NUMERICAL PROBLEMS

P.4.1 A man pushes a lawn mover with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

DATA. $F = 40\ N$

$\theta = 20^\circ$

$d = 20\ m$

Work done = W = ?

Sol: using ; $W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$W = 40\ N \times 20\ m \times \cos 20^\circ$

$= \boxed{7.5 \times 10^2\ J}$ $\because \cos 20^\circ = 0.94$ & $1\ N\times m = 1\ J$

P.4.2 A rain drop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100m, how much work is done by (a) gravity and (b) friction.

DATA. $m = 3.35 \times 10^{-5}$ kg

Height = h = 100 m

(a) Work done due to gravity = ?

(b) Work done due to friction = ?

Sol: we have

Work done due to gravity = W = mgh

$W = 100\ m \times 9.8\ m\,s^{-2} \times 3.35 \times 10^{-5}$ kg

$= \boxed{0.0328\ J}$

Work done due to both gravity and friction is the same. Here, the frictional force is acting on the rain drop against the gravitational force. So

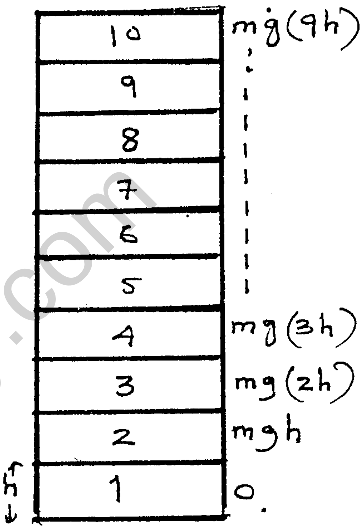
Work done by friction = $\boxed{-.0328\ J}$

P.4.3 Ten bricks, each 6 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?

DATA. Mass of each brick = 1.5 kg = m

Thickness of brick = h = 6 cm = 0.06 m

Work required to stack them one on the top of another = W = ?



Sol: There is no work done for the 1st brick. We have to put bricks on it one by one. Therefore, every brick gets some P.E w.r.t its height. so

$$W = 0 + mgh + 2mgh + 3mgh + 4mgh + 5mgh + 6mgh + 7mgh + 8mgh + 9mgh$$

$$W = 45mgh \quad \text{——— (1)}$$

Putting values, we have

$$W = 45 \times 1.5\,kg \times 9.8\,m\,s^{-2} \times 0.06\,m$$

$$= 39.69\,J \simeq \boxed{40\,J}$$

P.4.4 An object of mass 6 kg is travelling at a velocity of $5\,m\,s^{-1}$. What is its K.E? What will be its K.E if its velocity is double?

DATA. Mass of the object = m = 6 kg

velocity of " = V = $5\,m\,s^{-1}$

(a) K.E = ? (b) K.E = ? (When its velocity is double)

Sol: (a) We know that

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 6\,kg \times (5\,m\,s^{-1})^2$$

$$= 75\,kg\,m^2 s^{-2}$$

$$K.E = \boxed{75\,J}$$

$\because kg\,m\,s^{-2} = N$ & $N \times m = J$

(b) Now $V = 10\,m\,s^{-1}$, therefore

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 6\,kg \times (10\,m\,s^{-1})^2$$

$$= 300\,J = \boxed{3.0 \times 10^2\,J}$$

P.4.5 An electron strikes the screen of a cathode-ray tube with a velocity of $1.0\times10^{7}\,ms^{-1}$. Calculate its K.E. The mass of an electron is 9.1×10^{-31} kg.

DATA. Velocity of electron $= V = 1.0\times10^{7}\,ms^{-1}$

Mass of electron $= m = 9.1\times10^{-31}$ kg

K.E = ?

Sol: As we know that;

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\times 9.1\times10^{-31}\,kg\times(1.0\times10^{7}\,ms^{-1})^2$$

$$= 4.6\times10^{-17}\,kg\,m^2s^{-2}$$

$$K.E = \boxed{4.6\times10^{-17}\,J} \qquad (\because 1\,kg\,m^2s^{-2} = 1\,J)$$

P.4.6 If 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3} find
(a) the increase in P.E (b) the power delivered by the pump.

DATA. Volume of water $= V = 100\,m^3$

Height $= h = 10$ m

Time taken $= t = 20$ min $= 20\times60\,S = 1200\,S$

Density of water $= \rho = 1000\,kg\,m^{-3}$.

(a) Increase in P.E = ?

(b) Power delivered by the Pump = P = ?

Sol. (a) As Density$(\rho) = \dfrac{\text{Mass}(m)}{\text{Volume}(V)}$

$$\therefore m = V\times\rho \quad \text{———} (1)$$

$$= 100\,m^3\times1000\,kg\,m^{-3}$$

$$m = 10^5\,kg \quad \text{———} (2)$$

Also

$$\text{Work done} = P.E = mgh$$

$$= 10^5\,kg\times9.8\,ms^{-2}\times10\,m$$

$$\text{Increase in P.E} = \boxed{9.8\times10^6\,J} \quad \text{———} (3)$$

(b) Now Power$(P) = \dfrac{\text{Work}(W)}{\text{time}(t)}$

$$\therefore P = \frac{9.8\times10^6\,J}{1200\,S} = 8166.6\text{ watt}$$

$$P = 8.167\times10^3\text{ watt} \simeq \boxed{8.2\,KW}$$

P.4.7 A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at $80\ km\ h^{-1}$. What power (KW) must the engine develop?

DATA. $F = 400\ N$

Velocity $= v = 80\ km\ h^{-1} = \frac{80 \times 1000}{60 \times 60} = 22.22\ m\ s^{-1}$

Power $= P$ (in KW) $= ?$

Sol: As $P = \vec{F} \cdot \vec{v} = Fv\cos 0° = Fv$ — (1) ($\because$ Force & vel are along the same direction)

$P = 400\ N \times 22.22\ m\ s^{-1}$

$P = 8888$ watt

$= 8.9 \times 10^{3}$ watt $=$ **8.9 KW**

P.4.8 How large a force is required to accelerate an electron ($m = 9.1 \times 10^{-31}\ kg$) from rest to a speed of $2 \times 10^{7}\ m\ s^{-1}$ through a distance of 5 cm?

DATA. Mass of electron $= m = 9.1 \times 10^{-31}\ kg$

Initial velocity $= v_i = 0$

Final " $= v_f = 2 \times 10^{7}\ m\ s^{-1}$

Distance $= d = 5\ cm = 0.05\ m$

Force required $= F = ?$

Sol: Using work-energy principle;

$$F \times d = \frac{1}{2} \times m v_f^2 - \frac{1}{2} m v_i^2$$

Putting the values, we have;

$$F \times 0.05\ m = \frac{1}{2} \times 9.1 \times 10^{-31}\ kg \times (2 \times 10^{7}\ m\ s^{-1})^2 - 0$$

$$F = 3.6 \times 10^{-15}\ kg\ m^2\ s^{-2} \times m^{-1}$$

$$\therefore\ F = \boxed{3.6 \times 10^{-15}\ N}$$

P.4.9 A diver weighing 750 N drops from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5 m above the water surface, neglecting air friction?

DATA. Weight of diver = W = 750 N

$h_1 = 10m$

$h_2 = 5m$

Speed of diver = v = ?

Sol. As we know;

Loss of P.E = Gain in K.E

$mg(h_1 - h_2) = \frac{1}{2} \times mv^2$

$v^2 = 2g(h_1 - h_2)$

$v = \sqrt{2g(h_1 - h_2)}$ ——— (1)

Putting values, we have;

$v = \sqrt{2 \times 9.8\ ms^{-2} \times (10m - 5m)}$

$v = \boxed{9.9\ ms^{-1}}$ ——— (2)

P.4.10 A child starts from rest at the top of a slide of height 4 m (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of $6\ ms^{-1}$, what percentage of his total energy at the top of the slide is lost as a result of friction?

DATA. Height of slide = h = 4m

(a) speed at the bottom = v = ?

(b) %age of total energy lost = ? (if $v' = 6\ ms^{-1}$)

Sol: (a) As Loss of P.E = Gain in K.E

$mgh = \frac{1}{2} \times mv^2$

$v = \sqrt{2gh} = \sqrt{2 \times 9.8\ ms^{-2} \times 4m}$

$v = \boxed{8.8\ ms^{-1}}$ ——— (1)

(b) Loss of energy = K.E − K.E′

$= \frac{1}{2} \times mv^2 - \frac{1}{2} \times mv'^2$

$= \frac{1}{2} \times m \times (8.8)^2 - \frac{1}{2} \times m\,(6)^2$

$= (38.72\,m - 18\,m)\ J$ ——— (2)

$= (20.72\,m)\ J$ ——— (3)

% Loss of energy $= \frac{20.72\,m}{38.72\,m} \times \frac{100}{100}$

$= \boxed{54\%}$