Fluid Dynamics

Q # 1. Define the term 'Viscosity'.

Ans. The amount of force required to slide one layer of liquid over another layer is called as viscosity.

Q # 2. What do you know about drag force?

Ans. An object moving through a fluid experience a retarding force called the drag force. The drag force depends upon the velocity of object in a fluid, i.e., the drag force increases as the speed of the particle increases.

Q # 3. State the Stoke's law.

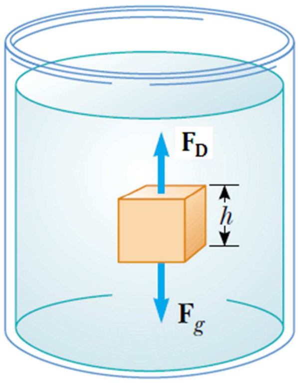
Ans. The drag force $\mathbf{F_D}$ on a sphere of radius r moving with speed $\mathbf{v}$ through a fluid of viscosity η is described as: $\mathbf{F_D} = 6\pi\eta r\mathbf{v}$

Q # 4. Write a note on terminal velocity of the object when it moves through an fluid.

Ans. When an object moves through a fluid, the force pulls the object downward. As the result the velocity of the object during its motion through fluid increases.

During downward motion of the object, a drag force acts on the body in upward direction that will tend to stop the motion of object. Thus the net force acting on the droplet is

$$\text{Net Force} = \text{Weight} - \text{Drag Force}$$



The drag force depends upon the velocity of object in a fluid, i.e., the drag force increases as the speed of the particle increases. As the speed of the droplet continues to increase, the drag force eventually approaches weight in magnitude. Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet becomes zero. Then the droplet will fall with constant speed called terminal velocity.

Q # 5. Derive the expression of terminal velocity for an object moving in a fluid.

Ans. Consider a spherical object of mass m moving a fluid of viscosity η with terminal velocity v_t. We know that the object will move with terminal velocity when its weight will equate the drag force i.e.,

$$mg = 6\pi\eta r v_t$$

$$v_t = \frac{mg}{6\pi\eta r} \text{ -------------- (1)}$$

As $mass(m) = density(\rho) \times volume\ (V)$

Also $volume\ V = \frac{4}{3}(\pi r^3)$

$\Rightarrow m = \frac{4}{3}\rho\pi r^3$

Substituting the values in equation (1), we get:

$$v_t = \frac{\left(\frac{4}{3}\rho\pi r^3\right) g}{6\pi\eta r} = \frac{2g\rho r^2}{9\eta}$$

This is the expression of terminal velocity of the object.

Q # 6. Differentiate among the laminar and turbulent flow.

Laminar Flow

The flow is said to be streamline or laminar, if every particle that passes a particular point moves along exactly the same path, as followed by particles which passed through that points earlier.

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

Q # 7. Describe the condition of an ideal fluid.

Ans. The conditions of an ideal fluid are:

1. The fluid is non-viscous
2. The fluid is incompressible
3. The fluid motion is steady

Q # 8. Derive the equation of continuity.

Ans. Consider a fluid flowing through a pipe of non-uniform size. Let the particles in fluid move along the streamlines as shown in the figure:

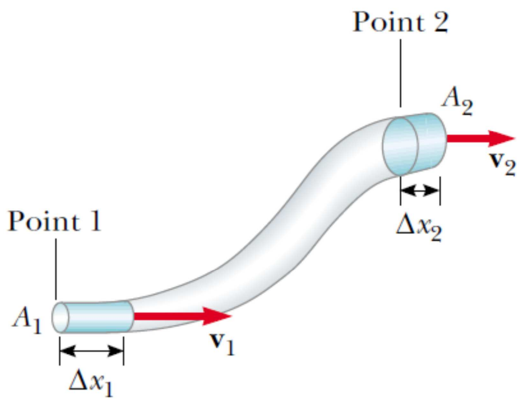
In a small time Δt, the fluid at the lower end of the tube moves a distance Δx_1, with a velocity v_1. If A_1 is the area of cross-section of this end, the mass of the fluid passing through fluid in particular interval can be find out by the expression:

$\Delta m_1 = \rho_1 V_1$ As $mass(m) = density(\rho) \times volume\,(V)$

$\Delta m_1 = \rho_1 A_1 \Delta x_1$ $\because V_1 = A_1 \Delta x_1$

$\Delta m_1 = \rho_1 A_1 v_1 \Delta t$ $\because \Delta x_1 = v_1 \Delta t$

Similarly the fluid move with velocity v_2 through the upper end of the pipe (area of cross-section A_2) in the same time Δt has the mass:



$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t$$

The mass that flows into the bottom of the pipe through A_1 in time Δt must be equal to the mass of the liquid that flows out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2$$

$$\Longrightarrow \rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

$$\Longrightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

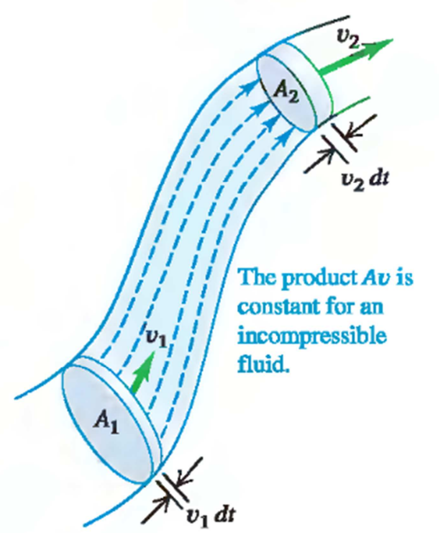
As the density is constant for the steady flow of incompressible fluid, the equation becomes:

$$A_1 v_1 = A_2 v_2$$

This is known as the equation of continuity. This equation describes that

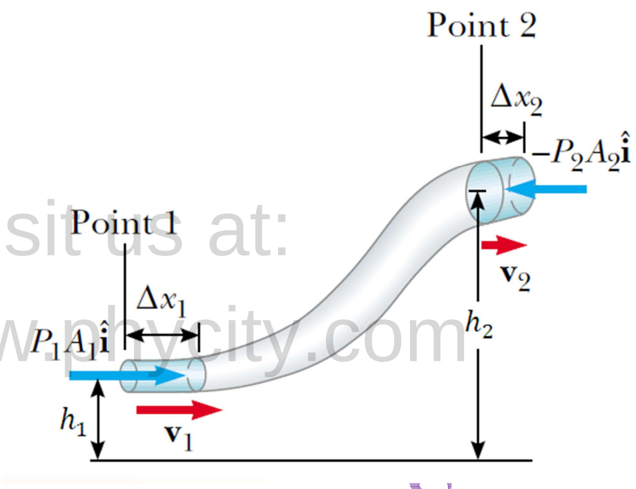
"The product of the cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant".

The equation of continuity is the statement of law of conservation of mass for the case of fluid motion.



Q # 9. Derive the expression of Bernoulli's equation for the case of an ideal fluid.

Ans. Consider a fluid that is incompressible, non viscous and flows in a steady state manner through a pipe of non-uniform size as shown in the figure:



The force on the lower end of the fluid is P_1A_1, where P_1 is the pressure and A_1 is the area of cross-sectional at the lower end. The work done on the fluid in moving through a distance Δx_1, will be:

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1$$

Similarly the work done on the fluid at the upper end is:

$$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2$$

Where P_2 is the pressure, A_2 is the area of cross-section of lower end and Δx_2 is the distance moved by the fluid in the same time interval t. The work done W_2 is taken to be negative as this work is done against the fluid force.

The net work done is

$$W = W_1 + W_2$$

$$W = P_1A_1\Delta x_1 - P_2A_2\Delta x_2$$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$W = P_1A_1v_1\,t - P_2A_2v_2t \quad \text{--------------} \quad (1)$$

From the equation of continuity,

$$A_1v_1 = A_2v_2$$

$$\Longrightarrow A_1v_1t = A_2v_2t = V \qquad (Volume\ of\ fluid\ under\ consideration)$$

The equation (1) will become:

$$W = (P_1 - P_2)V \quad \text{--------------} \quad (2)$$

If m is the mass and ρ is the density of fluid, then $V = \frac{m}{\rho}$

So the equation (2) will become:

$$W = (P_1 - P_2)\frac{m}{\rho}$$

Part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

$$Change\ in\ K.E. = \Delta K.E. = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$Change\ in\ P.E. = \Delta P.E. = mgh_2 - mgh_1$$

Where h_1 and h_2 are the heights of the upper and lower ends respectively.

Applying the law of conservation of energy to this volume of fluid, we get

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2)\frac{1}{\rho} = m\left[\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1\right]$$

$$(P_1 - P_2) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

Rearranging the above equation:

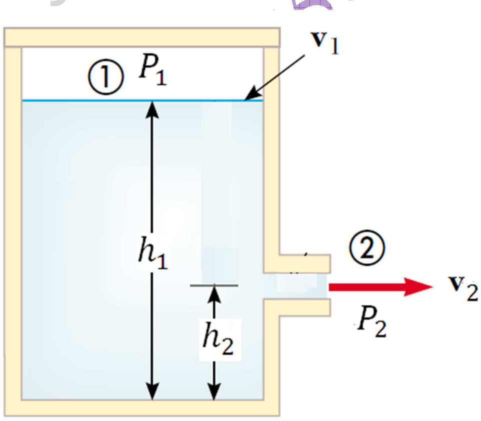
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Q # 10. State and prove the Torricelli's theorem.

Statement



The speed of efflux is equal to the velocity gained by the fluid in falling through a distance $(h_1 - h_2)$ under the action of gravity.

Proof

Suppose a large tank of fluid has a small orifice on it. Since the orifice is so small, the efflux speed $\mathbf{v_2}$ is much larger than $\mathbf{v_1}$ of the top surface of water. We can therefore, take $\mathbf{v_1}$ approximately equal to zero. Hence the Bernoulli's equation can be written as:

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

But $P_1 = P_2 = Atmospheric\ Pressure$

$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\frac{1}{2}v_2^2 = gh_2 - gh_1$$

$$v_2 = \sqrt{2g(h_2 - h_1)}$$

This is the expression of speed of efflux from the orifice. It is important to note that the speed of the efflux of liquid is the same as the speed of ball that falls through a height $(h_1 - h_2)$.

This is the Torricelli's theorem which states that:

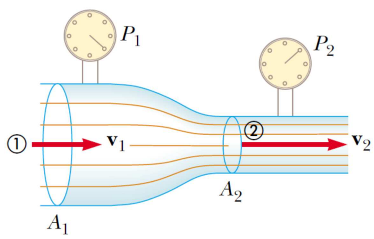
The speed of efflux is equal to the velocity gained by the fluid in falling through a distance $(h_1 - h_2)$ under the action of gravity.

Q # 11. Describe the relationship between speed and pressure of the fluid.

Ans. A result of Bernoulli's equation is that

" *Where the speed is high, the pressure will be low*"

Explanation



Suppose that water flows through a pipe system as shown in the figure. Clearly, the water flows faster at point 2 than it does at point 1.

Consider the flow speed at point 1 is 0.20 ms^{-1} and at point 2 it is $2\ ms^{-1}$. Applying the Bernoulli's equation and noting that the average P.E. is the same at both places, we have

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Substituting $v_1 = 0.20\ ms^{-1}$, $v_2 = 2\ ms^{-1}$ and $\rho = 1000\ kgm^{-3}$, we get:

$$P_1 - P_2 = 1980\ Nm^{-2}$$

This shows that the pressure in the narrow pipe where the streamline are closer together is much smaller than in the wider pipe.

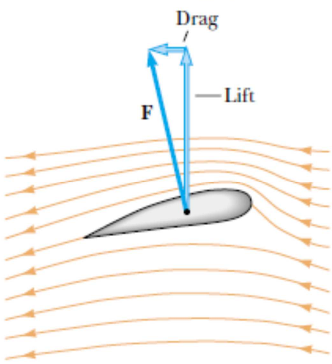
Q # 12. Explain the relationship between velocity and pressure for the following cases.

Lift on an Aero plane

Swing of the tennis ball in air

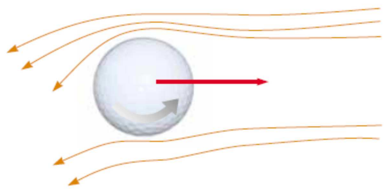
Ans.

Lift on an Aero plane



The lift on an aero plane is explained on the basis of relationship between pressure and velocity. The wing of the aero plane is designed to deflect the air so that the streamlines are closer together above the wing than below it. Thus, air is travelling faster on the upper side of the wing than on the lower. As the result, the pressure will be lower at the top of the wing, and the wing will be forced upward.

Swing of the tennis ball in air



When a tennis ball is hit by the racket in such a way that it spins as well as moves forward, the velocity of air on one side of the ball increases due to spin and hence the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

Q # 13. Derive the Venturi relation for an ideal fluid.

Ans. If one part of a pipe has a much smaller diameter than the other, we write Bernoulli equation in more convenient form. It is assumed that the pipe is horizontal ρgh terms becomes equal. Thus the Bernoulli equation is expressed as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

As the cross-sectional area A_2 is small as compared to the area A_1, then from equation of continuity $v_1 = \left(\frac{A_2}{A_1}\right) v_2$, will be small as compared to v_2. Thus we can neglect v_1 on the right hand side of the equation. Hence

$$P_1 - P_2 = \frac{1}{2}\rho\ v_2^2$$

This is known as Venturi relation, which is used in Venturi meter, a device used to measure speed of liquid flow.

Q # 14. How the blood pressure of a person is measured?

Ans. The blood pressure of a person is measured using a device called sphygmomanometer. It consist of an inflatable bag, that is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag.

When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the ball gradually decrease the external pressure.

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point, the blood flow through the vessel with very high speed. As a result the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. The flow of the blood switches from turbulent to laminar, and gurgle in the stethoscope disappears. This is the signal to record the diastolic pressure.

EXERCISE SHORT QUESTIONS

Q # 1. Explain what do you know about the term 'Viscosity'?

Ans. The amount of force required to slide one layer of liquid over another layer is called as viscosity. It is denoted by the Greek word η.

Q # 2. What is meant by drag force? What are the factors upon which the drag force acting upon a small sphere of radius r, moving down through a liquid, depends?

Ans. An object moving through a fluid experience a retarding force called the drag force. The drag force depends upon the velocity of object in a fluid, i.e., the drag force increases as the speed of the particle increases. The other factors upon which the drag force drag force depends are the shape and size of material.

Q # 3. Why fog droplets appear to be suspended in air?

Ans. When the magnitude of the drag force on the fog droplet becomes equal to its weight, the net force acting on the droplet is zero. In such a case, the droplet starts falling with a constant speed and appears to be suspended in air.

Q # 4. Explain what the difference between laminar and turbulent flow.

Laminar Flow

The flow is said to be streamline or laminar, if every particle that passes a particular point moves along exactly the same path, as followed by particles which passed through that points earlier.

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

Q # 5. State Bernoulli's relation for a liquid in motion and describe some of its applications?

Ans. The principle states that the sum of pressure, the kinetic and potential energy per unit volume for an ideal fluid remains constant at every point of its path.

Mathematically, it is described as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = constant$$

Where P is the pressure, v is the velocity and ρ is density of the fluid.

The Bernoulli relation is important in nozzle design and in flow measurements.

Q # 6. A person is standing near a fast moving train. Is there any danger that he will fall towards it?

Ans. When a person is standing near a fast moving train, then the air between them is also fast. According to Bernoulli, where the speed is high, pressure will be low. So the pressure between the person and train will be low as compared to the pressure of side way. So there will be a chance of force acting on the person from high pressure region to the low pressure region and that he may fall towards the train.

Q # 7. Identify the correct answer. What do you infer from Bernoulli's theorem?

i) **Where the speed of the fluid is high the pressure will be low .**

ii) **Where the speed of the fluid is high the pressure is also high.**

iii) **This theorem is valid only for turbulent flow of the liquid.**

Ans. The correct answer is (1) where the speed of the fluid is high the pressure will be low.

This is in accordance with the Bernoulli's equation, which states for a horizontal pipe,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Q # 8. Two row boats moving parallel in the same direction are pulled towards each other. Explain?

Ans. When two boats are moving parallel in the same direction, then the water between them is also flowing fast. According to Bernoulli, where the speed is high, pressure will be low. So the pressure between the tow boats decreases as compared to the pressure of side way. So the side way high pressure pushes the two boats towards each other.

Q # 9. Explain, how the swing is produced in a fast moving cricket ball?

Ans. When the cricket ball is thrown in such a way that is spins as well as moves forward, the velocity of the air on one side of the ball increases due to the spins and hence the pressure decreases. This gives an extra curvature to the ball known as swing which deceives opponent player.

Q # 10. Explain the working of a carburetor of a motor car using Bernoulli's principal.

Ans. The carburetor of a motor car uses a Venturi duct to give correct mixture of air and petrol to the engine. The petrol tank is attached with a pipe through a very small inlet. Air moves very fast through this pipe. As a result, pressure in the pipe decreases as compared to the pressure in the petrol tank which is atmospheric pressure. So the petrol moves from the tank to the air pipe (i.e., from high pressure to low pressure), and a correct mixture of petrol and air reaches the engine.

Q # 11. For what position will the maximum blood pressure in the body have the smallest value. (a) Standing up right (b) Sitting (c) Lying Horizontally (d)Standing on one's head?

Ans. The option (c) is correct. The blood pressure will have the smallest value when a person lying horizontally.

Q # 12. In orbiting space station, would the blood pressure in the major arteries in the leg ever be greater than the blood pressure in major arteries in the neck.

Ans. In an orbiting space station, due to weightlessness, the blood pressure in the major arteries in the leg will be equal to the blood pressure in major arteries in the neck.

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