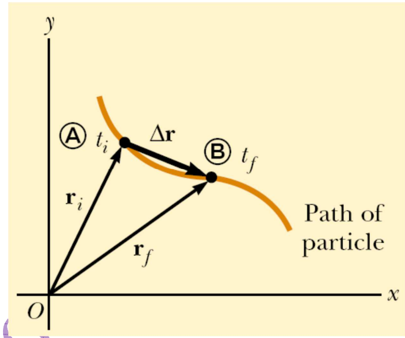
MOTION AND FORCE

Q # 1. Define following

i) Displacement

The change in position of the body from its initial to final position is called displacement. The displacement can also be described as:

"*the minimum distance between two points*".



It is a vector quantity and its direction is from initial point to the final point. The SI unit of displacement is meter.

If $\mathbf{r}_i$ and $\mathbf{r}_f$ are the position vectors of points A and B, respectively, then the displacement $\Delta\mathbf{r}$ between these two points will be:

$$\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$

ii) Velocity

The time rate of change of displacement is called the velocity. It is a vector quantity and its SI unit is ms^{-1}.

iii) Average Velocity

The ratio between the total displacement and the total time taken by the body is called average velocity. If $\Delta\mathbf{r}$ is the total displacement of the body in time t, then the average velocity $\mathbf{v}_{\text{av}}$ in time interval is described as:

$$\mathbf{v}_{\text{av}} = \frac{\Delta\mathbf{r}}{t}$$

iv) Instantaneous Velocity

The limiting value of velocity as the time interval approaches to zero is called instantaneous velocity. If $\Delta\mathbf{r}$ is the displacement covered by the object in time interval Δt, then $\mathbf{v}_{\text{int}}$ is expressed as:

$$\mathbf{v}_{\text{int}} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t}$$

v) Acceleration

The time rate of change of velocity of the body is called acceleration. It is a vector quantity and it is measured in ms^{-2}.

vi) Average Acceleration

The ratio between the total change in velocity and the total time taken by the body is called average velocity. If $\Delta\mathbf{v}$ is the total velocity of the body in time t, then the average acceleration $\mathbf{a}_{\text{av}}$ in time interval is described as:

$$\mathbf{a}_{\text{av}} = \frac{\Delta\mathbf{v}}{t}$$

vii) Instantaneous Acceleration

The limiting value of acceleration as the time interval approaches to zero is called instantaneous velocity. If $\Delta\mathbf{v}$ is the velocity the object in time interval Δt, then $\mathbf{a}_{\text{int}}$ is expressed as:

$$\mathbf{a}_{\text{int}} = \lim_{\Delta t\to 0}\frac{\Delta \mathbf{v}}{\Delta t}$$

Q # 2. Write down the applications of velocity-time graph.

Ans. the application of velocity time graph are as follow:

- The average acceleration of object can be determined from the slop of velocity-time graph.
- The area between the velocity time graph and time axis is numerically equal to the distance covered b the object.

Q # 3. State the Newton's laws of motion.

Newton's 1st laws of motion

A body at rest will remain at rest, and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force.

Newton's 2nd laws of motion

A force applied on a body produces acceleration in its own direction. The acceleration produced varies directly with the applied force and inversely with the mass of the body.

Mathematically, it is described as:

$$\mathbf{F} = m\mathbf{a}$$

where **F** is the applied force, m is the mass and **a** is the acceleration of the object.

Newton's 3rd laws of motion

Action and reaction are equal and opposite. Whenever an interaction occurs between two, each object exerts the same force on the other, but in opposite direction and for the same interval of time.

Q # 4. Define the term momentum?

The product of mass and velocity of an object is called the linear momentum. It is a vector quantity. The SI unit of momentum is kilogram meter per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Q # 5. Describe the Newton's second law of motion in terms of momentum.

Statement:

The time rate of change of momentum of a body is equal to the applied force.

Proof:

Consider a body of mass m is moving with an initial velocity $\mathbf{v}_i$. Suppose an external force **F** acts upon it for time t after which the velocity becomes $\mathbf{v}_f$. The acceleration **a** produced by this force is given by:

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, acceleration is given as

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating both equations, we get

$$\frac{\mathbf{F}}{m}=\frac{\mathbf{v}_f-\mathbf{v}_i}{t}$$

$$\Rightarrow \mathbf{F}=\frac{m(\mathbf{v}_f-\mathbf{v}_i)}{t}=\frac{m\mathbf{v}_f-m\mathbf{v}_i}{t}=\frac{\mathbf{p}_f-\mathbf{p}_i}{t}$$

Where $\mathbf{p}_i=m\mathbf{v}_i$ and $\mathbf{p}_f=m\mathbf{v}_f$ are the initial and final momentum of the body.

$$\Rightarrow \mathbf{F}=\frac{\Delta\mathbf{p}}{t} \qquad \because \mathbf{p}_f-\mathbf{p}_i=\Delta\mathbf{p}=\text{Change in linear momentum}$$

Hence proved that the rate of chance of linear momentum is equal to the applied force.

Q # 6. Define the term impulse.

When a force is acted on a body for a very short time Δt , the product of force and time is called impulse. It is a vector quantity and its unit is N s. Mathematically, it is described as:

$$\mathbf{I}=\mathbf{F}\times\Delta t$$

Where **I** is the impulse of force **F**.

Q # 7. Show that impulse of a force is equal to the change in linear momentum.

Ans. according to the Newton's second law of motion, the rate of change of linear momentum is equal to the applied force. Mathematically it is described as:

$$\mathbf{F}=\frac{\Delta\mathbf{p}}{\Delta t}$$

$$\mathbf{F}\times\Delta t=\Delta\mathbf{p} \qquad \text{----------------} \ (1)$$

As Impulse $\mathbf{I}=\mathbf{F}\times\Delta t$

Therefore, the equation (1) will become:

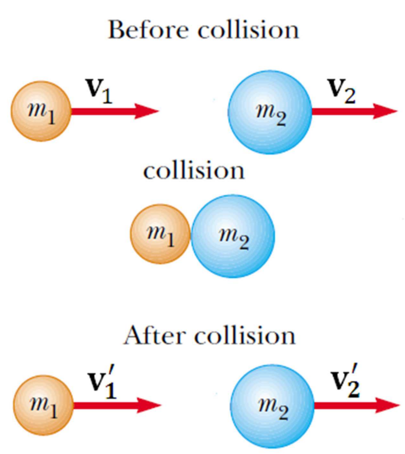
$$\mathbf{I}=\Delta\mathbf{p}$$

Hence proved that:

$$Impulse\ =\ Change\ in\ momentum$$

Q # 8. State and prove the law of conservation of linear momentum for an isolated system of two balls moving in the same direction.

Statement: The total linear momentum of an isolated system remains constant.



Proof: Consider an isolated system of two balls of masses m_1 and m_2 moving in same directions with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively. Both the balls collide and after collision, balls of mass m_1 moves with velocity $\mathbf{v}_1'$ and m_2 moves with velocity $\mathbf{v}_2'$ in the same direction as shown in the figure.

The change in momentum of mass m_1 will be:

$$\mathbf{F}\times t=m_1\mathbf{v}_1'-m_1\mathbf{v}_1 \qquad \text{----------------} \ (1)$$

Similarly, the change in momentum of mass m_2 will be:

$$\mathbf{F}'\times t=m_2\mathbf{v}_2'-m_2\mathbf{v}_2 \qquad \text{----------------} \ (2)$$

Adding equation (1) and (2), we get

$$(\mathbf{F}+\mathbf{F}')\times t=(m_1\mathbf{v}_1'-m_1\mathbf{v}_1)+(m_2\mathbf{v}_2'-m_2\mathbf{v}_2)$$

Since the action of the force $\mathbf{F}$ is equal and opposite to the reaction force $\mathbf{F}'$, we have $\mathbf{F}'=-\mathbf{F}\ \Rightarrow \mathbf{F}+\mathbf{F}'=0$. Therefore,

$$0=(m_1\mathbf{v}_1'-m_1\mathbf{v}_1)+(m_2\mathbf{v}_2'-m_2\mathbf{v}_2)$$

Or

$$m_1\mathbf{v}_1+m_2\mathbf{v}_2=m_1\mathbf{v}_1'+m_2\mathbf{v}_2'$$

which means that the total initial momentum of the system before collision is equal to the total momentum of the system after collision. Hence proved, the total linear momentum of an isolated system remains constant.

Q # 9. Differentiate among the elastic and inelastic collision.

Elastic collision

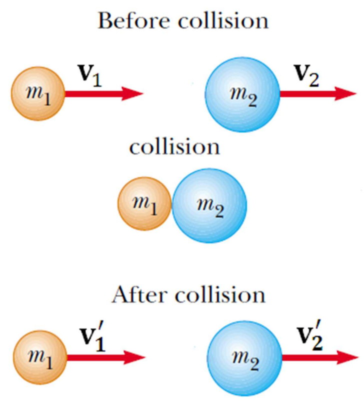
A collision in which the K.E. of the system is conserved is called elastic collision

Inelastic collision

A collision in which the K.E. of the system is not conserved is called inelastic collision.

Q # 10. Describe the elastic collision of balls in one dimension for the case of an isolated system.

Ans. Consider two smooth balls of masses m_1 and m_2 moving with velocities v_1 and v_2 respectively in the same direction. They collide and after collision, they move along the same straight line. Let their velocities after the collision be v_1' and v_2' as shown in the figure below:



By applying law of conservation of momentum, we have:

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

$$\Rightarrow m_1(v_1-v_1')=m_2(v_2'-v_2)$$

---------------- (1)

As the collision is elastic, so the K.E. is also conserved. From the conservation of K.E. we have:

$$\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2=\frac{1}{2}m_1{v_1'}^2+\frac{1}{2}m_2{v_2'}^2$$

$$\Rightarrow m_1\left({v_1}^2-{v_1'}^2\right)=m_2\left({v_2'}^2-{v_2}^2\right)$$

$$\Rightarrow m_1(v_1+v_1')(v_1-v_1')=m_2(v_2'+v_2)(v_2'-v_2) \qquad \text{----------------} \ (2)$$

Dividing equation (1) and (2), we get:

$$(v_1+v_1')=(v_2'+v_2) \qquad \text{----------------} \ (3)$$

$$v_2'=v_1+v_1'-v_2 \qquad \text{----------------} \ (4)$$

Putting the value v_2' from equation (4) in equation (1):

$$m_1(v_1-v_1')=m_2(v_2'-v_2)$$

$$\Rightarrow m_1(v_1 - v_1') = m_2(v_1 + v_1' - v_2 - v_2)$$
$$\Rightarrow m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1' - 2m_2v_2$$
$$\Rightarrow m_1v_1' + m_2v_1' = m_1v_1 - m_2v_1 + 2m_2v_2$$
$$\Rightarrow (m_1 + m_2)v_1' = (m_1 - m_2)v_1 + 2m_2v_2$$
$$\Rightarrow v_1' = \frac{(m_1-m_2)v_1}{(m_1+m_2)} + \frac{2m_2v_2}{(m_1+m_2)} \qquad \text{----------------} \quad (5)$$

Putting the value v_1' from equation (5) in equation (4), we get:

$$v_2' = v_1 + \frac{(m_1-m_2)v_1}{(m_1+m_2)} + \frac{2m_2v_2}{(m_1+m_2)} - v_2$$
$$\Rightarrow v_2' = \left[1 + \frac{(m_1-m_2)}{(m_1+m_2)}\right]v_1 + \left[\frac{2m_2}{(m_1+m_2)} - 1\right]v_2$$
$$\Rightarrow v_2' = \frac{2m_1v_1}{(m_1+m_2)} + \frac{(m_2-m_1)v_2}{(m_1+m_2)} \qquad \text{----------------} \quad (6)$$

The equation (5) and (6) gives the values of velocities of the balls after collision.

Special Cases

Case 1: When $m_1 = m_2$

Putting values $m_1 = m_2$ in equation (5) and (6), we get:

$$v_1' = \frac{(0)v_1}{(m_1+m_1)} + \frac{2m_1v_2}{(m_1+m_1)} = \frac{2m_1v_2}{2m_1} = v_2$$
$$v_2' = \frac{2m_1v_1}{(m_1+m_1)} + \frac{(0)v_2}{(m_1+m_1)} = \frac{2m_1v_1}{2m_1} = v_1$$

Thus, if the balls of same masses collies each other, they will interchange their velocities after collision.

Case 2: When $m_1 = m_2$ and $v_2 = 0$

Putting values $m_1 = m_2$ and $v_2 = 0$ in equation (5) and (6), we get:

$$v_1' = \frac{(0)v_1}{(m_1+m_1)} + \frac{2m_1(0)}{(m_1+m_1)} = \frac{2m_1v_2}{2m_1} = 0$$
$$v_2' = \frac{2m_1v_1}{(m_1+m_1)} + \frac{(0)v_2}{(m_1+m_1)} = \frac{2m_1v_1}{2m_1} = v_1$$

Thus, the ball of mass m_1, after collision, will come to stop and m_2 will takes of the velocity of m_1.

Case 3: When a light body collides with the massive body at rest.

In this case initial velocity $v_2 = 0$ and $m_2 \gg m_1$. Under these conditions m_1 can be neglected as compared to m_2

Putting values in equation (5) and (6), we get:

$$v_1' = \frac{(0-m_2)v_1}{(0+m_2)} + \frac{2m_2(0)}{(0+m_2)} = -v_1$$
$$v_2' = \frac{2(0)v_1}{(0+m_2)} + \frac{(m_2-m_1)(0)}{(0+m_2)} = 0$$

Thus, the body of mass m_1 will bounce back with the same velocity while m_2 will remain stationary.

Case 4: When a massive body collides with the light stationary body.

In this case initial velocity $v_2 = 0$ and $m_1 \gg m_2$. Under these conditions m_2 can be neglected as compared to m_1.

$$v_1' = \frac{(m_1 - 0)v_1}{(m_1 + 0)} + \frac{2(0)(0)}{(m_1 + 0)} = v_1$$

$$v_2' = \frac{2m_1 v_1}{(m_1 + 0)} + \frac{(0 - m_1)(0)}{(m_1 + 0)} = 2v_1$$

Thus, there will be no change in the velocity of massive body, and the lighter body will move in forward direction with twice the velocity of incident body.

Q # 11. Find out the expression of force on a wall due to water flow.

Ans. Suppose the water strikes a wall normally with velocity v and comes to rest after striking the wall. The change in velocity if $0 - v = -v$.

According to the Newton's second law of motion, the applied force is equal to the rate of change of momentum. If mass m of water strikes the wall in time t, then the force F on the water is:

$$F = -\frac{mv}{t}$$

From Newton's third law of motion, the reaction force exerted by the water on the wall is equal but opposite. Hence,

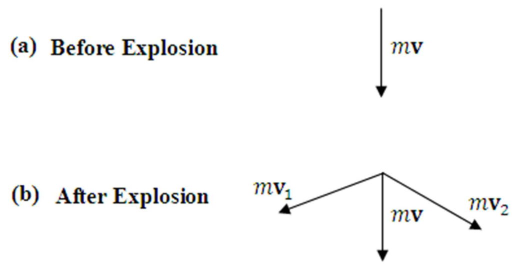
$$F = -\left(-\frac{mv}{t}\right) = \frac{mv}{t}$$

This is the expression of force exerted by the water on the wall.

Q # 12. Explain following cases by law of conservation of momentum.

Explosion of a falling bomb

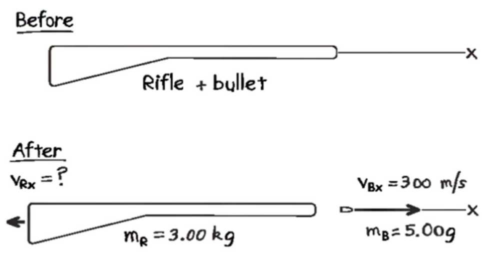
When a shell explodes in mid-air, its fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell.



Suppose a falling bomb explodes into two pieces. The momenta of the bomb fragments combine by the vector addition equal to the original momentum of the falling bomb.

Bullet fired from a Rifle

Consider a bullet of mass m fired from a rifle of mass M with velocity $\mathbf{v}$. initial momentum of the bullet and the rifle is zero. From the principle of conservation of linear momentum, when the bullet is fired, the total momentum of bullet and the rifle still remain zero, since no external force is acted on them. Thus, if $\mathbf{v}'$ is the velocity of the rifle then



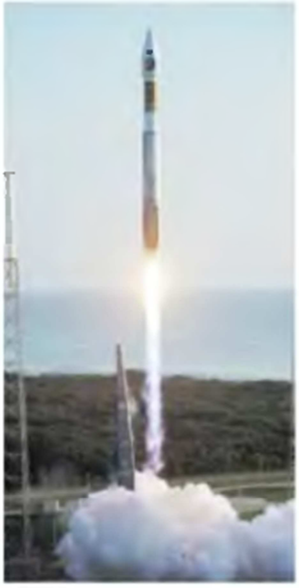
$$m\mathbf{v}\ (\text{bullet}) + M\mathbf{v}'\ (\text{rifle}) = 0$$

$$M\mathbf{v}' = -m\mathbf{v}$$

The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than the bullet, it follows that the rifle recoils with only a fraction of velocity of bullet.

Q # 13. Describe the rocket propulsion as a special case of law of conservation of momentum and Newton's 3rd law of motion.

Ans. When rocket is fired, it moves in forward direction by expelling burning gases through the engine at the rear. The rocket rains the momentum equal to the momentum of gas expelled from the engine but in opposite direction.

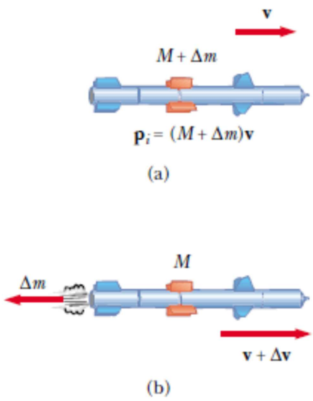


The moving rocket is considered as a system of variable mass. As the rocket moves forward, its fuel continues to be consumed and engines have to push less mass. Moreover, the rocket has to face less air resistance. Therefore, it continues to gain more and more momentum. So instead of moving at steady speed, the rocket gets faster and faster.

If m is the mass of the gas ejected per second with velocity $\mathbf{v}$ relative to the rocket, the change in momentum per second of ejecting gases is $m\mathbf{v}$. This equals the thrust produced by the engine on the body of rocket. So the acceleration $\mathbf{a}$ of the rocket is

$$\mathbf{a} = \frac{m\mathbf{v}}{M}$$

Where M is the mass of rocket. When the fuel in the rocket is burned and ejected, the mass of the rocket decreases and hence the acceleration increases.



Q # 14. What do you know about projectile motion? Find out the expression of horizontal and vertical distance at any instant of time.

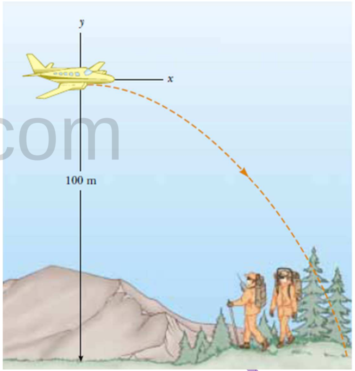
Projectile Motion

It is the two dimensional motion in which the object moves under constant acceleration due to gravity. During projectile motion, the object has constant horizontal component of velocity but changing vertical component of velocity.

Horizontal and Vertical Distance

Consider a ball is thrown horizontally from certain height. It is observed that the ball travel forward as well as falls downward, until it strikes something. There is no horizontal force acting on the object, so $a_x = 0$. Thus the horizontal velocity $\mathbf{v}_x$ will remain unchanged. The horizontal distance x covered by the object can be find out by using the 2nd equation of motion:

$$x = \mathbf{v}_x \times t + \frac{1}{2}a_x t^2$$
$$x = \mathbf{v}_x \times t$$



As the object is accelerated in downward direction under the force of gravity, therefore $a_y = g$. Since initial vertical velocity is zero i.e., $v_y = 0$. Therefore, vertical distance y covered by the object is:

$$y = v_y \times t + \frac{1}{2}a_y t^2$$
$$\Rightarrow y = \frac{1}{2}gt^2$$

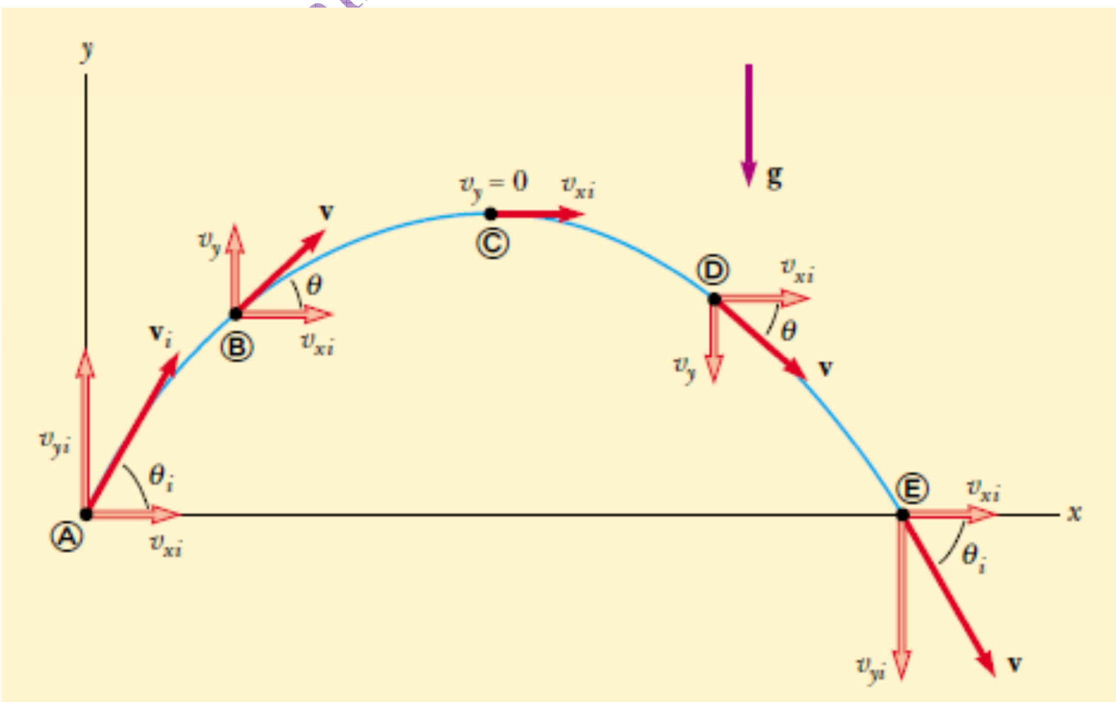
Q # 15. Find out the expression of instantaneous velocity for a projectile.

Ans. Consider a projectile is fired at an angle θ with horizontal. The motion of a projectile can be studied easily by resolving it in horizontal and vertical components. Let $v_i$and $v_i \sin\theta$ are the horizontal and vertical component of velocity, repectively. There is no force acting on the projectle acting on projectile in horzontal direction, therefore, $a_x = 0$. Therefore, by using the first equation of motion, we have:

$$v_{fx} = v_{ix} + a_x \times t$$
$$\Rightarrow v_{fx} = v_i \cos\theta$$

As the verticle component of velocity of the projectile is influenced by the force of gravity, therefore, for upward motion $a_y = -g$. The verticle component of velocity can be find out by using 1st equation of motion:

$$v_{fy} = v_{iy} + a_y \times t$$
$$\Rightarrow v_{fy} = v_i \sin\theta - gt$$



Magnitude

The magnitude of velocity at any instant of time is

$$v = \sqrt{v_{fx}^2 + v_{fy}^2}$$

Direction

The angle Φ which the resultant velocity makes with horizontal can be found from

$$\tan \Phi = \frac{v_{fy}}{v_{fx}}$$

Q # 16. Derive the expressions for

(a) Height of projectile **(b) Time of Flight** **(c) Range of Projectile**

Height of Projectile

Consider a projectile is thrown upward with initial velocity v_i making an angle θ with horizontal. Initially, the vertical component of velocity if $v_i \sin\theta$. At maximum height, the value of vertical component of velocity becomes zero. If t is the time t taken by the projectile to attain the maximum height h, then by using 3rd equation of motion:

$$2a_y h = v_{fy}^2 - v_{iy}^2$$

$$-2gh = 0 - v_i^2 \sin^2\theta$$

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

This is the expression of the height attained by the projectile during its motion.

Time of Flight

The time taken by the object to cover the distance from the place of its projection to the place where it hits the ground at the same level is called time of flight.

As the projectile goes up and comes back to the same level, thus covering no vertical distance i.e., $S = h = 0$. Thus the time of flight t can be find out by using 2nd equation of motion:

$$S = v_{iy} \times t + \frac{1}{2} a_y t^2$$

$$\Longrightarrow 0 = v_i \sin\theta\,.\,t - \frac{1}{2} g t^2$$

$$\Longrightarrow \frac{1}{2} g t^2 = v_i \sin\theta\,.\,t$$

$$\Longrightarrow t = \frac{2\, v_i \sin\theta}{g}$$

This is the expression of time of flight of a projectile.

Range of the Projectile

The distance which the projectile covers in the horizontal direction is called the range of the projectile.

In projectile motion, the horizontal component of velocity remains same. Therefore the range R of the projectile can be determine using formula:

$$R = v_{ix} \times t$$

where v_{ix} is the horizontal component of velocity and t is the time of flight of projectile.

$$R = v_i \cos\theta \times \left(\frac{2\, v_i \sin\theta}{g}\right)$$

$$\Rightarrow R = \frac{{v_i}^2}{g} 2 \sin\theta \cos\theta$$

$$\Rightarrow R = \frac{{v_i}^2}{g} \sin 2\theta$$

Thus the range of projectile depends upon the velocity of projection and angle of projection.

Maximum Horizontal Range

The horizontal range will be maximum when the factor $\sin 2\theta$ will be maximum. So,

Maximum value of $\sin 2\theta = 1$

$$\Rightarrow 2\theta = \sin^{-1}(1)$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

Hence for the maximum horizontal range, the angle of projection should be 45°.

Q # 17. Describe the motion of a ballistic missile as an applications of projectile motion.

Ans. An unpowered and unguided missile is called a ballistic missile. In ballistic flight, the projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity.

A ballistic missile moves in a way that is the result of superposition of two independent motions:

- A straight line inertial motion in the direction of launch
- A vertical gravity fall

According to the law of inertia, an object should move in straight at the constant speed. But the downward force of gravity will change its straight path into curved path.

At high speed and for long distances, the air resistance effect both horizontal and vertical components of velocity. Therefore, the ballistic missiles are used only for short ranges for which the initial velocity is not large. For long ranges, powered and remote control guided missiles are used.

Govt. Degree College Naushera
District Khushab

EXERCISE SHORT QUESTIONS

Q # 1. What is the difference between uniform and variable velocity? From the explanation of variable velocity, define acceleration. Give the SI unit of velocity and acceleration.

Uniform Velocity

A body is said to have a uniform velocity if it covers equal displacement in equal intervals of time.

Variable Velocity

A body is said to have a variable velocity if it covers unequal displacements in equal intervals of time.

Acceleration

The time rate of change of velocity of the body is called acceleration. Consider a body is moving with initial velocity $\mathbf{v_i}$ and after some time Δt its velocity becomes $\mathbf{v_f}$, then the acceleration **a** of the object will be:

$$\mathbf{a} = \frac{\mathbf{v_f} - \mathbf{v_i}}{t}$$

SI Unit of Velocity

The SI unit of velocity is meter per second or ms^{-1}

SI Unit of Acceleration

The SI unit of velocity is meter per second per second or ms^{-2}

Q # 2. An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

Ans. When the object is thrown vertically upward, it will move against the direction of gravity. The sign of acceleration **g** relative to velocity **v** will be taken as negative. It is because of the reason that the direction of **g** is opposite to the direction of **v** during upward motion.

If the object is moving downward, then the sign of **g** relative to **v** will be taken as positive because both **g** and **v** are in same direction.

Q # 3. Can the velocity of an object reverse the direction when the acceleration is constant? If so, give an example.

Ans. Yes, the velocity of a body can reverse its direction with constant acceleration. For example, when a body is thrown vertically upward under the action of gravity, the velocity of the object will go on decreasing because force of gravity is acting downward.

When the object reaches the maximum height, its velocity becomes zero, and then the object reverses its direction of motion and start moving vertically downward. During the whole process, the magnitude of the acceleration due to gravity remains constant.

Q # 4. Specify the correct statement:

a. **An object can have a constant velocity even its speed is changing.**
b. **An object can have a constant speed even its velocity is changing.**
c. **An object can have a zero velocity even its acceleration is not zero.**
d. **An object subjected to a constant acceleration can reverse its velocity.**

Ans. The statement (b) is correct.

An object can have constant speed even its velocity is changing. For the case of circular motion, the object moves with constant speed but its velocity changes due to change in direction continuously.

Q # 5. A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have a larger speed when it strikes the ground? Ignore the air friction.

Ans. Both balls will hit the ground with same speed.

When a ball is thrown upward with initial velocity $\mathbf{v_i}$, it will have same velocity $\mathbf{v_i}$ when it returns back to the same level. After that the ball will continue its motion in downward direction and hits the ground with velocity $\mathbf{v_f}$.

Thus if the second ball is thrown vertically downward with initial velocity $\mathbf{v_i}$ from the same height, it will hit the ground with the same final velocity $\mathbf{v_f}$.

Q # 6. Explain the circumstances in which the velocity v and acceleration a of a car are

(i) Parallel **(ii) Anti-parallel** **(iii) Perpendicular to one another**

(iv) v is zero but a is not zero **(v) a is zero but v is not zero**

Ans.

(i) When the velocity of the car is increasing along a straight line then **v** and **a** of the car will be parallel to each other.

(ii) When the velocity of the car is decreasing along a straight line then **v** and **a** of the car will be anti-parallel to each other.

(iii) When the car moves along circular path, then **a** will be directed towards the center of the circle while its velocity will be along the tangent. Thus **v** and **a** of the car will be perpendicular to each other when it moves on a circular path.

(iv) When the brake is applied on a moving car, it slows down and comes to rest due to negative acceleration in opposite direction. Thus **v** is zero but **a** is not zero.

(v) When the car is moving in straight line with uniform velocity, then **a** of the car is zero but **v** is not zero.

Q # 7. Motion with constant velocity is a special case of motion with constant acceleration. Is this statement is true? Discuss.

Ans. Yes this statement is true. When a body moves with constant velocity in the straight line, its acceleration is zero. Hence, the acceleration of the body will always remains constant during such motion. As the zero is a constant quantity, therefore this is a special case of motion.

Q # 8. Find the change in momentum for an object subjected to a given force for a given time and state the law of motion in terms of momentum.

Ans. Consider a body of mass m is moving with an initial velocity $\mathbf{v}_i$. Suppose an external force **F** acts upon it for time t after which the velocity becomes $\mathbf{v}_f$. The acceleration **a** produced by this force is given by:

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, acceleration is given as

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating both equations, we get

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

$$\Rightarrow \mathbf{F} = \frac{m(\mathbf{v}_f - \mathbf{v}_i)}{t} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t} = \frac{\mathbf{p}_f - \mathbf{p}_i}{t}$$

Where $\mathbf{p}_i = m\mathbf{v}_i$ and $\mathbf{p}_f = m\mathbf{v}_f$ are the initial and final momentum of the body.

$$\Rightarrow \mathbf{F} = \frac{\Delta\mathbf{p}}{t} \qquad \because \mathbf{p}_f - \mathbf{p}_i = \Delta\mathbf{p} = \text{Change in linear momentum}$$

This is Newton's second law of motion in terms of linear momentum.

Statement:

The time rate of change of momentum of a body is equal to the applied force.

Q # 9. Define impulse and show that how it is related to linear momentum?

Ans. Impulse

When a force is acted on a body for a very short time Δt , the product of force and time is called impulse. It is a vector quantity and its unit is N s. Mathematically it is described as:

$$\mathbf{I} = \mathbf{F} \times \Delta t$$

Where **I** is the impulse of force **F**.

Relationship between Impulse and Momentum

According to the Newton's second law of motion, the rate of change of linear momentum is equal to the applied force. Mathematically it is described as:

$$\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$$

$$\mathbf{F} \times \Delta t = \Delta\mathbf{p} \qquad \text{---------------} \quad (1)$$

As Impulse $\mathbf{I} = \mathbf{F} \times \Delta t$

Therefore, the equation (1) will become:

$$\mathbf{I} = \Delta\mathbf{p}$$

Hence

$$\textit{Impulse} = \textit{Change in Linear Momentum}$$

Q # 10. State the law of conservation of linear momentum, pointing out the importance of an isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

Ans. Statement

The total linear momentum of an isolated system remains constant.

Isolated System

It is a system on which no external agency exerts any force. In an isolated system, the bodies may interact with each other but no external force acts on them. Thus, in an isolated system, the linear momentum of the system remains conserve.

In ever day life, the effect of frictional forces and gravitational force is negligible. Thus law of conservation of momentum can be applied to the systems which are not completely isolated e.g., firing of gun, motion of rocket etc.

Q # 11. Explain the difference between elastic and inelastic collision. Explain how would a bouncing ball behave in each case? Give the plausible reason for the fact that K.E is conserved in most cases?

Elastic collision

A collision in which the K.E. of the system is conserved is called elastic collision

Inelastic collision

A collision in which the K.E. of the system is not conserved is called inelastic collision.

When a ball is dropped on floor, after the impact it attains the same height. It is because of the fact that small amount of K.E is converted into heat and sound energies.

Q # 12. Explain what is meant by projectile motion. Derive the expression for

(a) Time of flight (b) Range of projectile

Show that the range of the projectile is maximum when the projectile is thrown at an angle of 45° with the horizontal.

Projectile Motion

It is the two dimensional motion in which the object moves under constant acceleration due to gravity. During projectile motion, the object has constant horizontal component of velocity but changing vertical component of velocity.

Time of Flight

The time taken by the object to cover the distance from the place of its projection to the place where it hits the ground at the same level is called time of flight.

As the projectile goes up and comes back to the same level, thus covering no vertical distance i.e., $S = h = 0$. Thus the time of flight t can be find out by using 2nd equation of motion:

$$S = v_{iy} \times t + \frac{1}{2}a_y t^2$$

$$\Longrightarrow 0 = v_i \sin\theta \,.\, t - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{1}{2}gt^2 = v_i \sin\theta \,.\, t$$

$$\Rightarrow t = \frac{2\,v_i \sin\theta}{g}$$

This is the expression of time of flight of a projectile.

Range of the Projectile

The distance which the projectile covers in the horizontal direction is called the range of the projectile. In projectile motion, the horizontal component of velocity remains same. Therefore the range R of the projectile can be determine using formula:

$$R = v_{ix} \times t$$

where v_{ix} is the horizontal component of velocity and t is the time of flight of projectile.

$$R = v_i \cos\theta \times \left(\frac{2\,v_i \sin\theta}{g}\right)$$

$$\Rightarrow R = \frac{{v_i}^2}{g} 2 \sin\theta \cos\theta$$

$$\Rightarrow R = \frac{{v_i}^2}{g} \sin 2\theta$$

Thus the range of projectile depends upon the velocity of projection and angle of projection.

Maximum Horizontal Range

The horizontal range will be maximum when the factor $\sin 2\theta$ will be maximum. So,

Maximum value of $\sin 2\theta = 1$

$$\Rightarrow 2\theta = \sin^{-1}(1)$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

Hence for the maximum horizontal range, the angle of projection should be 45°.

Q # 13. At what point or points in its path does a projectile have its minimum speed, its maximum speed?

The speed of the projectile is minimum at the maximum height of projectile. It is because of the reason that, at maximum height the vertical component of velocity becomes zero.

The speed of the projectile is maximum at the point of projection and also just before it strikes the ground because the vertical component of velocity is maximum at these points.