NUMERICAL PROBLEMS:

The solutions to the problems are given below:

P. 1.1:- * one light year is the distance light travels in one year. $v = c = 3 \times 10^8$ m/s

$s = ?$ we know that: $s = vt = ct$

$\therefore \quad 1 L.Y. = 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 \times \frac{m}{\cancel{s}} \times \cancel{s}$

$\therefore \quad 1 L.Y. = 9.46 \times 10^{15}\, m = 9.5 \times 10^{15}\, m$

P. 1.2:- (a) $1\,year = 1 \times 1\,year$

$= 1 \times 365$ days

$= 365 \times 1\,day$

$= 365 \times 24\,hr$

$= 8760 \times 1\,hr$

$= 8760 \times 60 \times 60\,s$

$\therefore 1\,year = 3.1536 \times 10^7\,s$

(b) As $1\,year = 3.1536 \times 10^7\,s$

$= 3.1536 \times 10^7 \times 1s$

$= 3.1536 \times 10^7 \times 10^9\,ns$ $(\because 1s = 10^9 ns)$

$= 3.1536 \times 10^{16}\,ns$.

(c) $1s = 1 \times 1s$

$= 1 \times \frac{1}{60}\,min$

$= \frac{1}{60} \times 1\,min = \frac{1}{60} \times \frac{1}{60}\,hr$

$= \frac{1}{3600} \times 1\,hr = \frac{1}{3600} \times \frac{1}{24}\,day$

$= \frac{1}{86400} \times 1\,day = \frac{1}{86400} \times \frac{1}{365}\,years$

$= 3.17 \times 10^{-8}\,years$

$\therefore 1s = 3.17 \times 10^{-8}\,years.$

P. 1.3 :- Length of plate $= L = 15.3$ cm

width " " $= W = 12.80$

Area $= A = ?$

$\because$ Area = Length $\times$ width

$A = L \times W$

$= 15.3 \times 12.80 = 195.84\ cm^2$

on rounding off upto three digits : $A = 196\ cm^2$

P. 1.4 :- The given masses are :

Let: $m_1 = 2.189$ kg, $m_2 = 0.089$ kg, $m_3 = 11.8$ kg

$m_4 = 5.32$ kg.

Total mass : $m = m_1 + m_2 + m_3 + m_4$

$= 2.189 + 0.089 + 11.8 + 5.32 = 19.398$ kg

Applying precision rule : $m = 19.4$ kg.

P. 1.5 :- Given formula :- $T = 2\pi\sqrt{\frac{l}{g}}$

As: Length $= l = 100$ cm $= 1$ m

Time for 20 vib. $= 40.2$ s

Time period $= T = \frac{40.2}{20} = 2.01$ s

or

Squaring both sides.

$T^2 = 4\pi^2 \frac{l}{g}$

$g = 4\pi^2 \frac{l}{T^2}$

$= 4 \times (3.14)^2 \times \frac{(1)^2}{(2.01)^2}$

$\therefore g = 9.76\ m/s^2$.

L.C. (meter rod) $= 1$ mm $= 0.001$ m

L.C (Stop watch) $= 0.1$ S.

Absolute error in 'l' $= 0.001$ cm

%age " " " $= \frac{0.001}{1} \times \frac{100}{100} = 0.1\%$

Uncertainty in time $= \frac{\text{Least count}}{\text{No. of vib.}} = \frac{0.1}{20}$ s $= 0.005$ s

%age uncertainty $= \frac{0.005}{2.01} \times \frac{100}{100} = 0.25\%$

Total uncertainty in 'g' $= 1(0.1\%) + 2(0.25\%) = 0.6\%$

$\Rightarrow g = 9.76\ m/s^2$ with 0.6% error

or $g = 9.76 \pm 0.06^*\ m/s^2$

*(0.6% of 'g' $= \frac{0.6}{100} \times 9.76 = 0.06\ m/s^2$)

P.1.6 :- Dimensions and units of 'G' = ?

Given formula : $F = G \frac{m_1 m_2}{r^2}$

So : $G = \frac{F r^2}{m_1 m_2}$

$$\therefore [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[m][a][r^2]}{[m_1][m_2]}$$

$$= \frac{[M][LT^{-2}][L^2]}{[M^2]} = [M]^{1-2}[L]^3[T]^{-2}$$

$\therefore$ Dim. of $G = [G] = [M^{-1}L^3T^{-2}]$.

Units : $G = \frac{F r^2}{m_1 m_2} = \frac{N m^2}{Kg^2} = N m^2 Kg^{-2}$

$\therefore$ units of 'G' are : $N m^2 Kg^{-2}$.

P.1.7 :- The given eq. is : $v_f = v_i + at$

Dim. on LHS. $= [v_f] = [LT^{-1}]$

Dim. on RHS. $= [v_i] + [a][t]$

$= [LT^{-1}] + [LT^{-2}][T]$

$= [LT^{-1}] + [LT^{-1}]$

$= 2[LT^{-1}] = [LT^{-1}]$

$\because$ '2' is a dimensionless number.

As Dim. on LHS = Dim on RHS. Hence the given equation is dimensionally Correct.

P.1.8 :- Formula for speed of sound = ?

The speed 'v' depend on :

(a) Density of medium 'ρ'

(b) Modulus of elasticity $E = \frac{\text{stress}}{\text{strain}}$

$\because \rho = \frac{m}{V}$, $E = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\left(\frac{\Delta V}{V}\right)} = F/A$

we can write : $\left(\because \frac{\Delta V}{V} \text{ is dimensionless}\right)$

$$v \propto \rho^a \times E^b$$

or $v = \text{Constt.}\,(\rho^a \times E^b) \longrightarrow (1)$

using dimensions on both sides:

$$[LT^{-1}] = \frac{[m]^a}{[V]^a} \times \left[\frac{F}{A}\right]^b = \frac{[m]^a}{[V]^a} \times \frac{[m]^b[a]^b}{[A]^b}$$

$\because$ Constt. is not accounted in dimensions.

$$[LT^{-1}] = [M]^a \times [L^{-3}]^a \times [M]^b [L]^b [T^{-2}]^b \times [L^{-2}]^b$$

$$[LT^{-1}] = [M]^{a+b} [L]^{-3a+b-2b} [T]^{-2b}$$

or $[M]^0 [L]^1 [T]^{-1} = [M]^{a+b} [L]^{-3a-b} [T]^{-2b}$

Comparing powers of $[M], [L], [T]$ on both sides:

$0 = a + b \Rightarrow a = -b$

$1 = -3a - b$

$-1 = -2b \Rightarrow b = \frac{1}{2} \quad \therefore a = -(+\frac{1}{2}) = -\frac{1}{2}$

$\therefore a = -\frac{1}{2},\ b = \frac{1}{2}$

Putting these values in eq. (1) above we have:

$$v = \text{Constt.}\,(\rho^{-\frac{1}{2}} \times E^{\frac{1}{2}}) = \text{Constt.}\left(\frac{E^{1/2}}{\rho^{1/2}}\right)$$

$$\therefore v = \text{Constt}\sqrt{\frac{E}{\rho}}.$$

P.1.9 :- The given eq. is : $E = mc^2$ (Einstein's Eq.)

$$E = mc^2$$

Dim. on LHS. $= [E] = [\text{work}] = [Fd] = [ma][d]$

$= [MLT^{-2}][L] = [ML^2T^{-2}]$

Dim. on RHS. $= [m][c^2] = [ML^2T^{-2}]$

$\therefore$ Dim on LHS. = Dim on RHS.

$\therefore$ the given eq. is dimensionally correct.

P.1.10 :- As it is given that acc. of the particle moving along a circle depend on uniform speed 'v' and radius 'r'

$\therefore$ we can write :

$$a \propto r^n \times v^m$$

$$a = \text{constt.}\,(r^n \times v^m) \longrightarrow (1)$$

using dimensions on both sides :-

$$[LT^{-2}] = \text{constt.}\,[L]^n[LT^{-1}]^m$$

$$[L]^1[T]^{-2} = \text{constt.}\,[L]^{n+m}[T]^{-m}$$

comparing powers on both sides we get:

$$1 = n+m \quad , \quad -2 = -m \quad \Rightarrow m = 2$$

$$\therefore \; 1 = n+2 \Rightarrow n = 1-2 = -1$$

$$\therefore \; m = 2, \; n = -1 .$$

* using these values in eq. (1) we can derive eq. for centripetal acc. and centripetal force.

$$\therefore \; a = \text{constt.}\,(r^{-1} \times v^2)$$

$$= \text{constt.}\,\left(\frac{v^2}{r^1}\right)$$

$$\therefore \; a = \text{constt}\left(\frac{v^2}{r}\right) \qquad (\text{Eq. of acc.})$$

Let 'm' be the mass of the particle:

$$\therefore \; ma = m(\text{constt})\left(\frac{v^2}{r}\right)$$

$$\text{or} \qquad F = \frac{mv^2}{r} .$$

QUESTIONS

1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standards.

1.2 Give the drawbacks to use the period of a pendulum as a time standard.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

1.4 Three students measured the length of a needle with a scale on which minimum division is 1mm and recorded as (i) 0.2145 m (ii) 0.21 m (iii) 0.214m which record is correct and why?

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogous statement can you make regarding experimental data used in a computation?

1.6 The period of simple pendulum is measured by a stop watch. What type of errors are possible in the time period?

1.7 Does a dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

1.8 Write the dimensions of (i) Pressure (ii) Density

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that

$$[\lambda]=[L], \qquad [v]=[LT^{-1}] \qquad \text{and} \qquad [f]=[T]^{-1}$$

Decide which of the following is correct, $f = v\lambda$ or $f = \dfrac{v}{\lambda}$.

NUMERICAL PROBLEMS

1.1 A light year is the distance light travels in one year. How many metres are there in one light year: (speed of light = $3.0 \times 10^8\ ms^{-1}$).

(Ans: 9.5×10^{15}m)

1.2 a) How many seconds are there in 1 year?

b) How many nanoseconds in 1 year?

c) How many years in 1 second?

[Ans. (a) 3.1536×10^7s, (b) 3.1536×10^{16}ns (c) 3.1×10^{-8} yr]

1.3 The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

(Ans: 196 cm^2)

1.4 Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

(Ans: 19.4 kg)

1.5 Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment

Length of simple pendulum l = 100 cm.

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

(Ans: $9.76 \pm 0.06\ ms^{-2}$)

1.6 What are the dimensions and units of gravitational constant G in the formula $F = G\frac{m_1 m_2}{r^2}$

(Ans: $[M^{-1}L^3T^{-2}]$ Nm^2kg^{-2})

1.7 Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at $t = 0$, a is acceleration and v_f is the velocity at time t.

1.8 The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

(Ans: v = Constant $\sqrt{\frac{E}{\rho}}$)

1.9 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.10 Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

(Ans: n = -1, m = 2)