OSCILLATIONS

Q # 1. What do you know about the oscillatory/vibratory motion?

Ans. To and fro motion of a body about a fixed point is called the vibratory or oscillatory motion.

Q # 2. State the Hook's law.

Ans. For an object attached to an elastic spring, the displacement of the object from mean position is directly proportional to the applied force.

If an object of mass m attached to an elastic spring, then the displacement x produced in the object by the application of force F is described as:

$$F = kx$$

Where k is the spring constant.

Q # 3. Define following

- **Simple Harmonic Motion**
- **Restoring Force**

Ans. Restoring Force

The force that brings back the oscillatory object towards its mean position is called the restoring force. This restoring force F_r is equal and opposite to the applied force within the elastic limit of the spring.

$$F_s = -kx$$

where k is the spring constant and x is the displacement of the oscillatory object from its mean position.

Simple Harmonic Motion

It is a type of vibratory motion in which the acceleration of the body is proportional to displacement and is directed towards its mean position. Mathematically it is described as:

$$a \propto -x$$

Where a and x are the acceleration and displacement of the oscillatory object from mean position. The –ve sign indicates that acceleration of the object is directed towards the mean position.

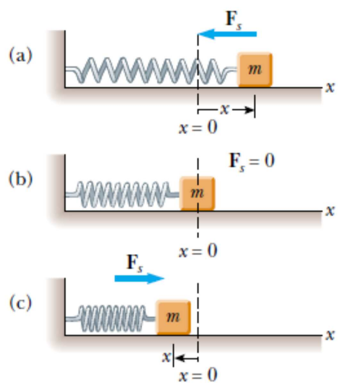
Q # 4. Show that an object attached to a horizontal mass spring system executes simple harmonic motion.

Ans. Consider a body of mass m is attached to a spring of spring constant k is executing the oscillatory motion. The restoring force acting on the object can be find out by using expression:

$$\mathbf{F}_s = -k\mathbf{x} \quad \text{---------------} \quad (1)$$

The acceleration **a** produced in the body of mass m due to restoring force can be calculated using second law of motion.

$$\mathbf{F} = m\mathbf{a} \quad \text{---------------} \quad (2)$$



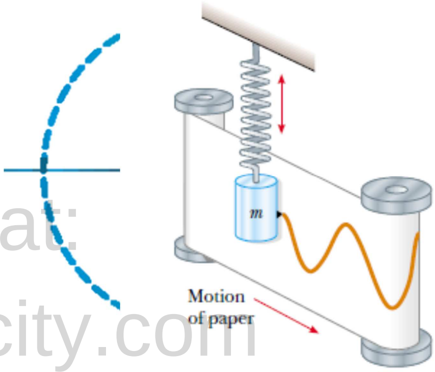
Comparing equation (1) and (2), we get:

$$-k\mathbf{x} = m\mathbf{a}$$

$$\mathbf{a} = -\frac{k}{m}\mathbf{x}$$

or $\mathbf{a} \propto -\mathbf{x}$

This shows that acceleration of the oscillating object is directly proportional to displacement and also acceleration is directed towards the mean position. Thus the object attached to a mass spring system executes simple harmonic motion.



Q # 5. Define the terms for an object executing simple harmonic motion.

- **Vibration**
- **Instantaneous Displacement**
- **Amplitude**
- **Time Period**
- **Frequency**
- **Angular Frequency**

Ans. Vibration

A vibration means one completer round trip of the body in motion.

Instantaneous Displacement

When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from mean position at any instant of time is known as instantaneous displacement.

Amplitude

The maximum value of displacement of vibratory body from its mean position is known as amplitude.

Time Period

It is the time required to complete one vibration.

Frequency

The number of vibrations executed by a body in one second is called frequency. It is measured in Hertz. The frequency f and the time period T of a vibrating body is related by the relation:

$$f = \frac{1}{T}$$

Angular Frequency

If T is the time period of a body executing simple harmonic motion, its angular frequency ω will be:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Angular frequency ω is basically a characteristic of circular motion. Here it has been introduce in to compare SHM with circular motion.

Q # 6. Derive the expression of displacement, velocity and acceleration for an object executing SHM by considering uniform circular motion.

Ans. Consider a point P moves in a circle of radius x_0, with uniform angular frequency $\omega = \frac{2\pi}{T}$. it can be visualized that when the point P moves along the circle of radius x_0, its projection (point N) execute simple harmonic motion on the diameter DE of the circle.

Thus the expression of displacement, velocity and acceleration for the object executing SHM can be derived using the analogy between the uniform circular motion of point P and SHM of point N on the diameter of the circle.

Displacement

It is the distance of projection of point N from the mean position O and any instant.

As from figure, it can be seen that

$$\angle O_1OP = \angle NPO = \theta$$

If x_0 is the amplitude and x is the displacement at any instant. Then from triangle NOP, we have

$$\sin\theta = \frac{\overline{ON}}{\overline{OP}} = \frac{x}{x_0}$$

$$x = x_0 \sin\theta \quad \text{------------ (1)}$$

Velocity

If the point P is moving in a circle of radius x_0 with uniform angular velocity ω, then the tangential velocity of point P will be:

$$v_P = x_0\,\omega$$

We want to find out the expression of velocity for point N, which is executing SHM. The velocity of N is actually the component of velocity v_P in the direction parallel to the diameter DE. Thus we can write the velocity v of point N as:

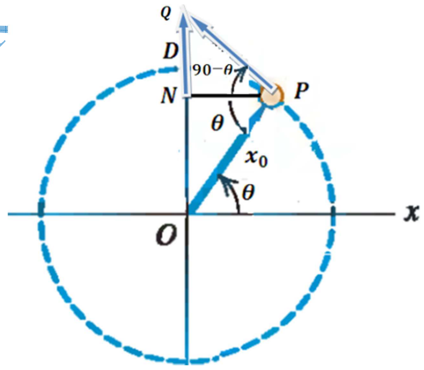
$$v = v_P \sin(90-\theta) = v_P\cos\theta$$

$$v = x_0\,\omega\cos\theta \quad \text{------------ (2)}$$

As from equation (1), we have:

$$\sin\theta = \frac{x}{x_0}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{x^2}{x_0^2}} = \sqrt{\frac{x_0^2-x^2}{x_0^2}} = \frac{\sqrt{x_0^2-x^2}}{x_0}$$



Putting this value in equation (2), we get:

$$v = x_0\,\omega\left(\frac{\sqrt{x_0^2-x^2}}{x_0}\right) = \omega\sqrt{x_0^2-x^2}$$

This is the expression of velocity of the object executing simple harmonic motion.

Acceleration

When the point P moves in a circle of radius x_0, the it will have an acceleration $a_P = x_0\,\omega^2$ that will be directed towards the center of the circle.

We want to find out the expression of acceleration of point N, that is executing SHM at the diameter of the circle.

It can be seen from the figure that the acceleration a of point N is the vertical component of acceleration a_P along the diameter DE.

$$a = a_P\sin\theta = x_0\,\omega^2\sin\theta \quad \text{--------- (3)}$$

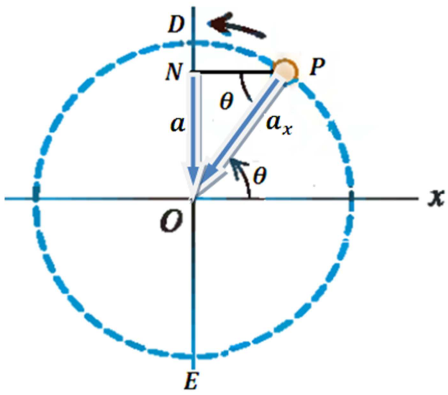
As from equation (1):

$$\sin\theta = \frac{x}{x_0}$$

Therefore the equation (3) will become:

$$a = x_0\,\omega^2\left(\frac{x}{x_0}\right)$$

$$a = \omega^2 x$$



Comparing the case of displacement and acceleration, it can be seen that the direction of displacement and acceleration are opposite to each other. Considering the direction of x as reference, the acceleration will be represented by:

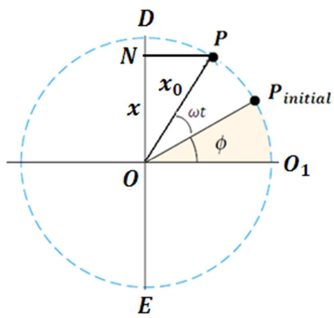
$$\mathbf{a} = -\omega^2 \mathbf{x} \quad \text{-------} \quad (4)$$

Q # 7. What do know about the term phase?

Ans. The angle θ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase.

Explanation

The displacement x and velocity v of the body executing SHM can be determined by using equations: $x = x_0 \sin\theta$ and $v = x_0\,\omega\cos\theta$, respectively. These equations indicates the displacement and velocity of simple harmonic oscillator are determine by the angle θ. This angle θ is obtained when the SHM is related with circular motion. It is the angle which the rotating radius OP makes with $x-$ axis at any instant. If the body starts its motion from mean position, its phase at this point would be 0. Similarly at the extreme position, its phase would be $\frac{\pi}{2}$.



Initial Phase Concept

In general at $t = 0$, the rotating radius can make any angle ϕ with $x-$axis as shown in the figure. In time t, the radius would rotate by $\theta = \omega t$. So now the radius OP will make an angle $(\omega t + \phi)$ with OO_1. The displacement $ON = x$ at instant t would be given by:

$$x = x_0 \sin(\omega t + \phi)$$

Now the phase angle is $\theta = \omega t + \phi$. At $t = 0$, $\theta = \phi$. So ϕ is the initial phase.

Special Case

If initial phase $\phi = \frac{\pi}{2}$, the expression of displacement will become:

$$x = x_0 \sin(\omega t + 90^\circ) = x_0 \cos\omega t$$

This equation describes the SHM for the object which starts its motion from extreme position.

Q # 8. Derive the expressions for angular frequency, time period, displacement and velocity for the case of horizontal mass spring system.

Ans. Consider a body of mass m is attached to a spring as shown in the figure. The acceleration of the object is described by the formula:

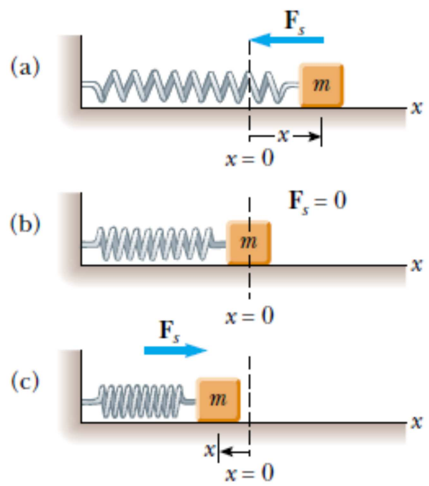
$$\mathbf{a} = -\frac{k}{m}\mathbf{x} \quad \text{-----------} \quad \mathbf{(1)}$$

The acceleration of the object executing SHM is described as:

$$\mathbf{a} = -\omega^2 \mathbf{x} \quad \text{-----------} \quad \mathbf{(2)}$$

Comparing equation (1) and (2), we have:

$$\omega = \sqrt{\frac{k}{m}}$$



Time Period

The time period of simple harmonic oscillator (SHO) is described as:

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

Instantaneous Displacement

The instantaneous displacement x of SHO is described by the formula:

$$x = x_0 \sin\theta$$

$$x = x_0 \sin(\omega t) = x_0 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

Instantaneous Velocity

The instantaneous velocity v of SHO is described by the formula:

$$v = \omega\sqrt{x_0^2 - x^2}$$

$$\Rightarrow v = \sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

$$\Rightarrow v = x_0\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_0^2}\right)} \quad \text{-----------} \quad \textbf{(3)}$$

The velocity of SHO become maximum at mean position $x = 0$. If v_0 is the velocity of SHO at mean position, then:

$$v_0 = x_0\sqrt{\frac{k}{m}}$$

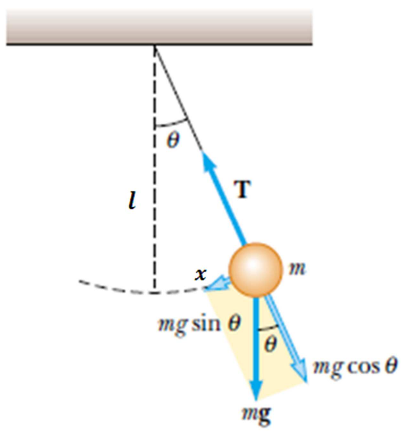
Thus, equation (3) will become:

$$\Rightarrow v = v_0\sqrt{1 - \frac{x^2}{x_0^2}}$$

Q # 9. Find out the expression of time period of simple pendulum.

Ans. Consider a simple pendulum which consists of bob of mass m is suspended from a rigid support by a string of length L as shown in the figure.

We want to find out the expression of time period for this simple pendulum.



When the bob is displaced from its mean position through a small angle θ and released, it starts to and fro motion about mean position. The weight mg of the object can be resolved in two rectangular components; $mg\cos\theta$ and $mg\sin\theta$. It is clear from the figure that the component of the weight $mg\cos\theta$ will balance the tension in the string. The restoring force acting on the object will be:

$$F = -mg\sin\theta$$

when the θ is very small, then $\sin\theta \approx \theta$. Thus,

$$F = -mg\theta \quad \text{-----------} \quad \textbf{(1)}$$

According to Newton's 2nd law of motion,

$$F = ma \quad \text{-----------} \quad \textbf{(2)}$$

Comparing (1) and (2), we get:

$$ma = -mg\theta$$

$$\Rightarrow a = -g\theta \quad \text{-----------} \quad \textbf{(3)}$$

The relationship between the arc length x and angular displacement θ is described by the formula:

$$x = l\theta$$

where l is the length of string.

$$\Rightarrow \theta = \frac{x}{l}$$

Putting value of θ in eqation (3), we get:

$$a = -\frac{g}{l}x \qquad \text{-----------} \quad \textbf{(4)}$$

The acceleration of the object executing SHM is described as:

$$a = -\omega^2 x \qquad \text{-----------} \quad \textbf{(5)}$$

Comparing equation (4) and (5), we have:

$$\omega = \sqrt{\frac{g}{l}}$$

Time Period

The time period of simple harmonic oscillator (SHO) is described as:

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

This expression shows that the time period of simple pendulum depends only on the length of the pendulum and the acceleration due to gravity. It is independent of mass.

Q # 10. Prove that the law of conservation of energy is satisfied for an object executing SHM.

Statement

The total energy of the object executing SHM remains constant.

Proof

Consider a vibrating mass spring system. When the mass m is pulled slowly, the spring is stretched by an amount x_0 against the elastic restoring force.

Derivation of Expression for P.E.

According to Hook's law:

$$F_h = kx_0$$

When displacement = 0 force = 0

When displacement = x_0 force = kx_0

Average force $F = \frac{0+kx_0}{2} = \frac{1}{2}kx_0$

The work done in displacing the mass m through x_0 is:

$$W = Fd = \left(\frac{1}{2}kx_0\right)(x_0) = \frac{1}{2}k{x_0}^2$$

This work appears as elastic potential energy of spring. Hence

$$P.E = \frac{1}{2}k{x_0}^2$$

This equation gives the maximum P.E at the extreme position. Thus

$$P.E_{max} = \frac{1}{2}k{x_0}^2$$

At any instant t, if the displacement is x, then P.E. at that instant is given by:

$$P.E = \frac{1}{2}kx^2$$

Derivation of Expression for P.E.

The velocity at any instant t is described as:

$$v = \sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

Hence the K.E. at that instant is

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\frac{k}{m}(x_0^2 - x^2)\right]$$

$$K.E. = \frac{1}{2}k(x_0^2 - x^2)$$

Total Energy ($T.E_1$) of Horizontal Mass Spring System at Mean Position

At mean position, $x = 0$. Therefore

$$P.E_1 = \frac{1}{2}kx^2 = \frac{1}{2}k(0) = 0$$

$$K.E_1 = \frac{1}{2}k(x_0^2 - x^2) = \frac{1}{2}k(x_0^2 - 0) = \frac{1}{2}kx_0^2$$

$$T.E_1 = P.E_1 + K.E_1 = 0 + \frac{1}{2}kx_0^2 = \frac{1}{2}kx_0^2 \quad \text{-----------} \quad \textbf{(1)}$$

Total Energy ($T.E_2$) of Horizontal Mass Spring System at Extreme Position

At mean position, $x = x_0$. Therefore

$$P.E_2 = \frac{1}{2}kx_0^2$$

$$K.E_2 = \frac{1}{2}k(x_0^2 - x^2) = \frac{1}{2}k(x_0^2 - x_0^2) = 0$$

$$T.E_2 = P.E_2 + K.E_2 = \frac{1}{2}kx_0^2 + 0 = \frac{1}{2}kx_0^2 \quad \text{-----------} \quad \textbf{(2)}$$

Total Energy ($T.E_3$) of Horizontal Mass Spring System at any instant

The total energy of the mass spring system at any instant of time is described as the sum of potential energy and kinetic energy at that instant. The P.E and K.E of SHO at any instant of time is:

$$P.E_3 = \frac{1}{2}kx^2$$

$$K.E_3 = \frac{1}{2}k(x_0^2 - x^2)$$

Thus

$$T.E_3 = P.E_3 + K.E_3 = \frac{1}{2}kx^2 + \frac{1}{2}k(x_0^2 - x^2) = \frac{1}{2}kx^2 + \frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$$

$$T.E_3 = \frac{1}{2}kx_0^2 \quad \text{-----------} \quad \textbf{(3)}$$

Hence from equations (1), (2) and (3), the total energy of the vibrating mass-spring system is constant. When the kinetic energy of the mass is maximum, the potential energy of mass-spring system will be zero, and vice versa. The variation of P.E and K.E with displacement is essential for maintaining oscillations. This periodic exchange of energy is a basic property of all oscillatory systems.

Q # 11. Differentiate among free and forced oscillations.

Free Oscillations

A body is said to be executing free vibrations when it oscillates without the interference of an external force. For example, a simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of pendulum.

Forced Oscillations

If an oscillating system is subjected to an external periodic force, then forced vibrations will take place. For example, the mass of a vibrating pendulum is struck repeatedly, the forced vibrations are produced.

Q # 12. What do you know about driven harmonic oscillator.

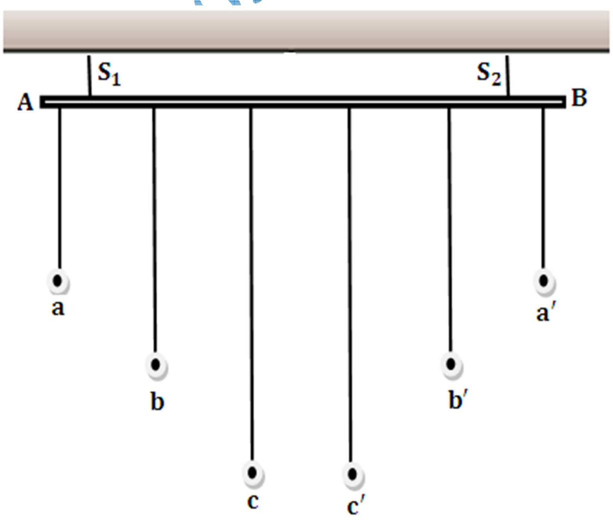
Ans. A physical system undergoing forced vibrations is known as driven harmonic oscillator.

Q # 13. Define the term resonance. Also describe few examples of resonance phenomenon occurring in daily life.

Ans. When the frequency of the applied force is equal to the natural frequency of simple harmonic oscillator, the amplitude of the motion may become extraordinary large. This phenomenon is called resonance.

Explanation

Consider a horizontal rod AB is supported by two strings S_1 and S_2. Three pairs of pendulums aa′, bb′ and cc′ are suspended to this rod. If one of these pendulums, say c, is displaced from its mean position, then its resultant oscillatory motion causes slight disturbance motion in rod AB. This causes the pendulum c′ to oscillate back with steadily increasing amplitude. However, the amplitude of the other pendulums remains small. The increase of the amplitude of pendulum c′ is due to effect of resonance, because the periods as well as the natural frequencies of pendulum c and c′ are equal.



Mechanical Resonance for the case of swing

A swing is the good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

March of soldiers on bridge

The column of soldiers, while marching on a bridge of long span is advised to break their steps. Their rhythmic march might set up oscillation of dangerously large amplitude in the bridge structure.

Electrical Resonance in Tuning of a Radio

Tuning of a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of electrical circuit of receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

Cooking of a Food in Microwave Oven

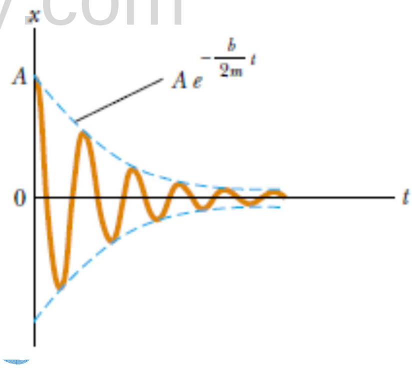
Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oxen. The waxes produced in this type of oxen have a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food.

Q # 14. Define the term damping.

Ans. Damping is the process whereby energy is dissipated from the oscillating system.

Q # 15. Write a note on damped oscillations?

Ans. The oscillation in which the amplitude decreases steadily with time are called damped oscillations.



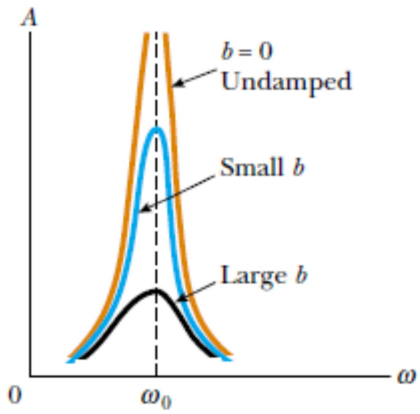
Explanation

In everyday life, the motion of any microscopic system is accompanied by frictional effects. For the case of SHM, the amplitude of simple harmonic oscillator gradually becomes smaller and smaller. The energy of oscillator is used up in doing work against the resistive forces.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent extensive oscillations.

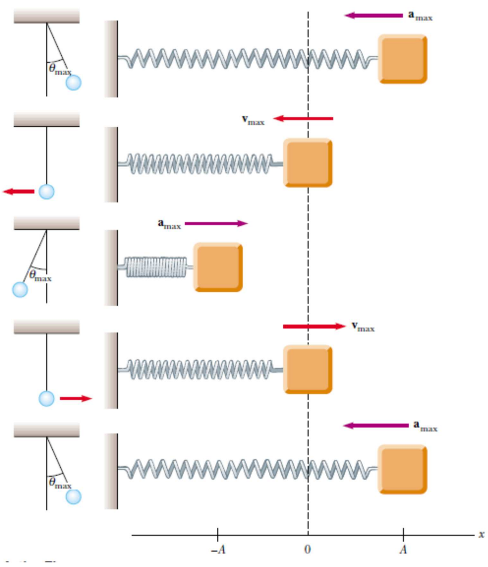
Q # 16. What do you understand by sharpness in resonance?

Ans. At resonance, the amplitude of the oscillator becomes very large. If the amplitude decreases rapidly at a frequency slightly different from the resonant frequency, the resonance will be sharp.



Q # 17. Describe the effect of damping on resonance and its sharpness.

Ans. Smaller the damping, greater will be the amplitude and more sharp will be the resonance. A heavily damped system has a fairly flat reason curve as is shown in an amplitude frequency graph.



EXERCISE SHORT QUESTIONS

Q # 1. Name the two characteristics of simple harmonic motion?

Ans. The characteristics of SHM are

i. Acceleration of the body is directly proportional to the displacement and is always directed towards mean position:

$a \alpha - x$

ii. Total energy of the particle eecuting SHM remains conserve

$E_{total} = K.E. + P.E. = const.$

Q # 2. Does frequency depend on the amplitude for harmonic oscillator?

Ans. No, frequency of the oscillator is independent of the amplitude of oscillation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The above expression shows that the frequency of harmonic oscillator does not depend upon its amplitude. It only depend on its mass and spring constant.

Q # 3. Can we realize an ideal simple pendulum?

Ans. No, we can't realize an ideal simple pendulum. An ideal simple pendulum should consist of a heavy but small metallic bob suspended from a frictionless rigid support by means of long, weighless and inextensible string. These conditions are impossible to attain in nature. So ideal simple pendulum can't be realized.

Q # 4. What is total distance travelled by an object moving with SHM in a time equal to its period, if its amplitude is A?

Ans. The total distance travelled by ab object moving with SHM in its time period is 4A, where A is amplitude of viberation.

Q # 5. What happens to period of simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

Ans. The time period of the simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where l is length of simple pendulum and g is gravitational constant.

Case 1: If Length is Doubled

If the length of simple pendulum is doubled, then the time period T' will be:

$$T' = 2\pi\sqrt{\frac{2l}{g}}$$

$$T' = \sqrt{2}\left[2\pi\sqrt{\frac{l}{g}}\right]$$

$$T' = 1.41T$$

So if the length of the simple pendulum is doubled, then its time period increase by 1.41 times of initial time period.

Case 2: If Mass is Doubled

If the mass of bob of simple pendulum is doubled, then there is no effect on time period, because the period is independent of the mass of simple pendulum.

Q # 6. Does the acceleration of simple harmonic oscillator remains constant during its motion? Is the acceleration ever zero? Explain.

Ans. No, the acceleration does not remain constant. The acceration a of simple harmonic oscillator is given by

$$a = -\text{constant } x$$

Or

$$a \; \alpha \; \text{-}x$$

This means that acceleration is proportional to the displacement and is always directed towards mean position. The acccleration becomes zero at mean position $(x = 0)$ and acceleration becomes maximum at extreme position.

Q # 7. What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

Ans. The angle θ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase.

It does not define angle between maximum displacement and driving force. It is the angle that the rotating radius makes with the reference direction.

Q # 8. Under what condition does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?

Ans. Addition of two simple harmonic motion produce a resultant, which is also simple harmonic, if the following conditions are fulfilled:

- Simple harmonic motion should be parallel
- Simple harmonic motion should have same frequency
- Simple harmonic motion should have constant phase difference

Q # 9. Show that in SHM, the accleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

Ans. The expressions of velocity and acceleration of the body executing SHM are as follow:

$$a = -\omega^2 x$$

$$v = \omega\sqrt{x_0{}^2 - x^2}$$

At mean position, where x=0

$$a = -\omega^2 x = -\omega^2 (0) = 0$$

$$v = \omega\sqrt{x_0{}^2 - x^2} = \omega\sqrt{x_0{}^2 - 0^2} = \omega\sqrt{x_0{}^2} = \omega x_0$$

At extreme position, where x=x_0

$$a = -\omega^2 x = -\omega^2 x_0$$

$$v = \omega\sqrt{x_0{}^2 - x_0{}^2} = \omega(0) = 0$$

Q # 10. In relation to SHM, explain the equation:

$y = A\sin(\omega t + \varphi)$

$a = -\omega^2 x$

Ans.

i) $y = A\sin(\omega t + \varphi)$

In this expression:

$y =$ Instantaneous displacement

$A =$ Amplitude

$\varphi =$ Initial Phase

$\omega t =$ Angle subtended in time t

(ii) $a = -\omega^2 x$

$a =$ Acceleration

$\omega =$ Angular Frequency

$x =$ Instantaneous displacement

Q # 11. Explain the relation between the total energy, Potential energy and kinetic energy for a body oscillating with SHM.

$E_{total} = P.E. + K.E.$

$E_{total} = \frac{1}{2}kx^2 + \frac{1}{2}kx_0^{\,2}\left(1 - \frac{x^2}{x_0^{\,2}}\right) = \frac{1}{2}kx_0^{\,2}$

Total energy of a body executing SHM remains constant if the frictional forces are absent. When the P.E. is maximum, then the K.E. of the system is zero and hence the total energy of the system is equal to the maximum P.E. of the system.

But when the body is at mean position, the P.E. is zero and the K.E. is maximum and hence the total energy of the system is equal to the maximum K.E. of the system.

Q # 12. Describe some common phenomenon in which resonance plays an important role.

Ans. There are some common phenomenon in which the resonance plays an important role such that:

- In radio sets
- In microwave oven
- Musical Instruments

Q # 13. In a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans. If the mass spring system is hung vertically and set into oscillation, the motion eventually stops due to friction and air resistance and some other damping forces.

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