IMPORTANT QUESTIONS WITH ANSWERS

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Q # 1. Differentiate among scalars and vectors.

Scalars	Vectors
(i) The physical quantities that are completely described by magnitude with proper unit are called scalars.	(i) The physical quantities that are completely described by magnitude with proper unit and direction are called vectors.
(ii) Mass, length, time and speed are examples of scalars.	(ii) Displacement, velocity, acceleration, force and momentum are examples of vectors.

Q # 2. What do you know about rectangular coordinate system? Describe its significance.

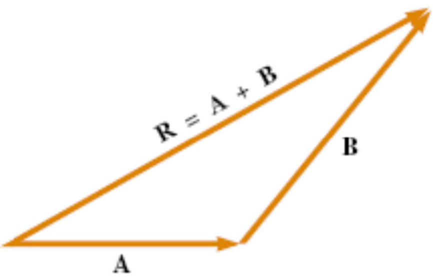
Ans. The lines which are drawn perpendicular to each other are called coordinate axis and a system of coordinate axis is called the rectangular or Cartesian coordinate system. A coordinate system is used to describe the location of a body with respect to a reference point, called origin.

Q # 3. Describe the Head to Tail rule.

Ans. The vectors can be added graphically by head to tail rule. According to this rule, the addition of two vectors **A** and **B** consists of following steps:

(i) Place the tail of vector **B** on the head of vector **A**.

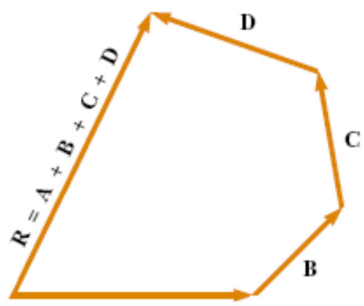
(ii) Draw a vector from the tail of vector **A** to the head of vector **B**, called the resultant vector.



Q # 4. What do you know about the Resultant Vector?

Ans. The vector which has the same effect as that of all component vectors is called resultant vector. Consider four vectors **A**, **B**, **C** and **D** are added by head to tail rule and **R** is their resultant vector, as shown in the figure.

The vector **R** has the same effect as the combined effect of vectors **A**, **B**, **C** and **D**.



Q # 5. Define following

(i) Negative of a Vector

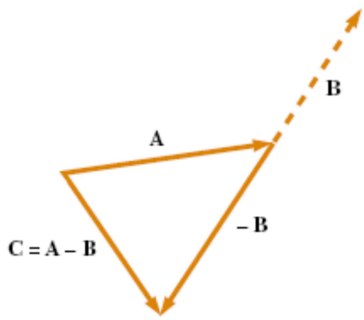
The vector which has the same magnitude as that of vector **A**, but opposite in direction is called negative of vector **A**.

(ii) Vector Subtraction

Subtraction of a vector is equivalent to the addition of one vector into negative of second vector. Consider two vectors **A** and **B**. In order to subtract **B** from **A**, the negative of vector **B** is added to vector **A** by head to tail rule.

The resultant **C** is given by

$$\mathbf{C} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$



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(iii) Equal Vector

Two vectors are said to be equal if they have same magnitude and direction.

(iv) Null Vector

A vector of zero magnitude and arbitrary direction is called null vector.

(v) Component of a Vector

A component of a vector is its effective value in a specific direction.

(vi) Rectangular Component

The components of a vector which are perpendicular to each other are called rectangular components.

(vii) Position Vector

The position vector describes the location of a point with respect to origin. In two dimension, the position vector '**r**' of point $P\ (a, b)$ is describe as

$$\mathbf{r} = a\hat{\imath} + b\hat{\jmath}$$

The magnitude of this position vector will be

$$r = \sqrt{a^2 + b^2}$$

In three dimensional Cartesian coordinate system, the position vector '**r**' of point $P\ (a, b, c)$ is describe as

$$\mathbf{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

The magnitude of this position vector will be

$$r = \sqrt{a^2 + b^2 + c^2}$$

Q # 6. Discuss the different cases of multiplication of a vector by a scalar (number).

Case -1

If any scalar $n > 0$ is multiplied by a vector '**A**', then the magnitude of the resultant 'n**A**' will become n times ($|\mathrm{nA}|$) but the direction remains same as that of **A**.

Case-2

If any scalar $n < 0$ is multiplied by vector, then the magnitude of the resultant vector will become n times and the direction will reverse.

Q # 7. What do you about Unit Vector? Describe its significance.

Ans. A vector having the unit magnitude is called the unit vector. It is used to indicate the direction of any vector. The unit vector in the direction of vector **A** is expressed as

$$\widehat{\mathrm{A}} = \frac{\mathbf{A}}{|\mathrm{A}|}$$

where $\widehat{\mathrm{A}}$ is the unit vector in the direction of vector **A** and $|\mathrm{A}|$ is its magnitude. In space, the direction of x, y and z-axis are represented by unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$, respectively.

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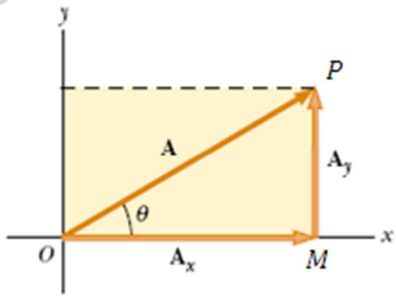
Q # 8. Find out the rectangular component of a vector.

Ans. Consider a vector **A**, represented by a line $\overline{OP}$ which makes an angle θ with the x-axis.

We want to find out rectangular components of vector **A**. For this, we draw a perpendicular from point 'P' on x-axis. Projection $\overline{OM}$ being along x-direction is represented by $A_x\hat{\imath}$ and projection $\overline{MP}$ along y -direction represented by $A_y\hat{\jmath}$. By head to tail rule:



$$\mathbf{A} = A_x\hat{\imath} + A_y\hat{\jmath} \quad \text{---------------} \quad \textbf{(1)}$$

For x component

$$cos\theta = \frac{\overline{OM}}{\overline{OP}} = \frac{A_x}{A}$$

$$\Longrightarrow A_x = Acos\theta$$

For y component

$$sin\theta = \frac{\overline{PM}}{\overline{OP}} = \frac{A_y}{A}$$

$$\Longrightarrow A_y = Asin\theta$$

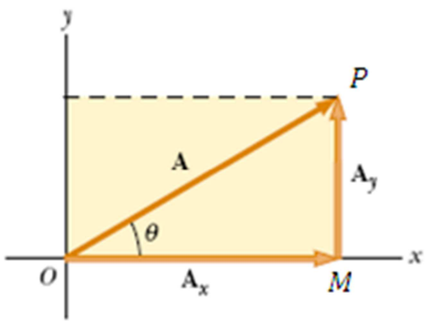
Putting values of A_x and A_y in eq. (1), we get

$$\mathbf{A} = Acos\theta\ \hat{\imath} + Asin\theta\ \hat{\jmath}$$

Q # 9. Determine a vector from its rectangular component.

Ans. Let A_x and A_y are the rectangular components of vector **A** which is represented by a line $\overline{OP}$ as shown in the figure below

We want to determine the magnitude and direction of vector **A** with x-axis.



Magnitude

The magnitude of vector **A** can be find using Pythagorean Theorem. In triangle OMP

$$(\overline{OP}\,)^2 = (\overline{OM}\,)^2 + (\overline{MP})^2$$

$$A^2 = A_x{}^2 + A_y{}^2$$

$$A = \sqrt{A_x{}^2 + A_y{}^2}$$

This expression gives the magnitude of resultant

Direction

In right angle triangle OMP

$$tan\theta = \frac{\overline{MP}}{\overline{OM}}$$

$$tan\theta = \frac{A_y}{A_x}$$

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$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

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This expression gives the direction of the vector A with respect to x-axis.

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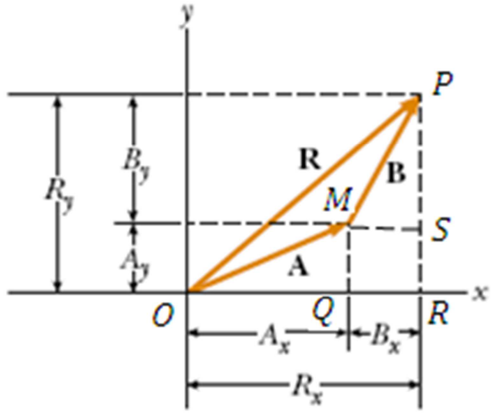
Q # 10. Describe the vector addition in terms of rectangular components.

Ans. Consider two vectors **A** and **B** represented by lines $\overline{OM}$ and $\overline{MP}$, respectively . By head to tail rule, the resultant vector is given by



$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

Let R_x and R_y are the rectangular components of resultant vector **R** along x and y-axis respectively, then we can write

$$\mathbf{R} = R_x\,\hat{\imath} + R_y\,\hat{\jmath} \qquad \text{-----------} \quad \textbf{(1)}$$

From figure,

$$\overline{OR} = \overline{OQ} + \overline{QR}$$

$$R_x = A_x + B_x \qquad \text{----------} \quad \textbf{(2)}$$

Also,

$$\overline{RP} = \overline{RS} + \overline{SP}$$

$$R_y = A_y + B_y \qquad \text{----------} \quad \textbf{(3)}$$

Putting values of R_x and R_y in eq. (1), we get

$$\mathbf{R} = R_x\,\hat{\imath} + R_y\,\hat{\jmath}$$

$$\mathbf{R} = (A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath}$$

Which is the expression of resultant in terms of rectangular components.

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Magnitude of Resultant

The magnitude of resultant can be expressed as

$$\mathrm{R} = \sqrt{{R_x}^2 + {R_y}^2}$$

Putting the values of $R_x$and R_y,

$$\mathrm{R} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Direction

The direction of resultant can be find out using expression,

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

Generalization

If **R** is the resultant vector of a large number of coplanar vectors represented by $A, B, C, \dots\dots$, then the expression for the magnitude of resultant will become

$$\mathrm{R} = \sqrt{(A_x + B_x + C_x + \cdots)^2 + (A_y + B_y + C_y + \cdots)^2}$$

The direction of resultant vector **R** with x-axis can be find out using expression

$$\theta = \tan^{-1}\left(\frac{A_y + B_y + C_y + \cdots}{A_x + B_x + C_x + \cdots}\right)$$

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Q # 11. Differentiate among scalar and vector product.

Scalar Product	Vectors Product
(i) When two vectors are multiplied to give a scalar quantity, then the product of vectors is called the scalar or dot product. The scalar product of two vectors **A** and **B** is written as **A. B** and is defined as $$\mathbf{A}.\mathbf{B} = AB\cos\theta$$ where A and B are the magnitudes of vector **A** and **B** and θ is the angle between them.	(i) When two vectors are multiplied to give a vector quantity, then the product of vectors is called the vector or cross product. The vector product of two vectors **A** and **B** is written as **A × B** and is defined as $$\mathbf{A}\times\mathbf{B} = AB\sin\theta\ \hat{\mathbf{n}}$$ where A and B are the magnitudes of vector **A** and **B** and θ is the angle between them and $\hat{\mathbf{n}}$ is the unit vector perpendicular to the plane containing **A** and **B**.
(ii) The work done W is the dot product of force **F** and displacement **d** is an example of scalar product. Mathematically, it is written as $$W = \mathbf{F}.\mathbf{d} = Fd\cos\theta$$	(ii) The turning effect of force is called the torque and is determined from the vector product of force **F** and position vector **r.** Mathematically, it is written as Torque $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$

Q # 12. Show that the scalar product is commutative.

Consider two vectors **A** and **B**. Place the both vector tail to tail as shown in Fig. (a)

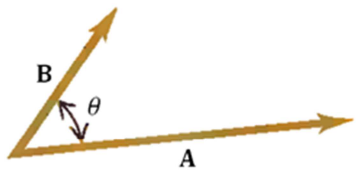

Fig. (a)

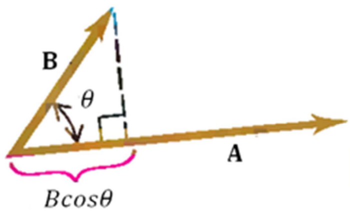

Fig. (b)

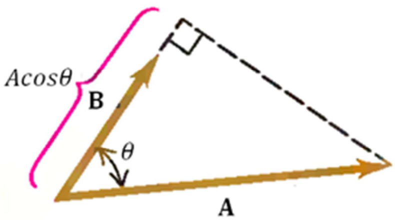

Fig. (c)

Then, from Fig. (b)

$$\mathbf{A}.\mathbf{B} = (Magnitude\ of\ \mathbf{A})(Projection\ of\ \mathbf{B}\ on\ \mathbf{A})$$

OR

$$\mathbf{A}.\mathbf{B} = (Magnitude\ of\ \mathbf{A})Componet\ of\ \mathbf{B}\ in\ the\ direction\ of\ \mathbf{A})$$

$$\mathbf{A}.\mathbf{B} = (\ A\ )(B\cos\theta) = AB\cos\theta \qquad \text{----------------------} \qquad \mathbf{(1)}$$

Similarly, from Fig. (c)

$$\mathbf{B}.\mathbf{A} = (\ B\ )(A\cos\theta) = BA\cos\theta = AB\cos\theta \qquad \text{----------------------} \qquad \mathbf{(2)}$$

Thus, from eq. (1) and (2)

$$\mathbf{A}.\mathbf{B} = \mathbf{B}.\mathbf{A}$$

Hence, the scalar product is commutative.

Q # 13. Show that the vector product is non-commutative.

Consider two vectors **A** and **B**. Place the both vector **A** and **B** tail to tail to define the plane of **A** and **B.** The direction of the vector product will be perpendicular to the plane of **A** and **B** and can be determined using right hand rule**.**

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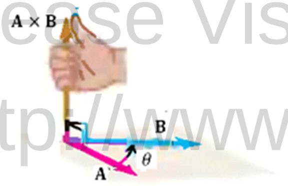

Fig. (a)

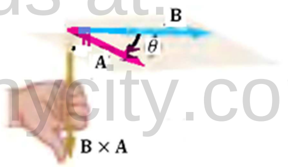

Fig. (b)

By applying the right hand rule on the vector products of $\mathbf{A}\times\mathbf{B}$ and $\mathbf{B}\times\mathbf{A}$ [Fig.(a) and Fig. (b)], **it** is clear that product vectors $\mathbf{A}\times\mathbf{B}$ and $\mathbf{B}\times\mathbf{A}$ are anti-parallel to each other i.e.,

$$\boldsymbol{A}\times\boldsymbol{B} \neq \boldsymbol{B}\times\boldsymbol{A}$$

Hence, the vector product is not commutative.

Q # 14. Compare the main characteristics of scalar and vector product.

Scalar Product	Vector Product
(i) Scalar product is commutative. That is, for vectors **A** and **B**, $\mathbf{A.B} = \mathbf{B.A}$	(i) Vector product is non-commutative. That is, for vectors **A** and **B**, $\boldsymbol{A}\times\boldsymbol{B} \neq \boldsymbol{B}\times\boldsymbol{A}$
(ii) Scalar product of two mutually perpendicular vectors is zero. ➢ If the two vectors are **A** and **B** mutually perpendicular to each other, then $\mathbf{A.B} = ABcos\ 90^\circ = 0$	(ii) Vector product of two mutually perpendicular vectors has maximum magnitude. ➢ If the two vectors are **A** and **B** mutually perpendicular to each other, then $\mathbf{A}\times\mathbf{B} = ABsin\ 90^\circ\ \hat{\mathbf{n}} = AB\ \hat{\mathbf{n}}$
(iii) The scalar product of two parallel vectors is equal to the product of their magnitudes. ➢ If the two vectors are **A** and **B** parallel to each other, then $\mathbf{A.B} = ABcos\ 0^\circ = AB$ ➢ If the two vectors are **A** and **B** anti-parallel to each other, then $\mathbf{A.B} = ABcos\ 180^\circ = -AB$	(iii) The vector product of two parallel and anti-parallel vectors is the null vector. ➢ If the two vectors are **A** and **B** parallel to each other, then $\mathbf{A}\times\mathbf{B} = ABsin\ 0^\circ\ \hat{\mathbf{n}} = \mathbf{O}$ ➢ If the two vectors are **A** and **B** anti-parallel to each other, then $\mathbf{A}\times\mathbf{B} = ABsin\ 180^\circ\ \hat{\mathbf{n}} = \mathbf{O}$
(iv) The self scalar product of vector **A** is equal to the square of its magnitudes. $\mathbf{A.A} = AAcos\ 0^\circ = A^2$	(iv) The self vector product of vector **A** is the null vector. $\mathbf{A}\times\mathbf{A} = AAsin\ 0^\circ\ \hat{\mathbf{n}} = \mathbf{O}$
(v) Scalar product of vectors **A** and **B** in terms of their rectangular components will be $\mathbf{A.B} = (A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}).(B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k})$ $\mathbf{A.B} = A_x\,B_x + A_yB_y + A_zB_z$	(v) Vector product of vectors **A** and **B** in terms of their rectangular components will be $\mathbf{A}\times\mathbf{B} = (A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}}) \times (B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}})$ $\mathbf{A}\times\mathbf{B} = (A_y\,B_z - A_zB_y)\,\hat{\mathbf{i}} + (A_z\,B_x - A_xB_z)\ \hat{\mathbf{j}} + (A_x\,B_y - A_yB_x)\,\hat{\mathbf{k}}$ $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
(vi) The angle between these vector can be find out by putting the value of $\mathbf{A.B}$ in above equation $\mathbf{A.B} = ABcos\theta = A_x\,B_x + A_yB_y + A_zB_z$ $cos\theta = \dfrac{A_x\,B_x + A_yB_y + A_zB_z}{AB}$	(vi) The magnitude of $\mathbf{A}\times\mathbf{B}$ is equal to the area of parallelogram formed with **A** and **B** as two adjacent sides

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Q # 15. Describe the right hand rule.

According to right hand rule, the right hand is placed on the first vector and fingers are curled towards the second vector, keeping the thumb erect. The erected thumb will give the direction of product vector.

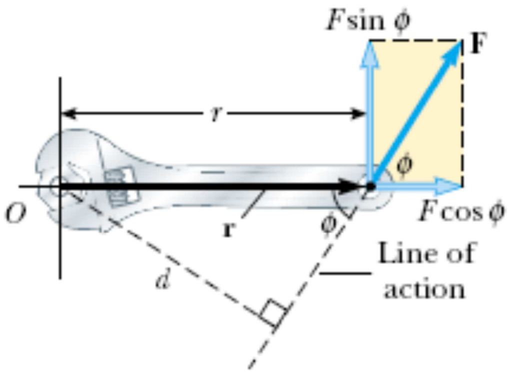
Q # 16. Define the term 'torque'.

Ans. The turning effect of a force is called torque. The torque '$\boldsymbol{\tau}$' acting on a body under the action of force '**F**' is described as

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Where **r** is the position vector of point of application of force with respect to pivot point 'O'. Anticlockwise torque is taken as positive, while the clockwise torque is considered as negative.

Q # 17. Derive the expression for torque produce in a rigid body under action of any force.



Ans. Let the force '**F**' acts on rigid body at point whose position vector relative to pivot 'O' is **r**.

We want to find out the expression torque about point 'O' acting on the rigid body due to force '**F**'.

The force '**F**' makes an angle 'Φ' with horizontal, therefore, it can be resolved in two rectangular components i.e., '$F\cos\Phi$' and '$F\sin\Phi$'. The torque due to '$F\cos\Phi$' about point 'O' is zero as its line of action posses through this point. Therefore, the '$F\sin\Phi$' component of forces is responsible for producing torque in the body about point 'O'.

Now the torque,

$$\tau = (\text{Force})(\text{Moment Arm})$$

$$\tau = (F\sin\Phi)(r)$$

$$\tau = rF\sin\Phi$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

This is the required expression of torque.

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Q # 18. Define the term "equilibrium". Write down different types of equilibrium.

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Ans. A body is said to be in state of equilibrium if it is at rest or moving with uniform velocity. There are two types of equilibrium.

(i) Static Equilibrium

If a body is at rest, then it is said to be in static equilibrium.

(ii) Dynamic Equilibrium

If the body is moving with uniform velocity, then it is said to be in dynamic equilibrium.

Q # 19. Write down different conditions of equilibrium.

Ans. There are two conditions of equilibrium.

First Condition of Equilibrium

The vector sum of all forces acting on any object must be zero. Mathematically,

$$\sum \mathbf{F} = 0$$

In case of coplanar forces, this conditions is expressed usually in terms of x and y components of forces. Hence, the 1st condition of equilibrium for coplanar forces will be

$$\sum \mathbf{F_x} = 0, \sum \mathbf{F_y} = 0$$

When the first condition of equilibrium is satisfied, there will be no linear acceleration and body will be in translational equilibrium.

Second Condition of Equilibrium

The vector sum of all torque acting on any object must be zero. Mathematically,

$$\sum \boldsymbol{\tau} = 0$$

When the second condition of equilibrium is satisfied, there is no angular acceleration and body will be in rotational equilibrium.

Q # 20. State the complete requirement for a body to be in equilibrium?

Ans. A body will be in the state of complete equilibrium, when the sum of all the forces and torques acting on the body will be equal to zero. Mathematically, it is described as

(i) $\sum \mathbf{F} = 0$ i.e. $\sum \mathbf{F_x} = 0, \sum \mathbf{F_y} = 0$

(ii) $\sum \boldsymbol{\tau} = 0$

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EXERCISE (SHORT QUESTIONS)

Q # 1. Define the terms (i) Unit Vector (ii) Position Vector (iii) Component of a Vector.

(viii) Unit Vector

A vector having the unit magnitude is called the unit vector. It is used to indicate the direction of any vector. The unit vector in the direction of vector **A** is expressed as

$$\hat{\mathrm{A}} = \frac{\mathbf{A}}{|\mathrm{A}|}$$

where $\hat{\mathrm{A}}$ is the unit vector in the direction of vector **A** and $|\mathrm{A}|$ is its magnitude.

(ix) Position Vector

The position vector describes the location of a point with respect to origin. In two dimension, the position vector '**r**' of point $P\ (a, b)$ is describe as

$$\mathbf{r} = a\hat{\imath} + b\hat{\jmath}$$

The magnitude of this position vector will be

$$r = \sqrt{a^2 + b^2}$$

In three dimensional Cartesian coordinate system, the position vector '**r**' of point $P\ (a, b, c)$ is describe as

$$\mathbf{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{\mathrm{k}}$$

The magnitude of this position vector will be

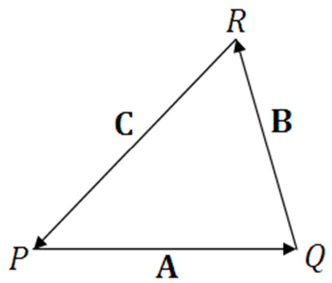
$$r = \sqrt{a^2 + b^2 + c^2}$$

(x) Component of a Vector

A component of a vector is its effective value in a specific direction.

Q # 2. The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

Ans. If the three vectors are such that they can be represented by the sides of a triangle taken in cyclic order, then the vector sum of three vectors will be zero.



Let three vectors **A**, **B** and **C** are the three vectors acting along the sides of triangle OPQ as shown in the figure.

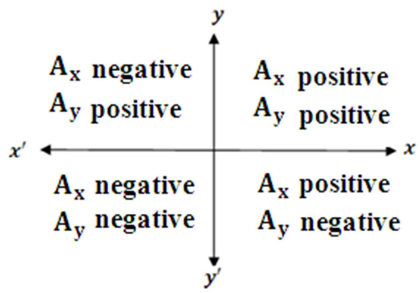
As the head of **C** coincides with the tail of **A**, so by head to tail rule, the resultant of these three vectors will be zero.

Q # 3. Vector A lies in xy plane. For what orientation will both of its rectangular components be negative? For what components will its components have opposite signs?

Ans. i) When the vector lies in 3rd quadrant, then both of its rectangular components of vector will negative.

ii) The components of a vector have opposite sign when the vector lies in 2nd or 4th quadrant.

If A_x and A_y are the rectangular components of vector **A**, then rectangular components of vectors in different quadrants will be:



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Q # 4. If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain.

Ans. If one of the components is not zero, then the magnitude of vector can't be zero. If A_x and A_y are the rectangular components of vector **A**, then its magnitude will be:

Magnitude of $\mathbf{A} = A = \sqrt{{A_x}^2 + {A_y}^2}$

If $A_x = 0$, then $A = \sqrt{0^2 + {A_y}^2} = A_y$

If $A_x = 0$, then $A = \sqrt{{A_x}^2 + 0^2} = A_x$

Q # 5. Can a vector have a component greater than the vector's magnitude?

Ans. The magnitude of the component of a vector can never be greater than the vector's magnitude because the component of a vector is its effective value in a specific direction. The maximum value of magnitude of component can be equal to the magnitude of the vector.

Q # 6. Can the magnitude of a vector have a negative value?

Ans. No, the magnitude of a vector cannot be negative, because the magnitude of vector **A** can be described by the formula:

Magnitude of **A** = $A = \sqrt{{A_x}^2 + {A_y}^2}$

Where A_x and A_y are the rectangular components of **A**. As the squares of real quantities always gives the positive values. Therefore, the magnitude of a vector will always be positive.

Q # 7. If $\mathbf{A} + \mathbf{B} = \mathbf{0}$, what can you say about the components of the two vectors.

Ans. Given that: $\mathbf{A} + \mathbf{B} = 0$

$\Rightarrow \mathbf{A} = -\mathbf{B}$

These vectors can be expressed in terms of rectangular components,

$A_x\hat{\imath} + A_y\hat{\jmath} = -(B_x\hat{\imath} + B_y\hat{\jmath})$

$A_x\hat{\imath} + A_y\hat{\jmath} = -B_x\hat{\imath} - B_y\hat{\jmath}$

Comparing the coefficients of unit vectors î and ĵ, we get:

$A_x = -B_x$ and $A_y = -B_y$

Hence the components of both vectors are equal in magnitude but opposite in direction.

Q # 8. Under what circumstances would a vector have components that are equal in magnitude?

Ans. The components of a vector will have equal magnitude when it makes an angle of 45° with x-axis. If a vector **A** makes an angle of 45°, then its rectangular components will be:

$A_x = A\cos 45^\circ = 0.707\,A$

$A_y = A\sin 45^\circ = 0.707\,A$

Q # 9. Is it possible to add a vector quantity to a vector quantity to a scalar quantity?

Ans. No it is not possible to add a vector quantity to a scalar quantity because the physical quantities of same nature can be added. Vectors and scalars are different physical quantities. It means that vectors can be added to vectors and scalars are added in scalars, but the vectors can't be added to scalar.

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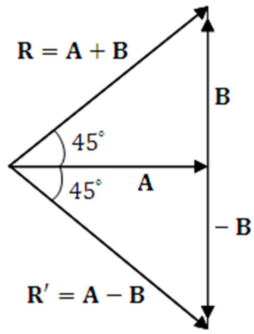
Q # 10. Can you add zero to a null vector?

Ans. No, zero can't be added to a null vector because zero is a scalar and scalars can't be added to vectors. Only the physical quantities of same nature can be added.

Q # 11. Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, the sum of two vectors having unequal magnitudes can't be zero. The sum of two vectors will be zero only when their magnitudes are equal and they act in opposite direction.

Q # 12. Show that the sum and the difference of two perpendicular vectors of equal lengths are also perpendicular and of same length.



Ans. Consider two vectors **A** and **B** of equal magnitude which are perpendicular to each other. The sum and the difference of both vectors gives the resultant **R** and **R′**, respectively, and are described below:

$\mathbf{R} = \mathbf{A} + \mathbf{B} = A\,\hat{\imath} + B\hat{\jmath}$

$\mathbf{R}' = \mathbf{A} - \mathbf{B} = A\,\hat{\imath} - B\hat{\jmath}$

Magnitude of $\mathbf{R} = R = \sqrt{A^2 + B^2}$ -------------- (1)

Magnitude of $\mathbf{R}' = R' = \sqrt{A^2 + B^2}$ -------------- (2)

From (1) and (2), it is clear that the sum and the difference of two perpendicular vectors of equal magnitude have the same lengths. Now taking dot product of **R** and **R′**, we get:

$\mathbf{R}\,.\,\mathbf{R}' = (A\,\hat{\imath} + B\hat{\jmath}).(A\,\hat{\imath} - B\hat{\jmath}) = A^2 - B^2$ $\because |\mathbf{A}| = |\mathbf{B}| \Rightarrow A = B$

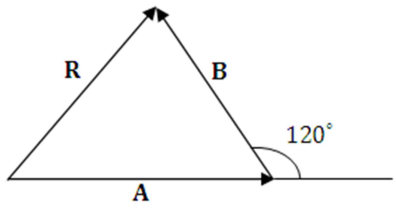
$\mathbf{R}\,.\,\mathbf{R}' = A^2 - A^2 = 0$

As $\mathbf{R}\,.\,\mathbf{R}' = 0$, therefore, the sum and the difference of two perpendicular vectors of equal magnitude are perpendicular to each other.

Q # 13. How would the two vector same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of same magnitude?

Ans. The two vectors of equal magnitudes are combined to give a resultant vector of same magnitude when they act at an angle of 120° with each other.

Consider two vectors **A** and **B** of equal magnitude which makes an angle of 120° with each other. Both vectors are added by head to tail rule to give resultant **R** as shown in the figure below:



From figure it is clear that

$\mathbf{R} = \mathbf{A} + \mathbf{B}$ and

$|\mathbf{R}| = |\mathbf{A}| = |\mathbf{B}|$

Q # 14. The two vectors to be combined have magnitude 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

(i) 100 N **(ii) 70 N** **(iii) 20 N**

Ans. The correct answer is 70 N.

1. The resultant of two vectors has maximum magnitude when they act in same direction. Thus if both vectors are parallel, then the magnitude of resultant will be: $60\ N + 35\ N = 95\ N$.

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2. The resultant of two vectors has minimum magnitude when they act in opposite direction. Thus if both vectors are anti-parallel, then the magnitude of resultant is $60\ N - 35\ N = 25\ N$.

Hence the sum can't be less than $25\ N$ and more than $95\ N$. Therefore, the only possible value for correct answer is $70\ N$.

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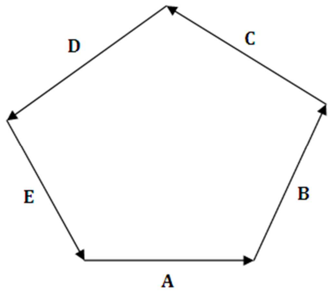
Q # 15. Suppose the sides of a closed polygon represent vector arranged head to tail. What is the sum of these vectors?

Ans. If there are five vectors A, B, C, D and E which are acting along the sides of close polygon as shown in the figure:



As the tail of the first vector meets with the head of last vector, so by head to tail rule:

$$\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

Hence the sum of vectors arranged along the sides of polygon will be zero.

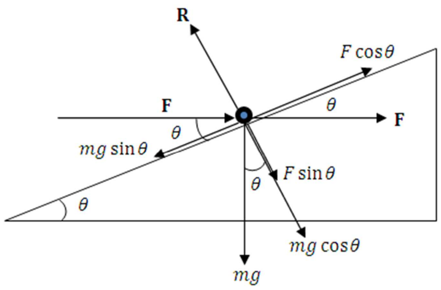
Q # 16. Identify the correct answer.

(i) Two ships X and Y are travelling in different direction at equal speeds. The actual direction of X is due to north but to an observer on Y, the apparent direction of motion X is north-east. The actual direction of motion of Y as observed from the shore will be

(A) East (B) West (C) South-east (D) South-West

Ans. The correct answer is (B) West

(ii) The horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in the figure below. The magnitude of the resultant force acting up and along the surface of the plane, on the object is



(a) $\mathbf{F\cos\theta - mg\sin\theta}$
(b) $\mathbf{F\sin\theta - mg\cos\theta}$
(c) $\mathbf{F\cos\theta + mg\sin\theta}$
(d) $\mathbf{F\sin\theta + mg\cos\theta}$
(e) $\mathbf{mg\tan\theta}$

Ans. The forces acting up and along the surface of plane is $\mathbf{F\cos\theta - mg\sin\theta}$, therefore the correct option is (a)

Q # 17. If all the components of the vectors, $\mathbf{A_1}$ and $\mathbf{A_2}$ were reversed, how would this alter $\mathbf{A_1} \times \mathbf{A_2}$.

Ans. If all the components of the vectors $\mathbf{A_1}$ and $\mathbf{A_2}$ are reversed, then both vectors will be represented as $-\mathbf{A_1}$ and $-\mathbf{A_2}$, respectively. Therefore,

$$-\mathbf{A_1} \times -\mathbf{A_2} = \mathbf{A_1} \times \mathbf{A_2}$$

Hence the vector product of two vectors will remain unchanged even when the components of the vectors are reversed.

Q # 18. Name the three different conditions that could makes $\mathbf{A_1} \times \mathbf{A_2} = 0$

Ans. The conditions that could make the $\mathbf{A_1} \times \mathbf{A_2} = 0$ are as follows:

- If $\mathbf{A_1}$ is the null vector
- If $\mathbf{A_2}$ is the null vector
- If the vectors $\mathbf{A_1}$ and $\mathbf{A_2}$ are parallel or anti-parallel with each other.

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Q # 19. Identify true or false statements and explain the reason.

(a) A body in equilibrium implies that it is neither moving nor rotating.

(b) If the coplanar forces acting on a body form a close polygon, then the body is said to be in equilibrium.

Ans. i) Statement (a) is false. Because a body may be in equilibrium if it is moving or rotating with uniform velocity.

ii) Statement (b) is correct. Since the vector sum of all the forces acting on the body along close polygon is zero, then the first condition of equilibrium will be satisfied and the body will be in state of equilibrium.

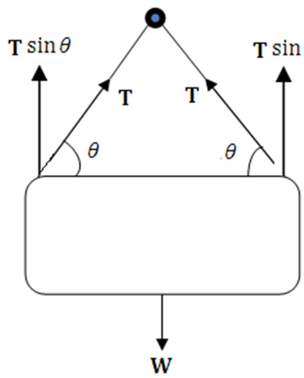
Q # 20. A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the string is minimum.

Ans. Consider a picture of weight w is suspended by two strings as shown in the figure.

From figure,

$$2T\sin\theta = W$$

$$\Rightarrow T = \frac{W}{2\sin\theta}$$



Case 1: For $\theta = 0^\circ$

$$T = \frac{W}{2\sin 0^\circ} = \frac{W}{0} = \infty$$

Case 2: For $\theta = 45^\circ$

$$T = \frac{W}{2\sin 45^\circ} = 0.7\,W$$

Case 3: For $\theta = 90^\circ$

$$T = \frac{W}{2\sin 90^\circ} = 0.5\,W$$

Hence it is clear that the tension will be minimum for $\theta = 90^\circ$

Q # 21. Can a body rotate about its center of gravity under the action of its weight?

Ans. No a body can't rotate about the center of gravity under the action of its weight. The whole weight of the body acts on the center of gravity. The torque due to weight will be zero because the moment arm is zero in this case. Hence, a body cannot rotate about center of gravity under the action of its weight.

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