CHP. 7
OSCILLATIONS.

TYPES OF MOTION:

(A) Translatory motion.

Def — "A body moves as a whole from one place to another & All of its particle move in the same direction with the same speed called translatory motion".

In this type of motion, the displacement of a body is known as linear displacement and is measured in metres (SI)

e.g: All the moving vehicles have translatory motion when moves along a straight line.

(B) Rotatory motion.

Def — "The motion of a body rotating about a fixed line (axis of rotation) is called the rotatory motion".

The displacement of a rotating body is known as angular displacement and is measured in revolutions, degrees and radians (SI)

e.g: Motion of a fly wheel.

N.B A body can simultaneously possesses translator as well as rotatory motion. For example, the wheels of all the moving vehicles have translatory as well as rotatory motion.

(C) Oscillatory or Vibratory motion.

Def — "The motion in which a body moves to and fro about a mean position is called oscillatory or vibratory motion".

The fixed point about which a body performs vibratory motion is called rest or equilibrium or mean position.

(P.T.O)

The vibratory or oscillatory motion is also called **Simple harmonic motion (SHM)** as its wave shape resembles Sine or cosine functions which are called harmonic functions. SHM is a particular case of periodic motion

- **Periodic motion.**

Def — "Such a motion which repeats itself after equal intervals of time is called periodic motion."

~~SHM may be linear or angular~~

(2) Examples of vibratory motion (OR SHM).

(i) Linear.

1 – The motion of spring-mass system in the horizontal or verticle plane (Fig a,b)

2 – A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways. (fig c)

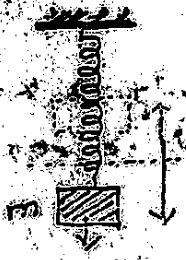


fig (a)

fig (b)

fig (c)

3 – Inward and outward motion of piston in a cylinder containing a gas when moved inward and released suddenly.

4. Motion of atoms or molecules in Solids

5. Plasma oscillations.

(ii) Angular

1 – Motion of simple pendulum about its mean position.

2 – When a steel ball is rolling in a curved dish, it oscillates about its rest position (fig (e)).

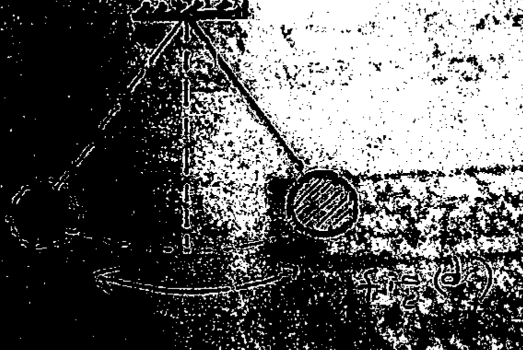


fig (d)

fig (e)

3- Motion of a magnet suspended in a uniform magnetic field.

4. Orbital motion

(b) Production of oscillations.

In order to get oscillations, a body is pulled away on one side from its rest or equilibrium position and then released. The body begins to oscillate (vibrate) due to restoring force. Under the action of this force, the body accelerates and it overshoots the rest position due to inertia. The restoring force pulls it back. As the restoring force is always directed towards the mean position and so the acc. is also directed towards the rest or mean position.

For example, a violin string produces sounds wave in air. So waves are produced due to vibrating bodies i.e; oscillations.

#7.1 SIMPLE HARMONIC MOTION

(a) Hooke's Law.

Statement — "It states that the displacement '$\vec{x}$' produced in an elastic body is directly proportional to the applied force '$\vec{F}$' provided its elastic limit is not crossed."

Mathematically,

$$\vec{F} \propto \vec{x}$$

$$\vec{F} = k\vec{x}$$

or $F = kx$ (Magnitude) — (1)

where 'K' is constant called **spring constant** for mass spring system. Its value depends upon the nature of the material of the spring. It is defined as the force per unit extension i.e; $K = \frac{F}{x}$ — (2)

The **SI unit** of spring constant is Nm^{-1}. 'K' is also known as 'stiffness factor for the spring.

(P.T.O)

(b) Elastic restoring force or return force.

Def — "When a force $\vec{F}$ is applied to an elastic body to produce a displacement $\vec{x}$ in it, an equal and opposite force acts on it due to elasticity. This force brings the body back to its mean position and is called elastic restoring force." *return force

Applied force $= F = Kx$ (Hooke's law)

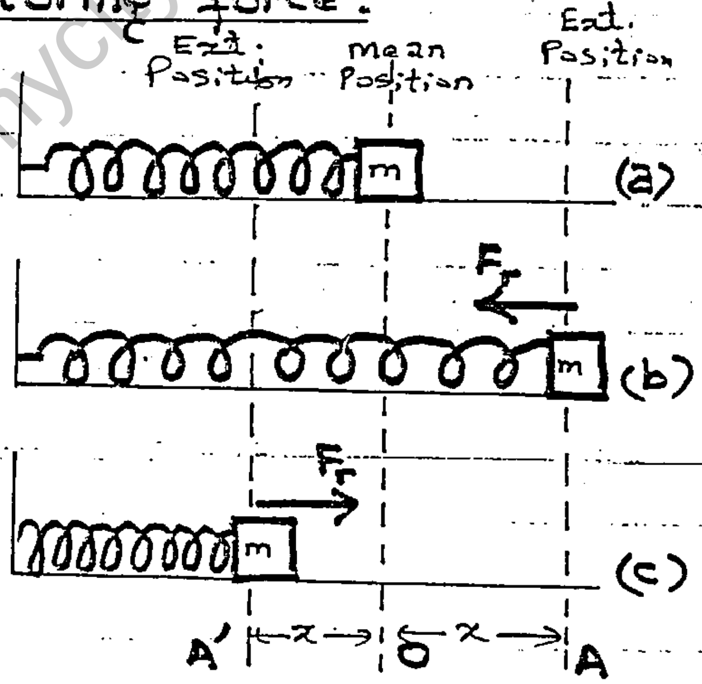
Restoring force $= F_r = -Kx$ —— (2)

Negative sign is given to this force because it is in the opposite direction to 'x'.

Applied force is also known as deforming or net or disturbing or accelerating or resultant force.

(c) Expression for acc. of a body executing SHM under elastic restoring force.

Consider a mass 'm' attached to one end of an elastic spring which can move freely on a frictionless horizontal surface as shown in fig (a). When the mass is pulled towards right through a distance 'a' from the equilibrium position by a force 'F', then according to Hook's law, applied force is directly proportional to displacement, i.e.;



fig

$$F \propto x$$

or $F = Kx$ —— (1)

Due to elasticity the spring opposes the applied force 'F' which produces the displacement 'x'. This opposing force 'F_r' which is equal and opposite to the applied force

(P.T.O)

within elastic limits of the spring :

$\therefore$ Restoring force $= F_r = -Kx$ ——— (5)

Let 'a' is the acc. produced by force 'F' in mass. spring system at any instant, then according to Newton's second laws of motion;

$$F = ma \quad \text{——— (6)}$$

comparing eq. (5) and (6), we have

$$ma = -Kx$$

$$a = -\left(\frac{K}{m}\right) \cdot x \quad \text{——— (7)}$$

$\therefore$ $\boxed{a \propto -x}$ ——— (8) $\quad \because \frac{K}{m} = \text{const}$

S.H.M. Def —— " such a type of vibratory motion in which the acc. at any instant of a body is directly proportional to the magnitude of displacement and is directed toward the mean position is called simple harmonic motion."

Examples of SHM.

1— The vibration of the string of a violin

2— The up and down motion of a loaded elastic string

3— Oscillation of simple pendulum

4— Motion of a swing

• DIFFERENT TERMS CONNECTED WITH S.H.M.

(a) Instantaneous displacement.

Def —"The distance moved by the object from its mean position on either side at any instant is known as instantaneous displacement."

It is denoted by 'x' and is a vector quantity. The **SI unit** is metre (m)

(b) Amplitude.

Def —" The magnitude of the max. displacement of a body on either side from the mean position is called amplitude."

(P.T.O)

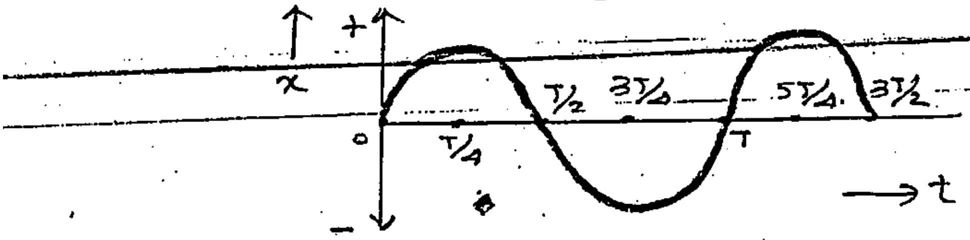
It is denoted by 'x_0'. It is a <u>scalar quantity</u>. The <u>SI unit</u> of amplitude is also metre (m).

(c) Wave form of S.H.M.

The inst. displacement of a body executing SHM is

$$x = r\sin\omega t = r\sin\left(\frac{2\pi}{T}\right)\times t \quad \text{——— (1)}$$

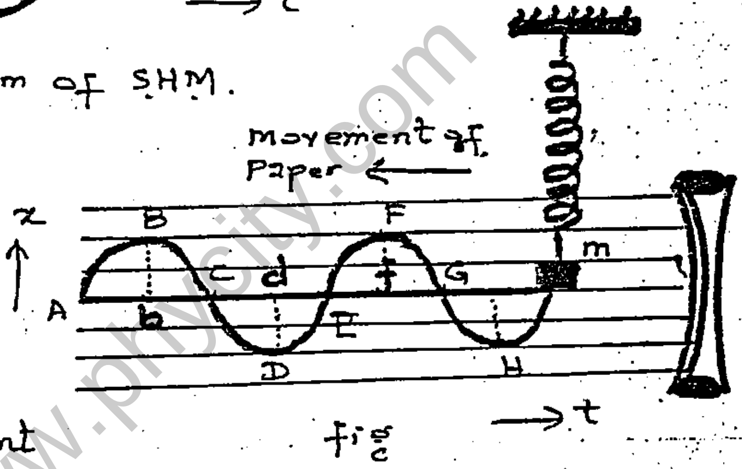
If we draw a curve between 'x' and 't', then we get a curve as shown in fig. This curve is a sin curve which



is known as wave form of SHM.

Experimental proof.

The experimental arrangement is shown in fig which can be used to record the variation in displacement with time for a mass spring system.

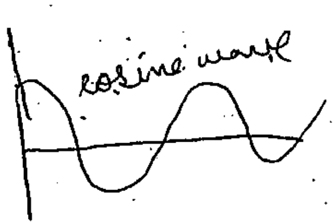


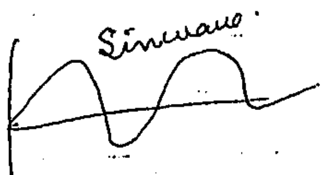
fig

A sheet of paper is placed behind the mass and there is an arrangement to move the paper at a constant speed from right to left. A time scale on the paper is shown by dotted lines. A pen is attached with the vibrating mass which lightly touches the paper. Thus, the pen records (marks) the displacement of the mass on the paper against time. The mass is raised and then released, it performs SHM and the wave form produced on the paper is very similar to <u>Sine curve</u>. It is generally as wave form of SHM.

The points 'B' and 'D' in the curve correspond to the extreme positions of the vibrating mass and points

(P.T.O)





'A', 'C' and 'E' show its mean position. Thus, the line ACE represents the mean position of the mass on the paper. The amplitude of vibration is thus a measure of the line 'Bb' or 'Dd'.

(d) Vibration.

Def — "One complete round trip of a body about its mean position is called a vibration."

The curve ABCDE in the fig correspond to the different positions of the mass during one complete vibration. i.e; The motion of a vibrating body from its one extreme position back again to the same extreme position is called one vibration. This will correspond to the portion of curve from points 'B' to 'F' or from points 'D' to 'H'.

(e) Time period.

Def — "The time required to complete one vibration is called time period."

It is denoted by 'T'. The **SI unit** of time period is second (s).

(f) Frequency.

Def — "The no. of vibrations completed by a body in one second is called frequency."

It is denoted by 'f'. The **SI unit** of frequency is hertz (Hz) or CPS or vib s^{-1}.

- Relation B/w Time period and frequency.

Time of f vibrations = 1 s

Time of 1 vibration $= \frac{1}{f}$

But, the time of one vibration = T

$$\therefore \quad T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Time period of a body performing S.H.M. is independent of its amplitude; Provided the amplitude is small. Such vibrations are called **isochronous**. SHM is a special case of periodic motion.

(P.T.O)

- Hertz (HZ).

Def — "The frequency of a body is one hertz when it completes one vibration in one second."

(g) Angular frequency.

"The frequency of a periodic circular motion is equal to '2π' times the no. of cps." It is denoted by 'ω'.

Mathematically,

$$\omega = \frac{2\pi}{T}$$

$$\therefore \omega = 2\pi f \quad \text{As } f = \frac{1}{T}$$

Angular frequency 'ω' is basically a characteristic of circular motion. From this we can find the values of instantaneous displacement and instantaneous velocity of a body executing SHM.

7.2 S.H.M. AND UNIFORM CIRCULAR MOTION:

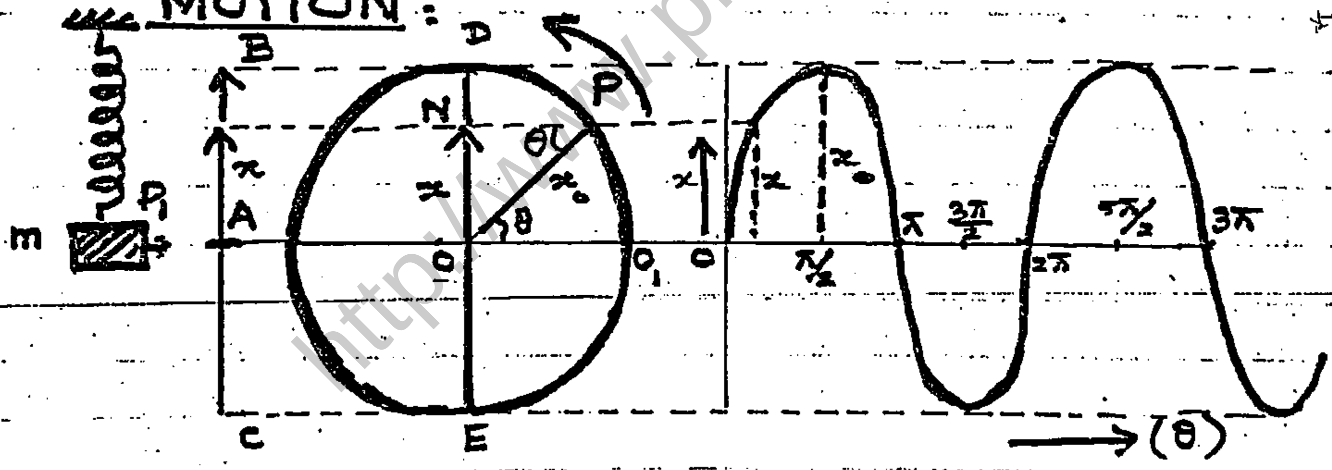


fig (a) fig (b) fig (c)

Consider a mass 'm' attached with the end of a vertically suspended spring, vibrates simple harmonically with period 'T', frequency 'f' and amplitude 'x_0'. The motion of the mass is displayed by the the pointer 'P_1' on the line 'BC' with 'A' as mean position and 'B', 'C' as

(P.T.O)

extreme position (fig (a)). Assuming 'A' as the position of the pointer at $t=0$, it will move so that it is at 'B', 'A', 'C' and back to 'A' at instants $T/4$, $T/2$, $3T/4$ and T respectively. This will complete one cycle of vibration with amplitude of vibration being $x_0 = AB = AC$.

Now consider a point 'P' moving on a circle of radius 'x_0' with a uniform angular frequency $\omega = \frac{2\pi}{T}$. Consider the motion of the point 'N', the projection of 'P' on the diameter 'DE' drawn parallel to the line of vibration of the pointer in fig (b). Note that the level of points 'D' and 'E' is the same as the points 'B' and 'C'. As 'P' describes uniform circular motion with a constant angular speed 'ω', 'N' oscillates to and fro on the diameter 'DE' with the period 'T'. Assuming 'O_1' be the position of 'P' at $t=0$, the position of the point 'N' at the instants 0, $T/4$, $T/2$, $3T/4$ and T will be at the points O, D, O, E and O respectively.

A comparison of the motion of 'N' with that of pointer 'P_1' shows that both the motions are identical. Thus the expression of displacement, velocity and acc. for the motion of 'N' also hold good for the pointer 'P_1', executing SHM.

(a) Displacement of "N".

"It is the distance of the projection 'N' at any instant from the mean position 'O'."

At this instant, the point 'P' or radius 'OP' fig (b) makes an angle $\angle O_1OP = \theta = \omega t$

but $\angle O_1OP = \angle OPN$ (Alternate angles)

P.T.O

From the right angled triangle ΔONP, we have

$$\frac{x}{x_0} = \frac{ON}{OP} = \sin\theta$$

or $$x = x_0 \sin\theta$$

$$\therefore \quad x = x_0 \sin\omega t \quad \text{--- (1)}$$

This will also be displacement of the pointer 'P_1' at the instant 't'.

• Graphical explanation

The value of 'x' as a function of 'θ' is shown in fig (c). This is the waveform of SHM. The angle 'θ' gives the states of the system in its vibrational cycle. For example, at the start of the cycle $\theta = 0$. Half way through the cycle is 180° (or π radian). When $\theta = 270^\circ$ (or $\frac{3\pi}{2}$ radians), the cycle is threefourth completed. We call 'θ' as the phase of the vibration. Thus when quarter of cycle is completed, phase of vibration is 90° (or $\frac{\pi}{2}$ radian). Thus phase is also related with circular motion which is an aspect of SHM.

(b) Instantaneous Velocity.

The linear velocity 'V_p' of a point 'P' in a uniform circular motion at any instant 't' will be directed along the tangent to the circle and its magnitude will be

$$V_p = r\omega$$

Here $\quad r = x_0$

$$\therefore \quad V_p = x_0\omega \quad \text{--- (2)}$$

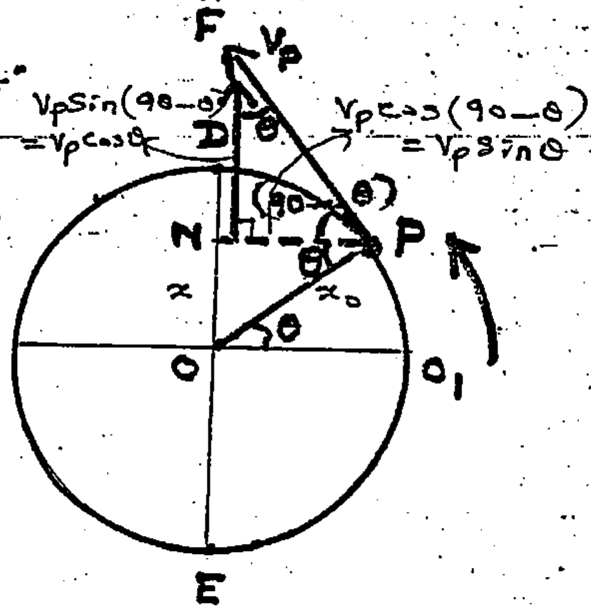


fig (d)

Velocity 'V_p' of the point 'P' on the circle is resolved into two rectangular components. fig (d)

(P.T.O)

As the motion of 'N' on the diameter 'DE' is due to motion of 'P' on the circle, the velocity of 'N' is actually the component of the velocity 'V_P' in a direction parallel to the diameter 'DE'. This component is

$$V_P \sin(90^\circ - \theta) = V_P \cos\theta = x_0 \omega \cos\theta \qquad \because V_P = x_0\omega$$

Thus, the magnitude of the velocity of 'N' or its speed is;

$$V_N = x_0 \omega \cos\theta = x_0 \omega \cos\omega t \quad \text{———} (3)$$

In ΔONP (right angled triangle),

$$\cos\theta = \cos\angle NPO = \frac{NP}{OP} = \frac{Base}{hyp}$$

$$NP = OP\cos\theta = OP\cos\omega t \quad \text{———} (4)$$

By Pythagorian theorem

$$(OP)^2 = (ON)^2 + (NP)^2$$

or $$x_0^2 = x^2 + (NP)^2 \qquad \because OP = x_0,\ ON = x$$

$$(NP)^2 = x_0^2 - x^2$$

$$NP = \sqrt{x_0^2 - x^2} \quad \text{———} (5)$$

Putting the value of 'NP' in eq. (4), we have

$$\cos\theta = \frac{\sqrt{x_0^2 - x^2}}{x_0} \quad \text{———} (6)$$

Putting this value in eq. (3), we have

$$V_N = \cancel{x_0}\,\omega \times \frac{\sqrt{x_0^2 - x^2}}{\cancel{x_0}} = \omega\sqrt{x_0^2 - x^2} \quad \text{———} (7)$$

(i) At the mean position, $x = 0$, $V_N = \omega x_0$ (max vel.)

(ii) At the extreme position, $x = x_0$, $V_N = 0$ (velocity is min)

• Direction of Velocity of 'N'.

The direction of the velocity of 'N' depends upon the value of the phase angle 'θ'. When 'θ' is between 0° to 90°, the direction is from O to D. When 'θ' is between 90° to 270°, its direction is from D to E. When 'θ' is between 270° to 360°, the direction of motion is from E to O.

As the motion of 'N' on the diameter DE is just similar to the pointer performing SHM. So the pointer 'P_1' performing SHM is given by eq. (7) in terms of 'ω'.

(P.T.O)

(C) Acceleration in terms of 'ω'.

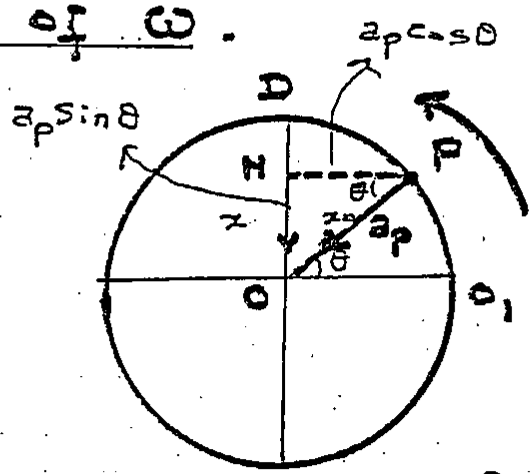


Fig (e)

When the point 'p' is moving on a circle, it has an acc.

$$a_p = \frac{v_p^2}{x_o}$$

$$\therefore\ a_p = \frac{x_o^2\omega^2}{x_o} = x_o\omega^2 \quad \text{—— (8)} \quad \because v_p = x_o\omega$$

This acc. is always directed towards the centre O of the circle.

At any instant 't' its direction will be along 'PO'.

Resolving 'a_p' into two rectangular component, we get

Horizontal component $= a_p\cos\theta$ ($\perp$ to DE)

Vertical component $= a_p\sin\theta$ (along DE)

As the point 'N' moves along the diameter DE due to motion of 'P', its acc. will be equal to the component of 'a_p' along the diameter 'DE'. Thus, the acc. of 'N' is given by;

$$a_N = -a_p\sin\theta = -x_o\omega^2\sin\theta \quad \text{—— (9)} \quad \because a_p = x_o\omega^2$$

where negative sign shows that the acc. is directed towards the mean position 'O', (i.e; directed from 'N' to 'O')

From fig (e), consider $\triangle ONP$,

$$\sin\theta = \frac{ON}{OP} = \frac{x}{x_o}$$

$$x = x_o\sin\theta \quad \text{—— (10)}$$

Putting this value in eq. (9), we have;

$$a_N = -\omega^2 x \quad \text{—— (11)}$$

$$\boxed{a_N \propto -x} \qquad \because \omega^2 = \text{constt.}$$

This shows that the projection 'N' of point 'p' performs SHM. The direction of acc. (a_N) and displacement (x) are always opposite at every instant 't'. Therefore, the motion of 'N' is just a replica of the pointer's motion.

(i) At mean position, $x = 0 \Rightarrow a_N = 0$ (Zero acc.)

(ii) At extream position, $x = x_o \Rightarrow a_N = -\omega^2 x_o$ (max. acc.)

(P.T.O)

7.3 PHASE :

Def — "The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM **Or** the angle that determines the state of motion of a point in its vibrational cycle is known as phase or phase angle."

This angle is obtained when SHM is coupled with circular motion.

Expression for displacement 'x'.

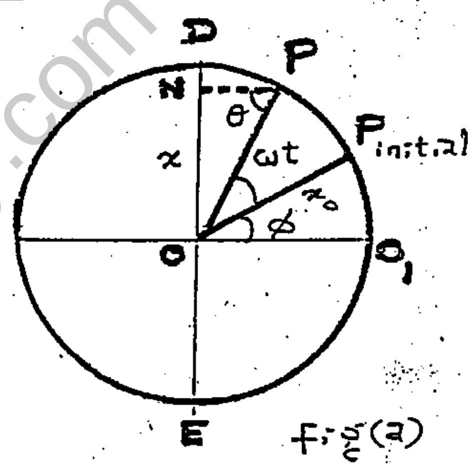


fig (a)

The displacement $x = x_o \sin\theta$ and velocity $V = x_o \omega \cos\theta$ of Projection 'N' of point 'P' executing SHM are determined by the angle $\theta = \omega t$. It is the angle which the rotating radius 'OP' makes with reference direction 'OO_1' at any instant 't'. In this **special case**, we assumed that to start with at $t = 0$, the position of rotating radius 'OP' is along 'OO_1', so that the point 'N' is at its mean position and displacement at $t = 0$, is zero.

But in general, we assume that 'OP' initially makes an angle 'φ' at $t = 0$ with the reference line 'OO_1' as shown in fig (a). In time 't', the radius will rotate from '$P_{initial}$' to 'P' by 'ωt'. Now the 'OP' would make an angle $(\omega t + \phi)$ with 'OO_1' at the instant 't'. and will have the displacement $ON = x$

From right angle triangle OPN,

$$\frac{ON}{OP} = \sin \angle OPN = \sin\theta$$

but $\angle OPN = \angle(\omega t + \phi)$ (Alternate angles)

$\because OP = r = x_o$

$$ON = OP \sin\theta$$

$$x = x_o \sin(\omega t + \phi) \quad \text{—— (1)}$$

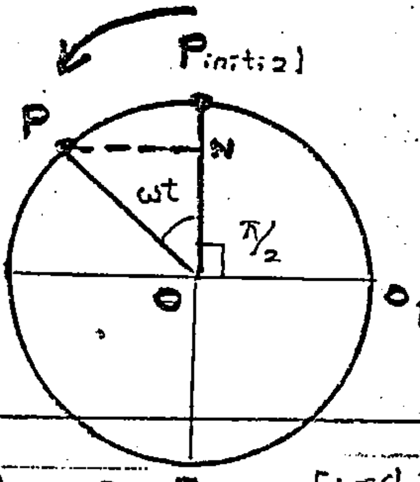
(P.T.O)

Now phase angle $= \theta = \omega t + \phi$

When $t = 0$, $\theta = \phi$

so ϕ is the initial phase.

If we take initial phase as $\pi/2$ or 90°, then displacement from eq. ① can be written as



$$x = x_0 \sin(\omega t + 90^\circ)$$

or $x = x_0 \cos \omega t$ ——— ②

$\because \sin(90 + \theta) = \cos\theta$ fig (b)

and

Phase angle $= \theta = \omega t + \pi/2$ $[\sin(A+B) = \sin A \cos B + \cos A \sin B]$

Eq. ② also gives the displacement of SHM; but in this case the point 'N' is starting its motion from extreme position instead of the mean position as shown in fig (b).

7.4 A HORIZONTAL MASS SPRING SYSTEM.

Consider a mass 'm' attached to one end of the spring whose other end is fixed with a rigid support as shown in fig. If mass is displaced towards right from its mean position by the application of force 'F' and released, thus it execute SHM under restoring force and inertia. Let 'x' be

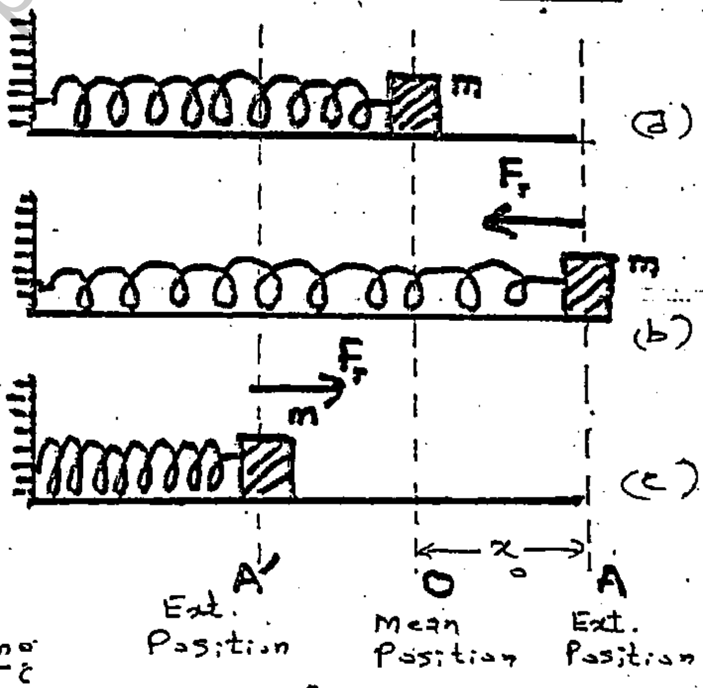


fig.

the displacement of the mass from its mean position 'O'. At this instant, the restoring force 'F_r' acting on it is

$$F_r = -Kx \quad \text{——— ①}$$

Also $$F = ma \quad \text{——— ②}$$

(P.T.O)

comparing eq. ① and ②, we have

$$ma = -Kx$$

$$a = -\frac{K}{m}x \quad \text{——— ③}$$

where 'K' and 'm' are constants.

The eq. ③ shows that acc. is directly proportional to displacement 'x' and its direction is towards the mean position. Thus the mass 'm' performs SHM between 'A' and 'A'' with 'x_o' as amplitude. It is similar to the motion of the projection 'N' of point 'P' moving in a circle, so

$$a = -\omega^2 x \quad \text{——— ④}$$

Comparing eq. ③ and ④, we get

$$-\omega^2 x = -\frac{K}{m}x$$

or $$\omega^2 = \frac{K}{m}$$

$$\omega = \sqrt{\frac{K}{m}} \quad \text{——— ⑤}$$

which is the vibrational angular frq. 'ω'.

with the dimension of

$$[\omega^2] = \left[\frac{K}{m}\right] = \left[\frac{F}{x}\cdot\frac{1}{m}\right] = \left[\frac{MLT^{-2}}{LM}\right] = [T^{-2}]$$

or $[\omega] = [T^{-1}]$

(a) Time period.

The time period of the mass executing SHM is given by

$$\theta = \omega t$$

For one vibration $\theta = 2\pi$

$$\therefore \quad 2\pi = \omega T \qquad \because t = T$$

or $$T = \frac{2\pi}{\omega} \quad \text{——— ⑥}$$

Putting the value of 'ω' from eq. ⑤ into eq. ⑥;

$$T = \frac{2\pi}{\sqrt{K/m}} = 2\pi\sqrt{\frac{m}{K}} \quad \text{——— ⑦}$$

(b) Frequency. Frequency of oscillation is given by

$$f = \frac{1}{T}$$

so

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{——— ⑧}$$

(P.T.O)

(c) Instantaneous displacement.

The inst. displacement of the mass 'm' is given by;

$$x = x_0 \sin \omega \times t \quad \text{——— (9)}$$

putting value of 'ω' from eq. (5) into eq (9), we have

$$x = x_0 \sin \sqrt{\frac{K}{m}} \times t \quad \text{——— (10)}$$

(d) Instantaneous velocity.

The inst. velocity 'V' of the point 'N' performing SHM in case of circular motion is given by;

$$V = \omega \times \sqrt{x_0^2 - x^2} \quad \text{——— (11)}$$

Putting the value of 'ω' from eq. (5) into eq. (11), we have

$$V = \sqrt{\frac{K}{m}} \times \sqrt{x_0^2 - x^2} \quad \text{——— (12)}$$

or
$$V = \sqrt{\frac{K}{m} \times x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)}$$

or
$$V = x_0 \sqrt{\frac{K}{m} \times \left(1 - \frac{x^2}{x_0^2}\right)} \quad \text{——— (13)}$$

(i) At the mean position, $x = 0$, $V_0 = x_0 \sqrt{\frac{K}{m}}$ ('V_0' max vel.)

(ii) At the extreame position, $x = x_0$, $V = 0$ (Zero vel.)

- Relation B/w max. vel. and inst. vel.

putting the value of $V_0 = x_0 \times \sqrt{\frac{K}{m}}$ in eq (13), we have

$$V = V_0 \sqrt{\left(1 - \frac{x^2}{x_0^2}\right)} \quad \text{——— (14)}$$

EXAMPLE 7.1: A block weighing 4 Kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.5 Kg body is hung from the same spring. If the spring is now stretched and then released, what is the period of vibration?

DATA. Mass of the block $= m_1 = 4 Kg$

Length of the stretched spring $= x = 0.16 m$

Mass of the body $= m_2 = 0.5 Kg$

Period of vibration $= T = ?$

(P.T.O)

Sol. By Hook's law;

$$F = Kx$$

or $K = \frac{F}{x} = \frac{m_1 g}{x}$ $\because F = W = mg$

$$K = \frac{4 \times 9.8}{0.16} = 245\ N\ m^{-1} \quad \text{—— (1)}$$

We also know that;

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{0.5}{245}} = \boxed{0.28\ S}$$

7.5 SIMPLE PENDULUM:

- **Construction**.

"A simple pendulum consists of a small heavy metallic bob suspended from frictionless support by a light and inextensible string fixed at its upper end in a uniform gravitational field".

The distance between the point of suspension and the centre of the bob is called the length of the pendulum denoted by l.

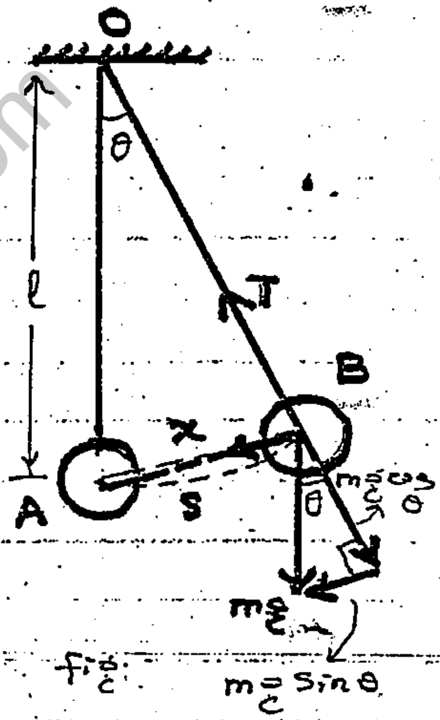


fig.

When such a pendulum is displaced from its mean position through a small angle 'θ' to the position 'B' and released, it starts oscillating to and fro over the same path.

An ideal pendulum cannot be practically realized.

- **Proof for Simple pendulum to execute SHM.**

Let the bob of the simple pendulum be displaced from its mean position 'A' to the position 'B' as shown in fig. The forces acting on the bob in this position are:

(i) Weight 'mg' of the bob acting vertically down and

(ii) Tension 'T' of the string along the direction 'BO'.

(P.T.O)

The weight 'mg' can be resolved into two components, one along the string in the direction 'OB' and other perpendicular to it along the tangent at 'B'.

- Component of mg along the string $= mg\cos\theta$
- Component of mg perpendicular to the string $= mg\sin\theta$

As there is no motion along the string $mg\cos\theta$ balances 'T', i.e;

$$T = mg\cos\theta \quad \text{---} \; (1)$$

The only force responsible for bringing the bob back to its mean position 'A' is $mg\sin\theta$. Therefore it represents the opposing restoring force 'F', Thus,

$$F = -mg\sin\theta \quad \text{---} \; (2)$$

Negative sign shows that the motion is towards mean position.

When the angle 'θ' is small and measured in radians;

$$\sin\theta \simeq \theta \text{ (rad)}$$

$\therefore$ eq. (2) becomes

$$F = -mg\theta \quad \text{---} \; (3)$$

By Newton's second law $\qquad \because F = ma$

$$\cancel{m}a = -\cancel{m}g\theta$$

$$a = -g\theta \quad \text{---} \; (4)$$

But $\quad \theta = \dfrac{\text{Arc AB}}{l} \qquad \left(\because s = r\theta \Rightarrow \theta = \dfrac{s}{r}\right)$

When 'θ' is small, $s = \text{arc AB} = x$

Hence $\quad \theta = \dfrac{x}{l} \quad \text{---} \; (5)$

Putting this value in eq. (4), we have

$$a = -\frac{gx}{l} \quad \text{---} \; (6)$$

$$\therefore \boxed{a \propto -x} \quad \text{---} \; (7) \qquad \because g/l = \text{constt}$$

Eq. (6) shows that acc. of the bob is directly proportional to the displacement 'x' and negative sign shows that it is directed towards the mean position. Thus, the motion of the simple pendulum is **SHM**.

(P.T.O)

• Time period of simple pendulum.

The acc. of the body executing SHM (i.e; projection of a point) is;

$$a = -\omega^2 x \quad \text{——} (8)$$

comparing eq. (7) and (8), we have

$$-\omega^2 x = -\frac{g}{l} x$$

$$\text{or} \quad \omega^2 = g/l$$

$$\text{or} \quad \omega = \sqrt{g/l} \quad \text{——} (9)$$

The time period of a body executing SHM is given by

$$T = \frac{2\pi}{\omega} \quad \text{——} (10)$$

Putting the value of 'ω' from eq (9) in eq (10), we have

$$T = \frac{2\pi}{\sqrt{g/l}} = 2\pi\sqrt{l/g} \quad \text{——} (11)$$

(Graph b/w T and $\sqrt{l}$ is a straight line)

Eq (11) shows that the time period of simple pendulum depends only on the length of the pendulum and acc. due to gravity. It is independent of the mass of bob attached.

• Second's Pendulum.

Def — " A second pendulum is a pendulum which completes one vibration in two seconds."

As time period of second's pendulum is 2s. Therefore, its **frequency** will be;

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{g/l} = \frac{1}{2} = 0.5 \text{ Hz} \quad (\text{Linear frq.})$$

$$\text{and} \quad \omega = 2\pi f = 2 \times 3.14 \times 0.5 = \pi \text{ rad/s} \quad (\text{Angular frq.})$$

It strikes as many times ~~as~~ in a day as there are second's in it.

• Use of simple pendulum.

Simple pendulum provides an accurate method for the determination of 'g' in the laboratory if we neglect biuoancy of air because both 'T' and 'l' can be directly measured.

(P.T.O)

EXAMPLE: What would be the length of a simple pendulum whose period is 1 s at a place where $g = 9.8\ m\,s^{-2}$? What is the frequency of such a pendulum?

DATA. Time period $= T = 1\ s$

Acc. due to gravity $= g = 9.8\ m\,s^{-1}$

Length of the pendulum $= l = ?$

Frequency of the pendulum $= f = ?$

Sol. (i) $T = 2\pi\sqrt{l/g}$

or $l = \dfrac{T^2 g}{4\pi^2}$

$= \dfrac{(1.0)^2 \times 9.8}{4 \times (3.14)^2} = \boxed{0.25\ m}$

(ii) $f = \dfrac{1}{T} = \dfrac{1}{1.0} = \boxed{1\ Hz}$

7.6 ENERGY CONSERVATION IN SHM:

(a)(i) **Max. P.E.** Consider the case of a vibrating mass spring system. When the mass is pulled through some distance 'x_0' against the elastic restoring force 'F'. It is assumed that stretching is done slowly so that acc. is zero, because change in velocity will be very small. According to Hook's law;

$$F = Kx_0$$

When displacement $= 0$ force $= F_i = 0$ (At mean)

When displacement $= x_0$ force $= F_f = Kx_0$ (At ext)

Thus; Average force $= F = \dfrac{F_i + F_f}{2} = \dfrac{0 + Kx_0}{2} = \dfrac{1}{2}Kx_0$

Work done in displacing the mass 'm' through 'x_0' is given by;

$$W = Fd = \frac{1}{2}Kx_0 \cdot x_0 = \frac{1}{2}Kx_0^2$$

This work appears as elastic P.E of the spring. Hence

$$(P.E)_{max.} = \frac{1}{2}Kx_0^2 \quad \text{———} (1)$$

(max. P.E at the ext. Position)

(ii) **P.E at any instant.**

At any instant 't', if the displacement is 'x', then P.E at any instant is given by

$$P.E = \frac{1}{2}Kx^2 \quad \text{———} (2)$$

(P.T.O)

(iii) Minimum P.E.

P.E is min. if the displacement, $x=0$ i.e; when the mass 'm' is at the mean position. Thus

$$(P.E)_{min} = \frac{1}{2} K(0)^2 = 0 \quad \text{——— (3)}$$

(b) (i) K.E at any instant.

The K.E of the mass at the instant 't' is given by;

$$K.E = \frac{1}{2} mv^2 \quad \text{——— (4)}$$

As we know the velocity of the mass attached to the spring at that instant is;

$$v = x_0\sqrt{\frac{K}{m}\left(1-\frac{x^2}{x_0^2}\right)} \quad \text{——— (5)}$$

Putting this value in eq. (4), we have

$$K.E = \frac{1}{2}\cancel{m}\left[x_0^2\left(\frac{K}{\cancel{m}}\right)\left(1-\frac{x^2}{x_0^2}\right)\right]$$

or

$$K.E = \frac{1}{2} K x_0^2 \left(1-\frac{x^2}{x_0^2}\right) \quad \text{——— (6)}$$

(ii) Max. K.E.

The K.E of the mass will be max. at the mean position i.e; when $x=0$, putting in eq. (6), we have

$$(K.E)_{max} = \frac{1}{2} K x_0^2 \quad \text{——— (7)}$$

(iii) Min. K.E.

The K.E is min when 'm' is at the extreme position 'A' or 'A′' when $x = x_0$, Putting in eq. (6), we have

$$(K.E)_{min} = \frac{1}{2} K x_0^2 \left(1-\frac{x_0^2}{x_0^2}\right) = \frac{1}{2} K x_0^2 (0) = 0 \quad \text{——— (8)}$$

(c) Total energy.

At any displacement 'x', the energy is partly P.E and partly K.E, So total energy is given by;

$$E_{total} = P.E + K.E$$

$$= \frac{1}{2} K x^2 + \frac{1}{2} K x_0^2 \left(1-\frac{x^2}{x_0^2}\right)$$

$$= \cancel{\frac{1}{2} K x^2} + \frac{1}{2} K x_0^2 - \cancel{\frac{1}{2} K x^2}$$

$$E_{total} = \frac{1}{2} K x_0^2 \quad \text{——— (9)}$$

(P.T.O)

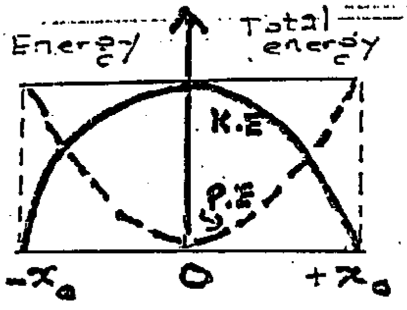


fig (a)

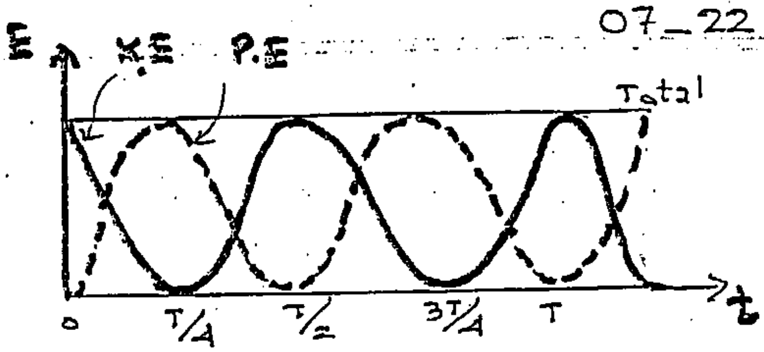


fig (b)

(d) Law of conservation of energy in SHM.

"The total energy of the vibrating mass and spring remains constant at every instant in its path. This is called as the law of conservation of energy in SHM."

Explanation.

When the mass is at mean position, its K.E is max. but the P.E of the spring is zero. When the mass is at extreme position either 'A' or 'A'', P.E of the spring is max, but the K.E of the mass is zero. In other words, it is wholly P.E at at the extreme positions and wholly K.E at the mean position. In any other position, it is partly potential and partly kinetic but the total energy remains the same.

The interchange of energy occurs continuously from one form to the other as the spring is stretched and compressed alternately. The variation of P.E and K.E with the displacement 'x' is essential for maintaining oscillations. The periodic interchange of energy is a basic property of all oscillating systems.

(e) Exchange of P.E and K.E in Simple pendulum.

In the case of a simple pendulum, when mass moves from top position to its mean position, its gravitational P.E is converted into K.E. Similarly, this K.E is converted into P.E as the bob rises to the top position. The energy is dissipated due to frictional force such as air friction and consequently the system does not oscillate indefinitely.

(P.T.O)

EXAMPLE 7-3. A spring, whose spring constt. is $80.0\ N m^{-1}$ vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with a velocity of $1.0\ m s^{-1}$.

DATA. Spring constant $= K = 80\ N m^{-1}$

Mass $= m = 1.0$ kg

Velocity of mass $= V = 1.0\ m s^{-1}$

Distance by which mass is pulled down $= x_0 = ?$

Sol. Using the formula;

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{80}{1}} = 8.94\ s^{-1} \quad \text{—— (1)}$$

Then using the formula;

$$x_0 = \frac{V}{\omega} \qquad \because V = x_0 \omega$$

$$x_0 = \frac{1.0}{8.94} = \boxed{0.11\ m}$$

7-7 FREE AND FORCED OSCILLATIONS (OR VIBRATIONS):

(a) Free Oscillations.

Def — "A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force."

Example. A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum, when it is slightly displaced from its mean position.

(b) Forced Oscillations.

Def — "A body is said to be executing forced vibrations, if it is subjected to an external force"

A physical system undergoing forced vibrations is known as driven harmonic oscillator.

(P.T.O)

Examples.

1 — If the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

2 — The vibrations of a factory floor caused by the running of heavy machinery is an example of forced vibration.

3 — All string instruments produce their loud music due to the forced vibrations of the wooden boards on which they are mounted.

4 — When the prones of a vibrating tuning fork are held over the open end of a resonance tube, forced vibrati are produced in the enclosed air column.

7.8 RESONANCE.

Def — "The marked increase in amplitude of a vibrating body under the periodic force whose period is equal to the natural period is called resonance."

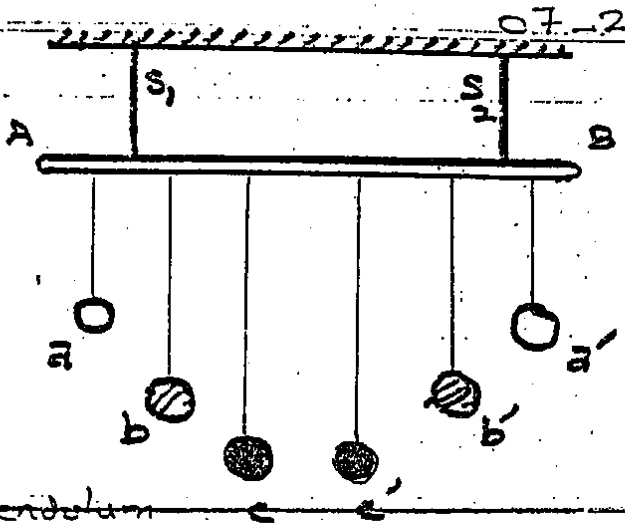
i.e; Resonance occurs when the frequency of the applied force is equal to one of the natural frequencie of vibration of the forced or driven harmonic oscillat

The Tacoma Narrous bridge disaster of 1939 was caused b the bridge being too slender for the wind conditions in the valley. A more tragic accident took place in Angers, France in 1850 when soldiers marching o a bridge caused sufficient vibration to break the bridge and over 200 of them were killed.

Experiment to demonstrate resonance.

Consider a horizontal rod AB is suppor by two strings S_1 and S_2 as shown in fig.

(P.T.O)



Three pairs of Pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs.

~~If one of these pendulum,~~ say 'c', is displaced in a direction perpendicular to the plane of the paper, a small periodic force acts on all the pendulums through the rod AB. The resultant oscillatory motion causes in rod AB, a very slight disturbing motion, whose period is the same as that of 'c'. Due to this slight motion of the rod, each of the remaining pendulums (aa', bb' and cc') undergo a slight periodic motion. This causes the pendulum c', whose length and, hence, period is exactly the same as that of 'c', to oscillate back and forth with steadily increasing amplitude. However, the amplitude of the other pendulums remain small through out the subsequent motion of 'c' and 'c'', because their natural periods are not the same as that of the disturbing force due to rod AB.

Resonant frequency.

Def — "The particular frq. which results in the max. amplitude of vibration is called the resonant frq."

Natural frequency.

Def — "When a body is disturbed from its equilibrium position with a specific frequency known as the natural frq. of the body."

Natural time Period.

Def — "when a body vibrates with a natural frq. then its period of vibration is called natural time period."

(P.T.O)

will also take place if the period of the applied force is any integral multiple of the natural period of the body

Applications of resonance.

1— Resonance can be used to determine the frq. of a given body. A second body, the natural frq. of which is known, is made to act on the given body. If it produces resonance, it is concluded that the given body has the same frq. as the second body.

2— It is used to find natural frequencies of the different bodies.

3— It is used to determine the speed of sound with resonance tube apparatus.

4— Mechanical and electrical system show a good response under phenomenon of resonance.

Examples of resonance.

1— Swing.

A swing is a good example of mechanical resonance. It is a pendulum with a single natural frq. depending on its length. In the swing, if pushes are given at the correct intervals, which coincide with the period of the swing, the amplitude of the swing can be made quite large. If the pushes are given irregularly, the swing will hardly vibrate.

2— March of Soldiers on the bridge.

If there is a big span of bridge (or a suspension bridge), then the columns of soldiers crossing the bridge are ordered to break their steps. Because if the frq. of their steps coincides with the natural frq. of the bridge, the bridge may be set into vibrations of large amplitude. Thus, the bridge may collapse due to resonance.

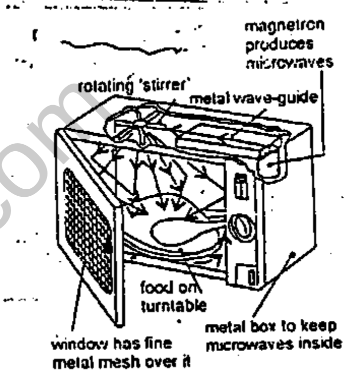
(P.T.O)

3— Tuning of a radio set.

The frq. of a radio set given by $f = \frac{1}{2\pi\sqrt{LC}}$ can be adjusted by adjusting the values of inductance 'L' and capacitance 'C'. If we adjust the frq. of our radio set equal to that of the radio station of our interest, we can get the programme of desired station by resonance.

4— ~~Cooking of food by microwave oven.~~

In microwave oven, the waves produced have a wavelength of 12 cm at a frq. of 2450 MHz. At this frq. the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food evenly and efficiently.



frq.

5— Many cities with tall buildings have already refused to allow supersonics to fly over them to avoid resonance in buildings.

6— The art of singing or speaking and mechanism of the human ear are excellent natural applications of resonance.

#7.9 DAMPED OSCILLATIONS:

In dealing with the oscillations, we ignored the resistive forces and dissipative effects. In practice, these effects cannot be neglected.

Def— "Such type of oscillation, in which the amplitude decreases steadily with time are called damped oscillations." **or** "Such a process in which energy is dissipated from the oscillating system is called damping and corresponding oscillations are called damped osscillations."

(P.T.O)

EXPLANATION.

In describing the motion of a simple pendulum, frictional effect was completely ignored. As the bob of a simple pendulum moves to and fro, three types of forces come into existance;

(i) Weight of the bob

(ii) Tension in the string

(iii) Viscous drag (i.e air resistance)

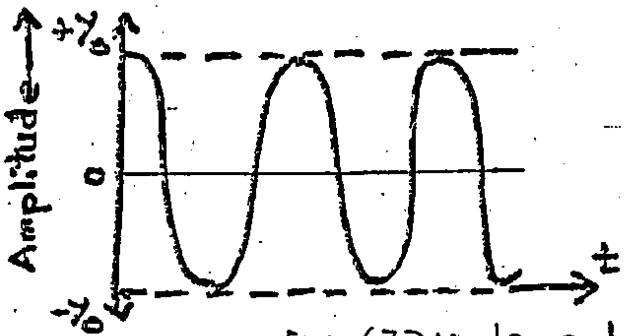


fig (a) Undamped

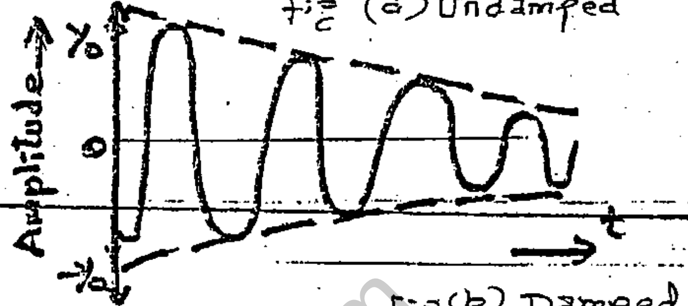


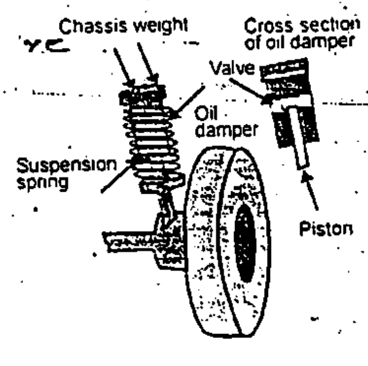
fig (b) Damped

• Graph B/w amplitude and time

Thus SHM is an idealization as shown in fig (a) which is an undamped harmonic oscissillation. In actual the amplitude of the motion of the bob gradually becomes smaller and smaller due to friction and air resistance. fig (b).

Applications of damped oscillations.

1 — An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillations. If the shock absorbers are defective then the car will be bouncing even along the smooth roads.



2 — Another application of damped oscillation is the shock absorber system of human body. Skier's body moves over the bumpy snow smoothly while his or her thighs and calves act like a damping spring. That is why skiing instructors are reputedly always saying "Bend Zee Knees".

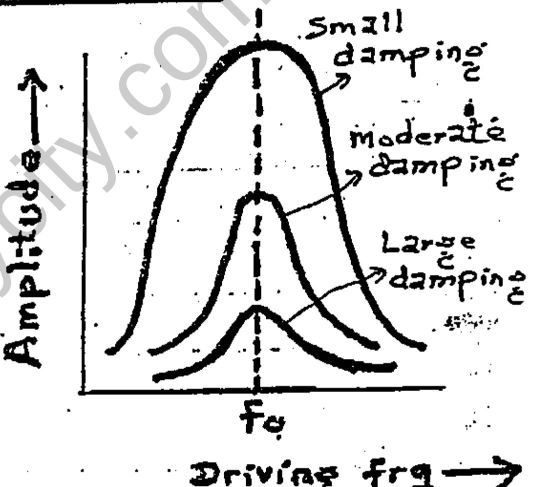
(P.T.O)

3— A new development in damped oscillations is that racing cars and hydrofoils can now be fitted with active suspensions. These involves computer operated hydraulic system in which bumps or waves are sensed, and the suspension system is adjusted accordingly.

4— Computers are used to control the angle of the wave through water to achieve remarkably stable movement of the hover craft.

7.10 SHARPNESS OF RESONANCE:

In case of resonance, the amplitude of vibration becomes very large when damping is small. Thus presence of damping prevents the amplitude from becoming sufficient large. The amplitude decreases rapidly at a frequency slightly different from resonance frq. Thus, a heavily damped system has a fairly flat curve as shown in an amplitude frq. graph.



In order to observe the damping effect, attach a very light ball such as a **pith ball**, and another of the same length carrying a heavy ball of equal size as **lead bob**. They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that the amplitude of the heavy ball (i.e lead ball) is much larger than that of the pith ball. This shows that the damping effect for the pith ball due to air resistance is much greater than for the lead bob. Thus, the sharpness of the resonance curve of a resonating system depends on energy loss due to friction. i.e; smaller the frictional loss of energy, the sharper the resonance curve.