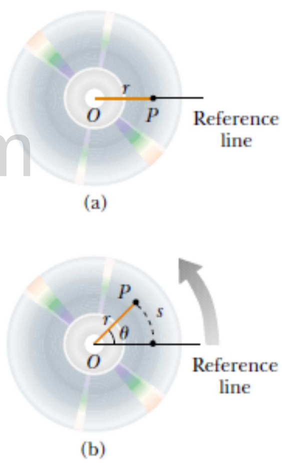
CIRCULAR MOTION

Q # 1. What do you know about the circular motion?

Ans. When a body is moving in a circle, its motion is called circular motion.

Q # 2. Define the term angular displacement. Also describe the relationship between and radian and degrees.



Ans. The angle through which a particle moves in a certain interval of time, while moving in a circle, is called its angular displacement. It is denoted by 'θ'. The angular displacement is a vector quantity and its SI unit is radian. The other units of angular displacement are degrees and revolution.

If a body moves in a circle of radius r, then the angular displacement θ covered by the object is described as:

$$\theta = \frac{S}{r} \qquad \text{-------------- (1)}$$

where S is the arc length corresponding the angular displacement θ.

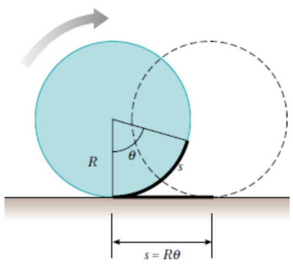
The direction of angular displacement is along the axis of rotation and is given by right hand rule.

"Grasp the axis of rotation in right hand with figure curling in the direction of rotation, the erected thumb points in the direction of angular displacement".

Relation between Radian and Degree

If the object complete its one revolution, then the total distance covered by the object is $S = 2\pi r$. Thus the eq. (1) will become:

$$1\,revolution = \frac{2\pi r}{r} = 2\pi\,radian \qquad \text{-------------- (2)}$$



Thus a body covers the angular displacement of $2\pi\,radian$ during one complete revolution. Also

$$1\,revolution = 360^\circ \qquad \text{-------------- (3)}$$

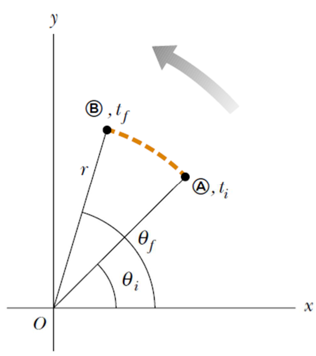
Or $\quad 1\,radian = \frac{360^\circ}{2\pi}$

Hence $\quad 1\,radian = 57.3^\circ$

Q # 3. Define the following terms corresponding to the circular motion

- **Angular Velocity**
- **Average Angular Velocity**
- **Instantaneous Angular Velocity**
- **Angular Acceleration**
- **Average Angular Acceleration**
- **Instantaneous Angular Acceleration**

Ans. Angular Velocity



The rate of change of angular displacement is called the angular velocity. It determines how fast or slow a body is rotating. It is denoted by ω. The angular velocity is a vector quantity and its SI unit is $radian/s$. The other units of angular velocity are revolution per second and degree per second.

Average Angular Velocity

The ratio of total angular displacement of the total interval of time during circular motion is called average angular velocity.

Let $\Delta\theta$ is the angular displacement during the time interval Δt, the average angular velocity during this interval is:

$$\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity

The angular velocity of the body at any instant of time, when the time interval approaches to zero, is called instantaneous angular velocity.

If $\Delta\theta$ is the angular displacement during the time interval Δt, when Δt approaches to zero, then its instantaneous angular velocity ω_{ins} is described by the relation:

$$\omega_{ins} = \lim_{\Delta t \to \infty} \frac{\Delta\theta}{\Delta t}$$

Angular Acceleration

The time rate of change of angular velocity is called angular acceleration. It is denoted by '∝'. The SI unit of angular acceleration is radian per second per second.

Average Angular Acceleration

The ratio of the total change in angular velocity to the total interval of time is called average angular acceleration.

Let ω_i and ω_f are the angular velocities at instants t_i and t_f, respectively. The average angular acceleration during interval $t_f - t_i$ is described as:

$$\propto_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

The angular acceleration of the body at any instant of time, when the time interval approaches to zero, is called instantaneous angular acceleration.

If $\Delta\omega$ is the angular velocity during the time interval Δt, when Δt approaches to zero, then its instantaneous angular acceleration $\propto_{ins}$ is described by the relation:

$$\propto_{ins} = \lim_{\Delta t \to \infty} \frac{\Delta\omega}{\Delta t}$$

Q # 4. Derive the relationship between the angular velocity and linear velocity.

Ans. Consider a object is moving in a circle of radius r. If the object covers the distance ΔS in moving from one point to the other in time Δt, then the relationship between angular displacement $\Delta\theta$ and distance ΔS covered by the object is described as:

$$\Delta S = r\,\Delta\theta$$

Dividing both sides of equation by Δt, we get:

$$\frac{\Delta S}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

Applying the limit when $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

As $\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = v$ (Instantaneous linear velocity)

And $\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \omega$ (Instantaneous angular velocity)

Therefore

$$v = r\omega$$

In vector form, the relationship between the linear and angular velocity is described as:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

Q # 5. Derive the relationship between the angular acceleration and linear acceleration.

Ans. Consider a object is moving in a circle of radius r. If the change in the linear velocity of object is Δv in time Δt, then the relationship between angular velocity $\Delta\omega$ and linear velocity Δv is described as:

$$\Delta v = r\ \Delta\omega$$

Dividing both sides of equation by Δt, we get:

$$\frac{\Delta v}{\Delta t} = r\ \frac{\Delta\omega}{\Delta t}$$

Applying the limit when $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

As $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a$ (Instantaneous linear acceleration)

And $\lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \propto$ (Instantaneous angular acceleration)

Therefore

$$a = r \propto$$

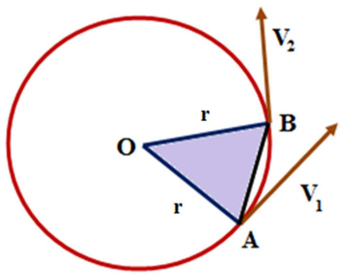
In vector form, the relationship between the linear and angular acceleration is described as:

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r}$$

Q # 6. What do you know about centripetal force? Also derive its expression.

Ans. The force needed to bend the normally straight path of the particle into the circular path is called centripetal force.

Derivation of expression of Centripetal Force



Consider a particle is moving in a circle of radius r from point A to point B. The velocity of the object changes its direction but its magnitude remains the same.

Let $\mathbf{v_1}$ and $\mathbf{v_2}$ are the velocities of the body executing circular motion at point A and B, respectively. If $\Delta\mathbf{v}$ is the change in velocity in time Δt, then the acceleration a of the particle will be:

$$a = \frac{\Delta v}{\Delta t} \qquad \text{-------------- (1)}$$

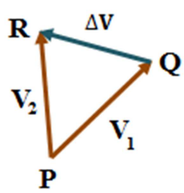
Since the speed v of the object remains the same during circular motion, so the time taken to cover the distance S will be:

$$\Delta t = \frac{S}{v}$$

So by putting the values in equation (1), we get:

$$a = \frac{\Delta v}{S/v} = \frac{v\,\Delta v}{S} \qquad \text{-------------- (2)}$$

Let us now draw a triangle PQR such that PQ is parallel and equal to v_1 and PR is parallel and equal to v_2. Since the tangent to the circle at point A and B (v_1 and v_2) are perpendicular to the radii OA and OB. Therefore the angle AOB equals the angle QPR. Therefore the triangles OAB and PQR are similar. Hence we can write:



$$\frac{\Delta v}{v} = \frac{AB}{r} \qquad \text{-------------- (3)}$$

If the point B is close to the point A on the circle , the arc AB is of nearly same length as the line AB. Since the line $AB = S$. Now by putting the values in equation (3), we get:

$$\frac{\Delta v}{v} = \frac{S}{r}$$

$$\Longrightarrow \Delta v = \frac{v\,S}{r}$$

Now by putting the value of Δv in equation (2), we get:

$$a = \frac{v}{S}.\frac{v\,S}{r} = \frac{v^2}{r}$$

This is the expression of centripetal acceleration.

Now by using the Newton's 2nd law of motion,

$$F_C = ma = \frac{m\,v^2}{r}$$

This equation gives the magnitude of centripetal force. In angular measure, the expression of the centripetal force will be:

$$F_C = \frac{m\,(r\omega)^2}{r} = \frac{m\,r^2\omega^2}{r} = mr\omega^2$$

Q # 7. What do you know about the moment of inertia for an object executing circular motion? Derive the relationship between moment of inertia and torque. Also derive the expression of moment of inertia for a rigid body.

Ans. The product of mass of the particle and square of its perpendicular distance from the axis of rotation is called the moment of inertia. Mathematically, the moment of inertia I is described as:

$$I = mr^2$$

Where m is the mass of the particle and r is the perpendicular distance from the axis of rotation.

Relationship between Torque and Moment of Inertia

Consider a mass attached to a light rod, which can rotate about a point. The mass of the rod is negligible.

Let a force F is acting on the mass perpendicular to the rod which can be find out by the expression:

$$F = ma \qquad \text{---------------} \quad \textbf{(1)}$$

Since tangential acceleration a_τ is related to angular acceleration $\propto$ by the equation:

$$a = r \propto$$

By putting the value of tangential acceleration in equation (1):

$$F = m\,r \propto \qquad \text{---------------} \quad \textbf{(2)}$$

Multiplying both sides of the equation by r, we get:

$$rF = m\,r^2 \propto \qquad \text{---------------} \quad \textbf{(3)}$$

Here $rF = \tau\ (Torque)$

And $m\,r^2 = I\ (Moment\ of\ Inertia)$

Thus equation (3) will become:

$$\tau = I \propto$$

This is the expression that related the torque with the moment of inertia. The moment of inertia plays the same role in angular motion as the mass in linear motion.

Moment of Inertia of a Rigid Body

Consider a rigid body which is made up of n small pieces of masses $m_1, m_2, \ldots \ldots \ldots, m_n$ at distances $r_1, r_2, \ldots \ldots \ldots, r_n$ from axis of rotation O. Let the body is rotating with angular acceleration $\propto$, then

The magnitude of torque acting on mass $m_1 = \tau_1 = m_1 r_1^2 \propto_1$

The magnitude of torque acting on mass $m_2 = \tau_2 = m_2 r_2^2 \propto_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

The magnitude of torque acting on mass $m_n = \tau_n = m_n r_n^2 \propto_n$

Now the total torque τ acting on the rigid body is described as:

$$\tau = \tau_1 + \tau_2 + \dots\dots\dots + \tau_n$$

$$\Rightarrow \tau = m_1 r_1^2 \propto_1 + m_2 r_2^2 \propto_2 + \dots\dots\dots + m_n r_n^2 \propto_n$$

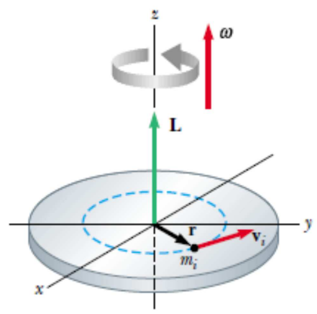
Since the body is rigid, so all the masses will rotate with same angular acceleration $\propto$,

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + \dots\dots\dots + m_n r_n^2) \propto$$

$$\Rightarrow \tau = \left(\sum_{i=1}^{n} m_i r_i^2\right) \propto$$

This is the expression of torque acting on a rigid body moving with angular acceleration $\propto$. Here $\sum_{i=1}^{n} m_i r_i^2 = I$ is the moment of inertia of the rigid body.

Q # 8. What do you know about the angular momentum for an object executing circular motion? Derive the relationship between moment of inertia and angular momentum. Also derive the expression of angular momentum for a rigid body.



Ans. The cross product of position vector and linear momentum of an object is known as angular momentum.

The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where r is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity and its magnitude is

$$L = rp\sin\theta = mvr\sin\theta$$

where θ is the angle between r and p.

Relationship between Angular Momentum and Moment of Inertia

If the particle is moving in a circle of radius r with uniform angular velocity ω, then the angle between position vector r and tangential velocity v is 90°. Hence

$$L = rp\sin 90^\circ = mvr$$

Since $v = r\omega$

$$\Rightarrow L = mr^2\omega$$

As $m\,r^2 = I\ (Moment\ of\ Inertia)$

Hence

$$L = I\omega$$

This expression gives the relationship between the angular momentum and moment of inertia.

Angular Momentum of a Rigid Body

Consider a symmetric rigid body which is made up of n small pieces of masses $m_1, m_2, \dots\dots\dots, m_n$ at distances $r_1, r_2, \dots\dots\dots, r_n$ from axis of rotation O.

We want to find out the expression of angular momentum for this rigid body, then

The magnitude of angular momentum for particle of mass $m_1 = L_1 = m_1 r_1^2 \omega_1$

The magnitude of angular momentum for particle of mass $m_2 = L_2 = m_2 r_2^2 \omega_2$

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The magnitude of angular momentum for particle of mass $m_n = L_n = m_n r_n^2 \omega_n$

Now the total angular momentum L acting on the rigid body is described as:

$$L = L_1 + L_2 + \ldots\ldots\ldots + L_n$$

$$\Rightarrow L = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + \ldots\ldots\ldots + m_n r_n^2 \omega_n$$

Since the body is rigid, so all the masses will rotate with same angular velocity ω,

$$L = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots + m_n r_n^2)\omega$$

$$\Rightarrow L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega = I\omega$$

Where $\sum_{i=1}^{n} m_i r_i^2 = I$ is the moment of inertia of the rigid body.

Q # 9. State and explain the law of conservation of angular momentum.

Statement: If no external torque acts on a system, the total angular momentum of the system remains constant.

If a system consist of n particles, which have the angular momentum $\mathbf{L_1}, \mathbf{L_2}, \mathbf{L_3}, \ldots\ldots\ldots, \mathbf{L_n}$, then according to the law of conservation of angular momentum

$$\mathbf{L} = \mathbf{L_1}, \mathbf{L_2}, \mathbf{L_3}, \ldots\ldots\ldots, \mathbf{L_n} = constant$$

where **L** is the total angular momentum of the system.

Explanation

The law of conservation of angular momentum can be explained by taking the example of a diver.

The diver pushes off the board with a small angular velocity ω_1. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed tuck position. In this case, the value of r is reduced thus the value of mr^2(moment of inertia) decreases. Hence the value of ω_2 must increase to keep the angular momentum constant. As the angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

Hence, the diver must spin faster when the moment of inertia becomes smaller to conserve angular momentum.

Direction of Angular Momentum

The angular momentum is a vector quantity with direction along the axis of rotation. The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Q # 10. What do you know about the rotational kinetic energy for an object executing circular motion? Derive the relationship between moment of inertia and rotational kinetic energy. Also derive the expression of rotational kinetic energy for a disc and hoop moving on an inclined plane.

Ans. Rotational Kinetic Energy

The energy due to spinning of a body about an axis is called rotational kinetic energy.

Relationship between Moment of Inertia and Rotational Kinetic Energy

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. The kinetic energy of an object moving with certain velocity v is described as:

$$K.E_{rot} = \frac{1}{2}mv^2$$

As for the case of circular motion, the linear velocity is related to angular velocity as: $v = r\omega$

Therefore

$$K.E_{rot} = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2$$

As $mr^2 = I\ (moment\ of\ inertia)$

$$K.E_{rot} = \frac{1}{2}I\omega^2$$

This expression relates the rotational kinetic energy with the moment of inertia.

Rotational Kinetic Energy of a Rigid Body

Consider a rigid body is spinning along the axis of rotation with uniform angular velocity ω. Let the object consist of n particles having masses $m_1, m_2, \dots \dots \dots, m_n$, which are at distances $r_1, r_2, \dots \dots \dots, r_n$ from axis of rotation.

The rotational kinetic energy for particle of mass $m_1 = K.E_1 = \frac{1}{2}m_1r_1^2\omega_1^2$

The rotational kinetic energy for particle of mass $m_2 = K.E_2 = \frac{1}{2}m_2r_2^2\omega_2^2$

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The rotational kinetic energy for particle of mass $m_n = K.E_n = \frac{1}{2}m_nr_n^2\omega_n^2$

Now the total rotational kinetic energy acting on the rigid body is described as:

$$K.E_{rot} = K.E_1 + K.E_2 + \dots \dots \dots + K.E_n$$

$$\Rightarrow K.E_{rot} = \frac{1}{2}m_1r_1^2\omega_1^2 + \frac{1}{2}m_2r_2^2\omega_2^2 + \dots \dots \dots + \frac{1}{2}m_nr_n^2\omega_n^2$$

Since the body is rigid, so all the masses will rotate with same angular velocity ω,

$$\Rightarrow K.E_{rot} = \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + \dots \dots \dots + m_nr_n^2)\omega^2$$

$$\Rightarrow K.E_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_ir_i^2\right)\omega^2 = \frac{1}{2}I\omega^2$$

Where $\sum_{i=1}^{n} m_ir_i^2 = I$ is the moment of inertia of the rigid body.

Rotational Kinetic Energy of Disc

The rotational kinetic energy of a rotating body can be find out by using the expression:

$$K.E_{rot} = \frac{1}{2}I\omega^2 \quad \text{---------------} \quad \mathbf{(1)}$$

For a disc the moment of inertia $I = \frac{1}{2}mr^2$

The equation (1) will become:

$$K.E_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$\Rightarrow K.E_{rot} = \frac{1}{4}mr^2\omega^2$$

As $r\omega = v$

$$\Rightarrow K.E_{rot} = \frac{1}{4}mv^2$$

This is the expression of rotational kinetic energy of a rotating disc.

Rotational Kinetic Energy of Hoop

The rotational kinetic energy of a rotating body can be find out by using the expression:

$$K.E_{rot} = \frac{1}{2}I\omega^2 \quad \text{---------------} \quad (1)$$

For a hoop the moment of inertia $I = mr^2$

The equation (1) will become:

$$K.E_{rot} = \frac{1}{2}(mr^2)\omega^2$$

$$\Rightarrow K.E_{rot} = \frac{1}{2}mr^2\omega^2$$

As $r\omega = v$

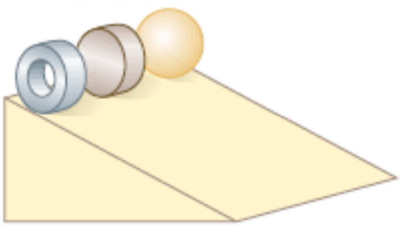
$$\Rightarrow K.E_{rot} = \frac{1}{2}mv^2$$

This is the expression of rotational kinetic energy of a rotating hoop.

Q # 11. Derive the expression of velocity of a disc and a hoop at the bottom of an inclined plane.

Velocity of the Disc at the Bottom of Inclined Plane

When a disc starts moving down an inclined plane of height h, their motion consists of both rotational and translational motions. If no energy is lost against friction, the total kinetic energy of the disc on reaching the bottom of inclined plane must be equal to its potential energy at the top.



$$P.E = K.E_{tran} + K.E_{rot}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\Rightarrow mgh = \frac{3}{4}mv^2$$

$$\Rightarrow gh = \frac{3}{4}v^2$$

$$\Rightarrow v = \sqrt{\frac{4gh}{3}}$$

This is the expression of the disc at the bottom of inclined plane.

Velocity of the Hoop at the Bottom of Inclined Plane

When a hoop starts moving down an inclined plane of height h, their motion consists of both rotational and translational motions. If no energy is lost against friction, the total kinetic energy of the disc on reaching the bottom of inclined plane must be equal to its potential energy at the top.

$$P.E = K.E_{tran} + K.E_{rot}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\Rightarrow mgh = mv^2$$

$$\Rightarrow gh = v^2$$

$$\Rightarrow v = \sqrt{gh}$$

This is the expression of the hoop at the bottom of inclined plane.

Q # 12. What do you know about satellite? Find out the expression for velocity and time period for a low flying satellite orbiting in an orbit.

Ans. Satellites are the objects that orbit around the earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the earth.

Velocity of satellite

Consider a satellite of mass m is moving in a circle of radius R around the earth. In circular orbit for a low flying satellite, the centripetal acceleration is provided by the gravity.

$$g = \frac{v^2}{R} \quad \text{----------------} \quad (1)$$

where v it the tangential velocity of the satellite. Solving equation (1), we have:

$$v = \sqrt{gR}$$

Near the surface of the earth, the gravitational acceleration $g = 9.8\ ms^{-1}$ and $R = 6.4 \times 10^6 m$.

$$\Longrightarrow v = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$\Longrightarrow v = 7.9\ kms^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit and is called critical velocity.

Time Period

The time period T of the satellite orbiting with critical velocity is given by the expression

$$T = \frac{2\pi R}{v} = \frac{2 \times 3.14 \times 6.4 \times 10^6}{7900}$$

$$T = 5060\ s = 84\ \ minutes\ approx.$$

Q # 13. Distinguish among the real and apparent weight.

Real Weight. The real weight of the earth is the gravitational pull of the earth on the object.

Apparent Weight

The weight of the object appears to be changed if the object moving up and down with an acceleration. Such a weight is called apparent weight.

Question: Discuss the relationship between the real and apparent weight for the following cases:

i) **The object is at rest ($a = 0$)**

ii) **Accelerating in upward direction**

iii) **Accelerating in downward direction**

iv) **Falling under the action of gravity**

Ans. Consider an object of mass m suspended by a string with spring balance inside a lift. The lift is capable of moving in upward and downward. The gravitational pull of the earth on the object is called the real weight of the object. The value of the weight of the object on the spring balance seemed to be varied depending upon its motion. Thus the reading of the on the spring balance describe the apparent weight of the object.

Case 1: When the lift is at rest

When the lift is at rest, then according to the Newton's 2nd law of motion, the acceleration of the object will be equal to zero. If w is the weight of the object and T is the tension in the string then we have

$$T - w = ma \quad \text{---------} \quad (1)$$

As $a = 0$, the equation (1) will become:

$$T = w$$

Hence when the lift is at rest, then the apparent weight of the object is equal to its real weight.

Case 2: When the lift is moving upward with acceleration

When the lift is moving upwards with an acceleration a, then

$T - w = ma$

$T = w + ma$

The object will then weight more than its real weight by an amount ma.

Case 3: When the lift is moving downward with acceleration

If the lift is moving downward with an acceleration a, then we have

$w - T = ma$

$T = w - ma$

Thus the tension in the string, which is the measure of apparent weight, is less than the real weight w by an amount ma.

Case 4: When the lift falls freely under gravity

When the lift falls under the action of gravity, then the acceleration $a = g$. Hence

$T = w - mg$

As the weight w of the body is equal to mg so

$T = mg - mg = 0$

The apparent weight of the object will be shown by the scale to be zero.

Q # 14. Explain why an object, orbiting the earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

Ans. When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless.

An earth's satellite is freely falling object. During its free fall, the velocity of the satellite is fast enough parallel to the earth's surface, such that the curvature of its path matches the curvature of earth. In fact, the space ship is falling towards the center of the earth all the time but due to spherical shape of earth, it never strikes the surface of the earth. Since the space ship is in free fall, all the objects within it appear to be weight less.

Q # 15. What will be the expression of orbital velocity for an object revolving in a circular orbit?

Ans. Consider a satellite of mass m_S going round the earth in a circular path of radius r with orbital velocity v. Let M the mass of the earth. The centripetal force F_C required to hold the satellite in orbit can be described as:

$$F_C = \frac{m_S v^2}{r}$$

This centripetal force is provided by the gravitational force of attraction between the earth and satellite. Thus we can write:

$$\frac{m_S v^2}{r} = \frac{G m_S M}{r^2}$$

$$\Rightarrow v^2 = \frac{GM}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

This is the expression of orbital velocity of a satellite for a circular orbit of radius r.

Q # 16. Find out the expression of frequency for producing the artificial gravity in a satellite equal to that of earth.

Ans. The weightlessness in satellite may affect the performance of astronaut in it. To overcome this difficulty, an artificial gravity is created in the satellite. For this, the satellite is set into rotation around its own axis.

Consider a satellite having outer radius R rotates around its own central axis with angular speed ω, then the angular acceleration a_c is

$$a_c = R\,\omega^2$$

But $\omega = \frac{2\pi}{T}$ where T is the period of the revolution of spaceship

$$\Rightarrow a_c = R\left(\frac{2\pi}{T}\right)^2 = R\,\frac{4\pi^2}{T^2}$$

As the frequency $f = \frac{1}{T}$, therefore

$$\Rightarrow a_c = R\,4\pi^2 f^2$$

$$\Rightarrow f^2 = \frac{a_c}{4\pi^2 R}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

The frequency f is increased to such an extent that a_c equals to g. Therefore,

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

This is the expression of frequency for producing the artificial gravity in satellite equal to that of earth.

Q # 17. What is a geostationary satellite? Write down the expression for radius of a geostationary orbit.

Ans. A geostationary satellite is the one whose orbital motion is synchronized with the rotation of the earth. We want to find out the expression of a geostationary orbit of a satellite.

The orbital speed necessary for the circular orbit, is given by the expression:

$$v = \sqrt{\frac{GM}{r}}$$

For a geostationary orbit, the orbital speed must be equal to the average speed of the satellite in one day, i.e.,

$$v = \frac{s}{t} = \frac{2\pi r}{T}$$

Where T is the period of revolution of the satellite, that is equal to one day. Thus

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

This is the expression of a geostationary orbit of a satellite.

Q # 18. How a communication system is developed?

Ans. A satellite communication system can be set up by placing several geostationary satellites in orbit over different points on the surface of the earth. One such satellite covers 120° of longitude. Since these geostationary satellites seem to hover over one place on the earth, continuous communication with any place on the earth's surface can be made.

Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites.

Q # 19. What do you know about INTELSAT's communication system?

Ans. The world's largest satellite system is manage by International Telecommunication Satellite Organization (INTELSAT). An INTELSAT VI satellite operate at microwave frequencies of 4, 6, 11 and 14 GHz and has a capacity of 30000 two way telephone circuits plus three TV channels.

Q # 20. Describe the Newton's and Einstein's view of gravitation.

Newton's View about Gravitation

According to Newton, the gravitation is the intrinsic property of matter. It means that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Einstein's View about Gravitation

According to Einstein's theory, space time is curved. In Einstein's theory, we don't speak the force of gravity acting on the bodies, instead we say that bodies and the light rays moves along geodesics in curved space time.

Q # 21. Why Einstein's theory is better than the Newton's theory.

Ans. Newton discovered the inverse square law of gravity but he offered no explanation of why gravity should follow an inverse square law.

On the other hand, Einstein's theory gives the physical picture of how gravity works. Einstein's theory also says that gravity follows an inverse square law, but it tells us why this should be so. That is why Einstein's theory is better than the Newton's theory.

Q # 22. Describe the Newton's and Einstein's idea about bending of light due to gravity.

Newton's idea about bending of light

Newton considered the light as a stream of tiny particles. Thus the tiny particles of light would be deflected by gravity.

Einstein's idea about bending of light

Einstein inferred that if gravitational acceleration and inertial acceleration are precisely equivalent, gravity must bend light, by a precise amount that can be calculated precisely.

In Einstein's theory, the deflection of light is predicted to be exactly twice as it was predicted by Newton's theory. When the bending of starlight caused by the gravity of Sun was measured during the solar eclipse in 1919, and it was found to match the Einstein's prediction rather than the Newton's.

EXERCISE SHORT QUESTIONS

Q # 1. Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, how will you find the other?

Ans. Tangential velocity is the linear velocity of a particle moving along a curve or circle. As the direction of the linear velocity is always along the tangent to the circle, that is why it is called tangential velocity.

The rate of change of angular displacement is called angular velocity. The direction of angular velocity is along the axis of rotation of the body.

If one of these two quantities are given for a wheel of known radius r, then we can find the other by using the relation:

$$v = r\omega$$

Where v and ω are the tangential and angular velocity, respectively.

Q # 2. Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

Ans. The force which keeps the body to move in the circular path and always directed towards the center of the circle is called the centripetal force.

The direction of a body moving in a circular path is always changing. To bend the normally straight path into circular path, a perpendicular force is needed, called centripetal force.

Q # 3. What is meant by moment of inertia? Explain its significance.

Ans. The product of mass of the particle and square of its perpendicular distance from axis of rotation is called moment of inertia. It is denoted by the symbol I and is expressed by the relation:

$$I = m\,r^2$$

The moment of inertia plays the same role in angular motion as the mass in linear motion.

Q # 4. What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans. The cross product of position vector and linear momentum of an object is known as angular momentum.

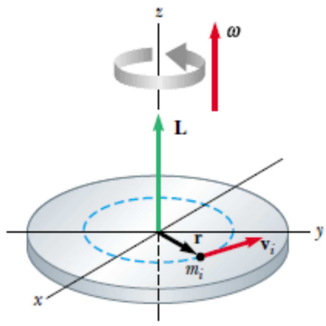
The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where r is the position vector of the particle at that instant relative to the origin O.

Law of Conservation of Angular Momentum

Statement: If no external torque acts on a system, the total angular momentum of the system remains constant.



Explanation

The law of conservation of angular momentum can be explained by taking the example of a diver.

The diver pushes off the board with a small angular velocity ω_1. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed tuck position. In this case, the value of r is reduced thus the value of mr^2(moment of inertia) decreases. Hence the value of ω_2 must increase to keep the angular momentum constant. As the angular momentum is conserved, so

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$I_1\omega_1 = I_2\omega_2$

Hence, the diver must spin faster when the moment of inertia becomes smaller to conserve angular momentum.

Q # 5. Show that orbital angular momentum $\mathbf{L_o} = \mathbf{mvr}$.

Ans. The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$\mathbf{L} = rp \sin\theta\, \hat{\boldsymbol{n}}$$

The magnitude of angular momentum will be:

$$L = rp\, sin\, \theta$$

$$\because\ p = mv$$

$$L = mvr\, sin\, \theta$$

Since the angle between radius r and tangential velocity v is 90^o, so

$$L = mvr \sin 90^o = mvr$$

Hence proved.

Q # 6. Describe what should be the minimum velocity, for a satellite, to orbit close to earth around it.

Ans. Consider a satellite of mass m is moving in a circle of radius R around the earth. In circular orbit for a low flying satellite, the centripetal acceleration is provided by the gravity.

$$g = \frac{v^2}{R} \quad \text{----------------} \quad (1)$$

where v it the tangential velocity of the satellite. Solving equation (1), we have:

$$v = \sqrt{gR}$$

Near the surface of the earth, the gravitational acceleration $g = 9.8\, ms^{-1}$ and $R = 6.4 \times 10^6 m$.

$$\Longrightarrow v = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$\Longrightarrow v = 7.9\, kms^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit and is called critical velocity.

Q # 7. State the direction of following vectors in simple situations; angular momentum and angular velocity.

Ans. The directions of angular momentum and angular velocity are used to described by right hand rule:

Grasp the axis of rotation in right hand with the figures curling in the direction of rotation, then the erected thumb will give the direction of angular velocity and angular momentum.

Q # 8. Explain why an object, orbiting the earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

Ans. An object is put into an orbit around the earth will move in a curved path under the action gravity. The curvature of the path is such that it matches the curvature of the earth, and object does not touch the earth surface. As the object continues to fall around the Earth, so it is said to be freely falling object.

When a body is falling freely, it moves with an acceleration g. and the bodies moving with acceleration g appear weightless.

Q # 9. When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Ans. When the mud flies off the tyre of a moving bicycle, it always flies along the tangent to the tyre. This is due to the reason that the linear velocity is always tangent to the circle, and the mud will fly in the direction of linear velocity.

Q # 10. A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will have greater speed on reaching the bottom?

Ans. The formulae for the velocity of the disc and the hoop are given by:

$$v_{disc} = \sqrt{\frac{4gh}{3}} \text{ and } v_{hoop} = \sqrt{gh}$$

So it is clear from the above relations that the disc will be moving with greater speed on reaching the bottom.

Q # 11. Why a diver does changes its body position before and after diving in the pool?

Ans. When the diver jumps from the diving board, his legs and arm are fully extended. The diver has large moment of inertia I_1 but the angular velocity w_1 is small. When the diver curls his body, the moment of inertia reduces to I_2. In order to conserve the angular momentum, the value to angular velocity increases to w_2.

$$L = I_1 w_1 = I_2 w_2 = const.$$

In this way, the diver can make more somersaults before entering the water.

Q # 12. A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulll the dumb bells towards his chest. What will be the effect on rate of rotation?

Ans. Initially, the arms of the students are fully extended, so he has large moment of inertia I_1 but angular velocity w_1 is small. When the student curls his body, the moment of inertia reduces to I_2. In order to conserve the angular momentum, the value of angular velocity increases to w_2.

$$L = I_1 w_1 = I_2 w_2 = const.$$

Thus the rate of rotation will increase.

Q # 13. Explain how many minimum number of geo-stationary satellites are required for global coverage of TV transmission.

Ans. A geostationary satellite covers 120^0 of longitude. So the whole earth can be covered by three correctly positioned geostationary satellites.

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