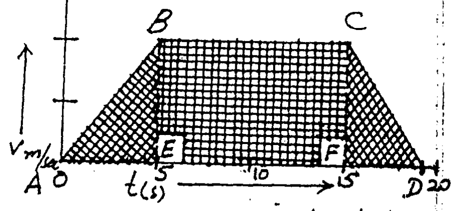
If the body is moving with variable acceleration, the velocity time graph is a curve. The instantaneous acceleration at A is equal to the slope of tangent at A.



-: EXAMPLE#3.1: -

The velocity-time graph of a car moving on a straight road is shown in fig. Describe the motion of the car and find the distance covered.

-: SOLUTION: -



As the velocity of the object changes from $v_i = 0$ m/s to $v_f = 20$ m/s in 5 sec from point A to B according to the graph. So its average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t} = \frac{20-0}{5} = 4\,m/s^2$$

From point B to point C, the car has zero acceleration due to constant velocity.
In moving from point C to D, the velocity starts decreasing and finally become zero in 4 sec. So the retardation in the car is

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0-20}{4} = -5m/sec^2$$

The distance covered by the car is equal to the shaded area between the velocity time graph and time axis and is given as

$$S_{AD} = S_{AB} + S_{BC} + S_{CD}$$
$$= \text{Area of } \triangle ABE + \text{Area of } \square BCFE + \text{Area of } \triangle CDF$$
$$= \frac{1}{2} \times \text{base} \times \text{Altitude} + \text{Length} \times \text{width} + \frac{1}{2} \times \text{base} \times \text{Altitude}$$
$$= \frac{1}{2} \times 20 \times 5 + 20 \times 10 + \frac{1}{2} \times 20 \times 4$$
$$= 50 + 200 + 40 = 290m.$$

-: REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION:-

The equations of motion for an object moving with uniform acceleration when its velocity changes from v_i to v_f in time t and it travels a distance s are given as.

$$v_f = v_i + at$$
$$S = V_{av} \times t = \left(\frac{v_f + v_i}{2}\right) \times t$$
$$S = v_i t + \frac{1}{2} a t^2$$
$$v_f^2 = v_i^2 + 2as$$

These equations are useful for rectilinear motion of an object.

☆ In the absence of air friction when the body falls with a uniform acceleration g under the effect of gravity, the equations of motion for freely falling body are given as

$$V_f = v_i + gt$$

$$S = v_i t + \frac{1}{2} g t^2$$

$$2gS = v_f^2 - v_i^2 \quad \text{or} \quad v_f^2 = v_i^2 + 2gS$$

ACCELERATION DUE TO GRAVITY

Def:- The acceleration produced in a freely falling bodies due to pull of earth is called acceleration due to gravity. Its numerical value all over the surface of earth is $9.8\,m/s^2$. It is maximum ($9.83\,m/s^2$) at the poles and minimum ($9.78\,m/s^2$) at the equator.

-: NEWTON'S LAWS OF MOTION :-

These laws were clearly stated and published by Sir Isaac Newton in his famous book "**Principia**" in 1687.

and are applicable to those objects moving at a very low speeds as compared to the speed of light i.e. $3 \times 10^8\,m/sec$.

while relativistic mechanics developed by Einstien is applicable to the objects moving with speeds comparable to the speed of light e.g. elementary particles in an accelerator.

-: NEWTON'S FIRST LAW OF MOTION:-

According to this law " A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force."

It is also called the law of inertia because a body can maintain its state of rest or of uniform motion due to its principle feature called inertia. It is difficult for the body to change its static or dynamic state itself due to inertia which depends upon its mass.

This law is only applicable in inertial frame of refrence like frame of refrence ($a=0$) stationed on earth.

-: NEWTON'S SECOND LAW OF MOTION:-

-: STATEMENT:- This law states that " The acceleration produced in an object by an applied force is directly propotional to the force and inversly proportional to the mass of the object"

Let a force F produces an acceleration a in a body of mass m then

$$a \propto F \quad , \quad a \propto \frac{1}{m}$$

$$\Rightarrow a \propto \frac{F}{m} \quad \text{or} \quad F \propto ma$$

$$\Rightarrow F = K\, ma$$

Where K is a constant of proportionality and in S.I system, it's value is equal to unity

So $K = 1$

$\therefore F = ma$

-: NEWTON'S THIRD LAW OF MOTION:-

STATEMENT:-

"Action and Reaction are equal in magnitude but oppositly directed."

In case of interaction between the two objects, each body will experience a force on itself equal to that which it exerts on the second object. These action and reaction forces never balance each other because they are acting on two different bodies.

Walking on the ground, Firing of bullet launching of rocket are the key examples of third law of motion.

-: MOMENTUM :-

Def:- The quality possessed by an object due to which it is very difficult to stop its motion when it is either moving with larger velocity or having greater mass is called quantity of motion or momentum.

Mathematically it is defined as the product of mass and velocity and is denoted by p.

$$\therefore \vec{p} = m\vec{v}$$

It is a vector quantity and is directed along the direction of velocity:-

UNIT:- S.I unit of momentum is $kg\,m\,sec^{-1}$ or N-sec

DIMENSIONS:- Its dimensions are $[MLT^{-1}]$

MOMENTUM AND NEWTON'S SECOND LAW OF MOTION

-: STATEMENT :- This law states as

" The rate of change of momentum of an object is equal to the applied force."

-: EXPLANATION :-

Let us consider an object of mass m is moving with velocity v_i when an external force acts on it for time t, it changes its velocity from v_i to v_f. The acceleration produced is given as

$$a = \frac{v_f - v_i}{t} \quad \text{---} (1)$$

but according to Newton's Second Law of motion $a = F/m$ ——(2)

Comparing equation (1) and equation (2)

$$\frac{F}{m} = \frac{v_f - v_i}{t}$$

$$F \times t = m(v_f - v_i)$$

$$F \times t = mv_f - mv_i$$

$$F \times t = P_f - P_i$$

$$F \times t = \Delta P \quad \text{———} (3)$$

Equation (3) indicates that the product of force and time is equal to the change in momentum. From equation (3) we have

$$F = \frac{\Delta P}{t}$$

which is according to the statement of the law. Hence proved.

-: IMPULSE :-

Def:- When a very large force acts on an object for a very short interval of time. The product of force and short interval of time is called impulse.

It is denoted by I

Let a force $\vec{F}$ acts on an object for time t then impulse I is given as

$$\vec{I} = \vec{F} \times t$$

Impulse is a vector quantity and its units are $kg\ m\ sec^{-1}$ or N-sec.

-: EXAMPLE :- A sixer hit by a player.

EXAMPLE 3.2

A 1500kg car has its velocity reduced from 20m/sec to 15m/sec in 3.0 s. How large was the average retarding force?

-: SOLUTION :-

According to Newton's second law of motion in terms of momentum

$$F \times t = \Delta P = P_f - P_i$$

$$\Rightarrow F \times t = mv_f - mv_i$$

$$F = \frac{mv_f - mv_i}{t}$$

$$F = \frac{1500 \times 15 - 1500 \times 20}{3}$$
$$= 7500 - 10000 = -2500\ N$$

-: ISOLATED SYSTEM:-

Def:- "A system in which the molecules collide with one another without the force of any external agency is called isolated system:" e.g. a gas enclosed inside a glass container forms an isolated system.

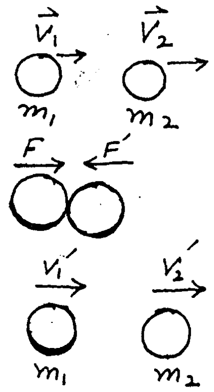
-: LAW OF CONSERVATION OF MOMENTUM:-

Statement:- This law states that ☆
"The total linear momentum of an isolated system remains constant"

-: Explanation:-

Let us consider an isolated system of two smooth hard balls of masses m_1 and m_2 moving with velocities V_1 and V_2 respectively, along a same straight line in same direction.

If $V_1 > V_2$, a collision takes place between the two ball. Let after collision the velocities of the balls of masses m_1 and m_2 becomes V_1' and V_2' respectively. During collision if mass m_1 exerts a force F then the change in momentum of mass m_1



according to Newton's Second Law of motion in terms of linear momentum is given as

$$F \times t = \Delta P = P_f - P_i$$

$$F \times t = m_1 v_1' - m_1 v_1 \quad \text{——} (1)$$

Similarly for mass m_2

$$F' \times t = m_2 v_2' - m_2 v_2 \quad \text{——} (2)$$

Adding equation (1) and equation (2)

$$F \times t + F' \times t = m_1 v_1' - m_1 v_1 + m_2 v_2' - m_2 v_2$$

$$(F + F')t = m_1 v_1' + m_2 v_2' - m_1 v_1 - m_2 v_2 \quad \text{——} (3)$$

As F and F' are action and reaction forces having equal magnitude but oppositly directed therefore the left side of eq (3) will be equal to zero.

Hence $$0 = m_1 v_1' + m_2 v_2' - m_1 v_1 - m_2 v_2$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

which Conclude that the initial momentum of system of balls before collision is equal to their final momentum after collision. or the total linear momentum of the system remains Conserved before and after collision which is same as mentioned in the statement.

EXAMPLE 3.3

Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6 m/sec and 4 m/sec respectively. what must be the velocity of smaller mass after collision, if the velocity of the bigger ball is 3 m/sec.

-: **SOLUTION** :- As the balls are moving in opposite direction. So their velocities will have opposite sign. Let the velocity of 2kg mass is +ve and that of 3kg mass is negative So their momentum before collision is

$$P_1 = m_1 v_1 - m_2 v_2$$
$$= 2 \times 6 - 3 \times 4 = 0$$

Momentum after collision is

$$P_2 = m_1 v_1' - m_2 v_2'$$
$$= 2 \times v_1' - 3 \times 3$$
$$= 2 v_1' - 9$$

According to the law

$$P_1 = P_2$$
$$0 = 2v_1' - 9 \Rightarrow 2v_1' = 9$$
or $$v_1' = 4.5 \, m/s$$

-: COLLISION :-

Def:- When two objects approach each other in such a way that some sort of interaction exists between them with or without the presence of external force then collision is said to be taken place between them.

There are two main types of collision

(i) Elastic collision

(ii) Inelastic collision

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(i) -: ELASTIC COLLISION:-

Def:- A collision in which there is no loss of kinetic energy is called a perfectly elastic collision

OR

A Collision in which the linear momentum and kinetic energy of the system remain conserved is called elastic collision

-: INELASTIC COLLISION:-

Def:- A collision in which the K.E. of the system is not conserved is called the inelastic collision

The K.E. in such system is either lost in friction or converted in to heat or sound energies.

ELASTIC COLLISION IN ONE DIMENSION

Consider two non rotating, smooth spherical balls of masses m_1 and m_2 moving with velocities v_1 and v_2 respectively along a straight line in same direction. If $v_1 > v_2$ then the incident ball (m_1) collide with the target body (m_2) and their velocities become v_1' and v_2' after collision respectively. Since collision is elastic one so by using the law of conservation of momentum

Total momentum before collision =

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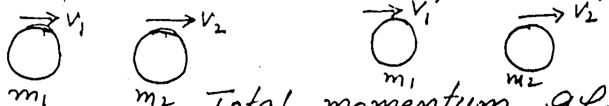
Before collision After collision



Total momentum after collision

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{——(A)}$$

$$m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \quad \text{——(1)}$$

Also as the K.E is also Conserved So

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2$$

$$m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2)$$

$$m_1 (v_1 + v_1')(v_1 - v_1') = m_2 (v_2' + v_2)(v_2' - v_2) \quad \text{—(2)}$$

dividing equation (2) by equation (1)

$$\frac{\cancel{m_1} (v_1 + v_1') \cancel{(v_1 - v_1')}}{\cancel{m_1} \cancel{(v_1 - v_1')}} = \frac{\cancel{m_2} (v_2' + v_2) \cancel{(v_2' - v_2)}}{\cancel{m_2} \cancel{(v_2' - v_2)}}$$

$$v_1 + v_1' = v_2' + v_2$$

$$v_1 - v_2 = v_2' - v_1' \quad \text{—— (3)}$$

where $v_1 - v_2$ is the relative velocity of m_1 with respect to m_2 and is Called the speed of approach. while $v_2' - v_1'$ is the relative velocity of m_2 with respect to m_1 and is Called the speed of separation Both these velocities have the same magnitudes but oppositly directed.

In order to determine the values of v_1' and v_2' after collision, Consider equation (3) i.e $v_1 - v_2 = v_2' - v_1'$

$$v_2' = v_1 + v_1' - v_2 \quad \text{—— (4)}$$

using the value of v_2' in equation (A)

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 (v_1 + v_1' - v_2)$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_1 + m_2 v_1' - m_2 v_2$$

$$m_1 v_1 - m_2 v_1 + m_2 v_2 + m_2 v_2 = m_1 v_1' + m_2 v_1'$$

$$(m_1 - m_2) v_1 + 2 m_2 v_2 = v_1' (m_1 + m_2)$$

$$\Rightarrow v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \frac{2 m_2 v_2}{m_1 + m_2} \quad \text{---} (5)$$

From equation 3 we have

$$v_1 - v_2 = v_2' - v_1'$$

$$v_1' = v_2' - v_1 + v_2 \quad \text{---} (6)$$

using the value of v_1' in equation 4

$$m_1 v_1 + m_2 v_2 = m_1 (v_2' - v_1 + v_2) + m_2 v_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_2' - m_1 v_1 + m_1 v_2 + m_2 v_2'$$

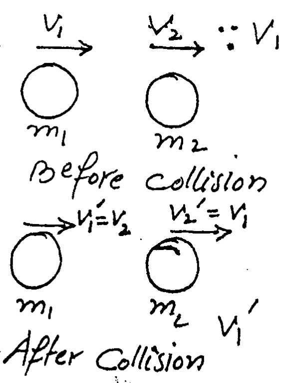
$$m_1 v_1 + m_1 v_1 + m_2 v_2 - m_1 v_2 = m_1 v_2' + m_2 v_2'$$

$$2 m_1 v_1 + (m_2 - m_1) v_2 = v_2' (m_1 + m_2)$$

$$\Rightarrow v_2' = \frac{2 m_1 v_1}{m_1 + m_2} + \frac{(m_2 - m_1) v_2}{(m_1 + m_2)} \quad \text{---} (7)$$

Equation 5 and equation 7 give the values of v_1' and v_2' respectively after collision. Now we discuss the following cases.

CASE I $\quad m_1 = m_2 = m$



$$\because v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \frac{2 m_2 v_2}{m_1 + m_2}$$

$$= \left(\frac{\cancel{m} - \cancel{m}}{m + m}\right) v_1 + \frac{2m \; v_2}{m + m}$$

$$= \frac{0}{2m} \times v_1 + \frac{2m}{2m} v_2$$

$$v_1' = 0 + v_2 \Rightarrow \boxed{v_1' = v_2} \quad \text{---} (8)$$

Also $$V_2' = \frac{2m_1 V_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right) V_2$$

$$\Rightarrow V_2' = \frac{2m V_1}{m+m} + \left(\frac{\cancel{m}-\cancel{m}}{m+m}\right) V_2$$

$$= \frac{2m}{2m} V_1 + \frac{0}{2m} \times V_2$$

$$\boxed{V_2' = V_1} \quad (9)$$

From equations 8 and equation 9 it is evident that both the objects are exchanging their velocities after collision.

CASE II $\quad m_1 = m_2 = m$

$V_2 = 0$ **Target is at rest**

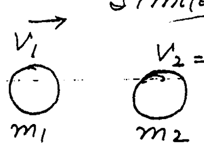
$$\because V_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right) V_1 + \frac{2m_2 V_2}{m_1+m_2}$$

$$V_1' = \left(\frac{\cancel{m}-\cancel{m}}{m+m}\right) V_1 + \frac{2m}{m+m} \times 0$$

$$V_1' = \frac{0}{2m} V_1 + \frac{2m}{2m} \times 0 = 0$$

$$\Rightarrow \boxed{V_1' = 0} \quad (10)$$

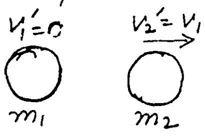
Similarly



Before Collision

$$V_2' = \frac{2m_1 V_1}{m_1+m_2} + \left(\frac{m_2-m_1}{m_1+m_2}\right) V_2$$

$$= \frac{\cancel{2m} V_1}{\cancel{2m}} + \frac{m-m}{m+m} \times 0$$



After collision

$$V_2' = V_1 + 0$$

$$\boxed{V_2' = V_1} \quad (11)$$

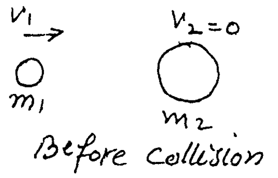
Equations 10 and equation 11 shows that

For equal masses the incident body m_1 after collision will come to rest and m_2 moves with a velocity equal to that of m_1. Hence both the balls are exchanging their velocities after collision.

CASE III When a light body collides with a massive ball at rest.

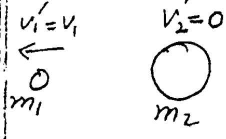
According to the condition

$m_1 \simeq 0$ i.e. $m_1 <<< m_2$, $V_2 = 0$ m_1 can be neglected



$$\because \quad V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1 + \frac{2 m_2 V_2}{m_1 + m_2}$$

$$= \left(\frac{0 - m_2}{0 + m_2}\right) V_1 + \frac{2 m_2 \times 0}{0 + m_2}$$



$$V_1' = \frac{-\cancel{m_2}}{\cancel{m_2}} \times V_1 + 0$$

$$\boxed{V_1' = -V_1} \quad (12)$$

Also

$$V_2' = \frac{2 m_1 V_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2} V_2$$

$$= \frac{2 \times 0 \times V_1}{0 + m_2} + \frac{m_2 - 0}{0 + m_2} \times 0$$

$$= 0 + \frac{\cancel{m_2}}{\cancel{m_2}} \times 0$$

$$\Rightarrow \boxed{V_2' = 0} \quad (13)$$

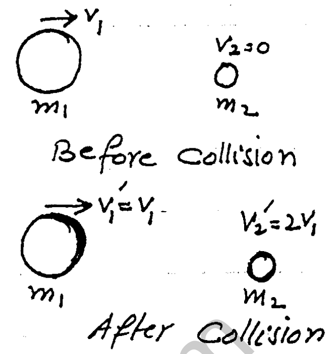
Equation 12 and 13 shows that the mass m_1 will bounce back with same velocity while m_2 remain stationary

CASE IV When massive body collides with Light stationary body

As $m_1 >>> m_2$ i.e $m_2 \simeq 0$

m_2 Can be neglected, $V_2 = 0$

So



$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1 + \frac{2m_2 V_2}{m_1 + m_2}$$

$$V_1' = \left(\frac{m_1 - 0}{m_1 + 0}\right) V_1 + \frac{2 \times 0 \times 0}{m_1 + 0}$$

$$= \frac{\cancel{m_1}}{\cancel{m_1}} V_1 + 0 \Rightarrow \boxed{V_1' = V_1} \quad (14)$$

Also
$$V_2' = \frac{2m_1 V_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) V_2$$

$$= \frac{2 \times m_1 V_1}{m_1 + 0} + \frac{0 - m_1}{m_1 + 0} \times 0$$

$$V_2' = \frac{2 \cancel{m_1} V_1}{\cancel{m_1}} + 0 \Rightarrow \boxed{V_2' = 2V_1} \quad (15)$$

The above equations indicates that there will be no change in the velocity of incident body after collision but the target body will move with a velocity twice that of incident body:-

EXAMPLE 3.4

A 70 g ball collides with an other ball of mass 140 g. The initial velocity of the first ball is 9 m sec^{-1} to the right while the second body is at rest. If the collision were perfectly elastic. what would be the velocity of two balls after collision.

-: Solution:- $m_1 = 70g$ $v_1 = 9\ m/sec$ $v_1' = ?$

$m_2 = 140g$ $v_2 = 0$ $v_2' = ?$

$$\because \quad v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

Also $$v_2' = \frac{2m_1\ v_1}{m_1 + m_2} + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

As $v_2 = 0$

$$\therefore \quad v_1' = \frac{m_1 - m_2}{m_1 + m_2} \times v_1$$

$$= \frac{70 - 140}{70 + 140} \times 9$$

$$= \frac{-7\cancel{0}}{21\cancel{0}} \times 9 = -\frac{63}{21} = -3\ m/s$$

Also $$v_2' = \frac{2\ m_1}{m_1 + m_2} v_1 \quad (\because v_2 = 0)$$

$$= \frac{2 \times 70}{70 + 140} \times 9 = \frac{\cancel{14\cancel{0}}^{2} \times \cancel{9}^{3}}{\cancel{21\cancel{0}}_{\cancel{3}_{1}}}$$

$$\Rightarrow \boxed{v_2' = 6\ m\,sec^{-1}}, \quad \boxed{v_1' = -3\ m\,sec^{-1}}$$

EXAMPLE 3.5

A 100g golf ball is moving to the right with a velocity of $20\ m\,sec^{-1}$. It makes a head on Collision with an 8 kg steel ball, initially at rest. Compute the Velocities of the balls after Collision

-: SOLUTION :-

$m_1 = 100g = 0.1\ kg$ $v_1 = 20\ m/sec$

$m_2 = 8kg$, $v_2 = 0$ $v_1' = ?$, $v_2' = ?$

when the target body is at rest then

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} \times v_1$$

$$= \frac{0.1 - 8}{0.1 + 8} \times 20$$

$$\boxed{v_1' = -19.5 \text{ m/sec}}$$

Also $$v_2' = \frac{2m_1}{m_1 + m_2} \times v \qquad (\because v_2 = 0)$$

$$= \frac{2 \times 0.1}{0.1 + 8} \times 20$$

$$\boxed{v_2' = 0.5 \text{ m/sec}}$$

FORCE DUE TO WATER FLOW

When water from a horizontal pipe strikes on the wall normally, it exerts a force on the wall. Let m be the mass of water strikes on the wall and its velocity changes from $v_i = v$ to $v_f = 0$ in time t then the force exerted by the wall on water according to the second law of motion in terms of momentum is equal to the rate of change of momentum of water and is given as

$$F = \frac{\Delta P}{t} = \frac{m(v_f - v_i)}{t} = \frac{m(0 - v)}{t}$$

$$F = -\frac{mv}{t} = -\frac{m}{t} \times v$$

The reactional force exerted by the water on the wall is equal in magnitude but oppositly directed. So

$$F = -\left(-\frac{mv}{t}\right) = \frac{mv}{t}$$

$$F = \frac{m}{t} \times v$$

Hence the force exerted by the water is equal to the product of mass of water striking in unit time to the change in its velocity.

For example the water flows out from a pipe at $3\,kg\,s^{-1}$ and its velocity changes from $5\,m\,s^{-1}$ to zero on striking the wall Then

$$F = \frac{m}{t} \times (v_f - v_i) = \frac{3}{1} \times (5-0) = 15N$$

EXAMPLE #3.6

A hose pipe ejects water at a speed of $0.3\,m\,s^{-1}$ through a hole of area $50\,cm^2$. If the water strikes the wall normally. Calculate the force on the wall, assuming the velocity of the water normal to the wall is zero on striking.

-: SOLUTION :- $v_i = 0.3\,m\,s^{-1}$, $v_f = 0$,

$A = 50\,cm^2 = 5\times10^{-3}\,m^2$, $m = ?$, $d = 1000\,kg\,m^{-3}$ $F = ?$

The flow of water ($v_i = 0.3\,m\,s^{-1}$) indicates that it covers a distance of $0.3\,m$ in 1 second

So $d = l = 0.3\,m$, $t = 1\,sec.$

From the equation of volume

$$V = \text{Area} \times \text{distance}$$
$$= A \times l = 5\times10^{-3} \times 0.3$$
$$= 1.5\times10^{-3}\,m^3$$

$\because \quad d = \frac{m}{V} \Rightarrow m = d \times V$

$m = 10^3 \times 1.5 \times 10^{-3} = 1.5\ Kg$

So the force exerted by the water on the wall is given as

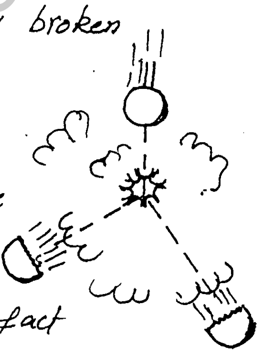
$$F = \frac{m}{t} \times (\Delta V)$$

$$= \frac{1.5}{1} \times (0.3 - 0)$$

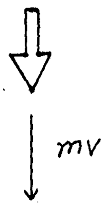
$$= 0.45 N$$

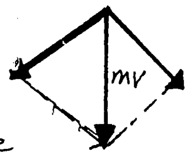
MOMENTUM AND EXPLOSIVE FORCES:-

In order to explain the change in momentum due to explosive forces, let a shell is exploded in the mid air. As a result it fragments are scattered in different directions. Let it is broken in to two fragments then the momentum of these fragments by vector addition equals to the initial momentum of the shell.



To verify the above fact farther, Consider the example of firing of bullet from a rifle. Let a bullet of mass m is fired with a velocity v from a rifle of mass M. The rifle recoils back with a velocity V. Initially as the rifle and the bullet are at rest. Hence their initial momentum is zero. As no external force acts on this system therefore the final





momentum of this isolated system should be equal to zero. Hence

$$0 = MV + mv$$

$$MV = -mv \qquad \text{——— (d)}$$

$$V = \frac{-mv}{M}$$

The equation(d) indicates that the momentun of rifle is equal and opposite to that of bullet. As mass of the rifle is greater than bullet. So it recoils back with only a fraction of velocity of the bullet.

-: ROCKET PROPULSION :-

Rockets propulsion is the familiar example of Newton's third law of motion and occurs due to the ejection of hot gases through engine at their rear. These gases are expelled as a result of burning of liquid oxygen and liquid hydrogen in its Combustion chamber. These are accelerated upward after gaining the momentum equal to that of hot ejected gases due to the burning of fuel Continuously. The liquid and solid fuel is very effective at greater altitudes where the air is either less or not present. In order to provide enough thrust to overcome gravity, a rocket Consumes about 10,000 kg/sec of fuel and eject the burning gases at a speed of 4000m/sec. That is why 80% of launching rocket Consists of fuel only. Multistage rockets are designed to overcome the problem ~~of mass of fuel.~~

If m be the mass of the gas ejected per second with velocity v relative to the rocket, the change in momentum of the ejected gases is mv which is equal to the thrust of rocket, the acc. of the rocket is given by

$$a = \frac{mv}{M}$$

where M is the mass of rocket. The equation indicates that accelerated motion of rocket is due to reduction in its mass after burning fuel.

-: PROJECTILE MOTION :-

-: **Def** :- The motion of an object confined in a plane under the effect of force of gravity when it is launched at a certain angle with horizontal is called projectile motion.

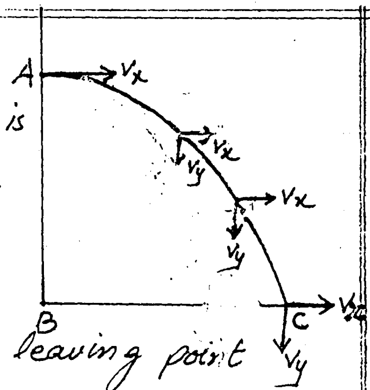
-: PROJECTILE :-

Def :- An object thrown in the upward direction at a certain angle α and moves under the effect of force of gravity without experiencing any air friction is called projectile.

EXAMPLES :-

(i) Bullet fired from a gun
(ii) Foot ball kicked by a player.
(iii) A bomb dropped by a plane.

EXPLANATION:-



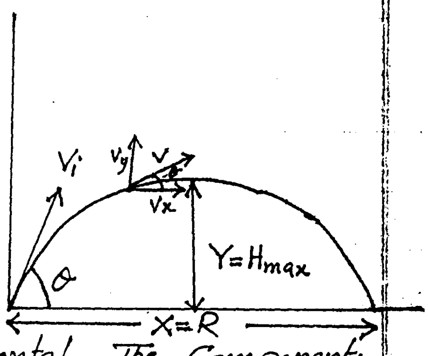
In order to describe the projectile motion, let a ball is thrown horizontally at point A. The velocity of the ball at point A is totaly horizontal. It is observed that after leaving point A, it tends to move farward as well as fall down ward. The x component of its velocity V_x remains constant in absence of air friction or any other external horizontal force ($a_x = 0$). So the horizontal distance travelled by the projectile (ball) is

$$X = V_x \times t \qquad (\because S = vt)$$

The vertical motion of the ball is under the effect of gravity. For downward motion $a_y = g$. Hence the vertical displacement is given as

$$Y = V_{iy}t + \frac{1}{2}a_y t^2$$

$$Y = \frac{1}{2}gt^2 \qquad (\because V_{iy} = 0)$$



Now let us Consider the Case when the ball is thrown up ward with an initial velocity V_i by making a certain angle θ with the horizontal. The Components of its initial velocity are given as

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-: HEIGHT OF PROJECTILE :-

Def:- Maximum vertical distance attained by a projectile during its motion is called maximum height or height of projectile.

At maximum height, the vertical component of velocity vanishes i.e. $v_{fy} = 0$

Also $a_y = -g$, $v_{iy} = v_i \sin\alpha$

By using the third equation of motion

$$2 a_y Y = v_{fy}^2 - v_{iy}^2$$

$$2(-g) H = 0 - v_i^2 \sin^2\alpha$$

$$-2gH = -v_i^2 \sin^2\alpha$$

$$H = \frac{v_i^2 \sin^2\alpha}{2g}$$

-: TIME OF FLIGHT :-

Def:- The time taken by the body to cover a distance from the point of projection to a point where it hits the ground is called time of flight and is denoted by T:-

As the net vertical distance covered by the body from the point of projection to a point at the level of projection is equal to zero

$$\therefore \quad Y = 0$$

Also $a_Y = -g$, $v_{iy} = v_i \sin\alpha$

$$t = T$$

By using the 2nd equation along vertical direction

$$Y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = v_i \sin\alpha \, T + \frac{1}{2}(-g) T^2$$

$$\frac{1}{2} g T^2 = v_i \sin\alpha \, T$$

$$\frac{gT}{2} = v_i \sin\alpha$$

$$T = \frac{2 v_i \sin\alpha}{g}$$

-: RANGE OF PROJECTILE :-

Def:- Horizontal distance traveled by an object during it flight is called range of projectile and is denoted by R

Range is determined by multiplying the horizontal component of the velocity V_x of projection with time of flight T i.e.

$$R = V_x \times T$$

$$= v_i \cos\alpha \times \frac{2 v_i \sin\alpha}{g}$$

$$= \frac{v_i^2}{g} 2\sin\alpha\cos\alpha$$

$$R = \frac{v_i^2}{g} \sin 2\alpha$$

If v_i and g are constant then R depends on $\sin 2\alpha$. R will be maximum when $\sin 2\alpha$ will be max i.e.

$$\sin 2\alpha = 1$$

but $\sin 90 = 1$

$$\therefore \sin 2\alpha = \sin 90$$

$$\Rightarrow \quad 2\theta = 90^\circ$$

$$\text{or} \quad \theta = 45^\circ$$

So for maximum range the particle should be projected at an angle of 45° with the horizontal or x-axis.

APPLICATION TO BALLISTIC MISSILES

BALLISTIC MISSILE

Def:- Ballistic missile is basically an unpowered and unguided missile that moves due to its inertia and under the effect of gravity when it is pushed slightly.

* The path followed by it is called missile trajectory.

When a ballistic missile is projected in to a space, then it should move with uniform velocity along the direction of its launch due to inertia but the downward force of gravity alters its straight path in to a curved trajectory. The path followed by a missile is parabola for short range and flat earth and should be elliptical for spherical earth. For long range air friction is more effective and sometimes greater than the force of gravity at higher speed. Hence we cannot neglect the aerodynamic forces.

when a missile is fired at a spot, it undergoes a complex path due to air friction or dragging wind. The angle of projection can not be measured precisely at the moment of launching. The latest equation of trajectory can be implemented only to short range ballistic missiles. For greater precision and long range, only powered and remote. Control guided missiles are used.

EXAMPLE # 3.7

A ball is thrown with a speed of $30 m\,sec^{-1}$ in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

-: SOLUTION :-

$$V_i = 30\,m/sec$$

$$\theta = 30^\circ \quad , \quad g = 9.8\,m\,sec^{-2}$$

$$H = ? \quad , \quad T = ? \quad , \quad R = ?$$

$$\because \quad H = \frac{V_i^2 \sin^2\theta}{2g} = \frac{(30)^2 \times (\sin 30)^2}{2 \times 9.8}$$

$$H = \frac{900 \times 0.25}{19.6} = 11.5\,m$$

$$\text{Also} \quad T = \frac{2V_i \sin\theta}{g} = \frac{2 \times 30 \times \sin 30}{9.8}$$

$$\boxed{T = 3.1\,S}$$

The equation of range is

$$R = \frac{V_i^2}{g}\sin 2\theta = \frac{(30)^2}{9.8} \times \sin 2 \times 30$$

$$= \frac{900 \times 0.866}{9.8} = 79.5\,m \text{ Ans}$$

EXAMPLE # 3.8

In example 3.7 Calculate the max. range and height reached by the ball if the angles of projections are (i) 45° (ii) 60°

-: SOLUTION :-

(i) As we know that

$$H = \frac{v_i^2 \sin^2\alpha}{2g}$$

Here $\alpha = 45°$

$$\therefore \quad H = \frac{(30)^2(\sin 45)^2}{2\times 9.8} = \frac{900\times(.707)^2}{19.6}$$

$$H = 22.9 m.$$

As $$R = \frac{v_i^2}{g}\sin 2\alpha$$

$$= \frac{(30)^2}{9.8}\times \sin 2\times 45$$

$$= \frac{900}{9.8}\times 1$$

$$= 91.8 m$$

(ii) For this part $\alpha = 60°$

Hence

Height $$H = \frac{v_i^2 \sin^2\alpha}{2g} = \frac{(30)^2\times(\sin 60)^2}{2\times 9.8}$$

$$= \frac{900\times(0.866)^2}{19.6}$$

$$= 34.4 m$$

Range $$R = \frac{v_i^2}{g}\sin 2\alpha$$

$$= \frac{(30)^2}{9.8}\times \sin 2\times 60°$$

$$= \frac{900}{9.8}\times 0.866$$

$$R = 79.5 m$$