NUMERICAL PROBLEMS :-

*– The solutions of the problems are given here.

P. 11.1 :- As given : $T = 0^\circ C + 273 = 273\,K$, $P = 1\,atm.$

$\langle v \rangle = ?$ for N_2 molecules.

Sol. :- Using formula: $T = \frac{2}{3K} \langle \frac{1}{2} m v^2 \rangle$

or $T = \frac{\cancel{2}}{3K} \times \frac{1}{\cancel{2}} m \langle v^2 \rangle = \frac{m}{3K} \langle v^2 \rangle$

or $\langle v^2 \rangle = \frac{3KT}{m}$, $K = 1.38 \times 10^{-23}\, JK^{-1}$

mass of N_2 molecule $= \frac{\text{molecular mass}}{\text{Avo. No. } (=N_A)}$

$m = \frac{28\,g}{6.022 \times 10^{23}} = \frac{28\,kg}{6.022 \times 10^{26}}$

$\therefore$ $m = 4.65 \times 10^{-26}\, kg$

putting these values in above eq. we get:

$\langle v^2 \rangle = 2.43 \times 10^{5}\ m^2/s^2$

$\therefore$ $\langle v \rangle = 493\ m/s.$

P. 11.2 :- As given: mass of molecules $= m_1, m_2$

Speeds " " $= \langle v_1 \rangle, \langle v_2 \rangle$

temp. $= T =$ const.

Sol. :- Using formula : $T = \frac{2}{3K} \langle \frac{1}{2} m v^2 \rangle$

For 1st gas : $T_1 = \frac{2}{3K} \times \frac{1}{2} m_1 \langle v_1^2 \rangle$

" 2nd " : $T_2 = \frac{2}{3K} \times \frac{1}{2} m_2 \langle v_2^2 \rangle$

$\because$ $T_1 = T_2$

So $\frac{\cancel{2}}{\cancel{3K}} \times \frac{1}{\cancel{2}} m_1 \langle V_1^2 \rangle = \frac{\cancel{2}}{\cancel{3K}} \times \frac{1}{\cancel{2}} m_2 \langle V_2^2 \rangle$

$\Rightarrow$ $m_1 \langle V_1^2 \rangle = m_2 \langle V_2^2 \rangle$

$\frac{\langle V_1^2 \rangle}{\langle V_2^2 \rangle} = \frac{m_2}{m_1}$

or $\frac{\langle V_1 \rangle}{\langle V_2 \rangle} = \frac{\sqrt{m_2}}{\sqrt{m_1}}$. (Generally $\langle V \rangle \propto \frac{1}{\sqrt{m}}$)

P. 11.3 :- As given : $V_1 = V$, $V_2 = \frac{1}{2}V = ?$

$P = \text{Constt} = 1.25 \times 10^5\ N m^{-2}$, $W = 100\ J$

Sol. :- $\Delta V = V_1 - V_2 = V - \frac{V}{2} = \frac{1}{2}V$

$\Delta V = \frac{1}{2}V = ?$ ($\because$ Final vol. = change in vol.)

we know that : $W = P\Delta V$ or $\Delta V = \frac{W}{P}$

or $\Delta V = \frac{100}{1.25 \times 10^5} = 8 \times 10^{-4}\ m^3$

$\therefore \Delta V = 8 \times 10^{-4}\ m^3$.

P. 11.4 :- As given :- Decrease in internal energy

$= \Delta U = -300\ J$, $\Delta W = -120\ J$ (on the system)

Sol. :- Using formula of $\Delta Q = ?$

1st law i.e. $\Delta Q = \Delta U + \Delta W = (-300) + (-120)$

$\therefore \Delta Q = -420\ J$ ($\because$ Heat Lost is −ve)

P. 11.5 :- As given : $T_1 = 227°c + 273 = 500K$

(i) $\eta = ?$, $T_2 = 127°c + 273 = 400K$

(ii) $Q_1 = ?$, (iii) $Q_2 = ?$ $W = 10,000\ J$

Sol. :- using formula : (i) $\eta(\%) = (1 - \frac{T_2}{T_1}) \times 100$

$\eta(\%) = (1 - \frac{400}{500}) \times 100 = (\frac{500-400}{500}) \times 100 = 20\%$

$\therefore \eta(\%) = 20\%$

(ii) $\because \eta = \frac{W}{Q_1}$ or $\frac{20}{100} = \frac{10000}{Q_1}$

or $\frac{1}{5} = \frac{10000}{Q_1} \Rightarrow Q_1 = 50000\ J$

(iii) $\because W = Q_1 - Q_2$ or $Q_2 = Q_1 - W$

or $Q_2 = 50000 - 10000 = 40000\ J$

$\therefore Q_2 = 40,000\ J$.

P. 11.6 :- As given: $\Delta T = 100^\circ C = 100\,K$.

$Q_1 = 746\,J$, $Q_2 = 546\,J$, $T_1 = ?$, $T_2 = ?$

Sol. :- Using formula : $\eta = 1 - \frac{Q_2}{Q_1}$

or $\eta = 1 - \frac{546}{746} = \frac{746-546}{746} = 0.268$

$\therefore \eta = 0.268$

As $\Delta T = T_1 - T_2$ or $T_1 = \Delta T + T_2$

or $T_1 = 100 + T_2$ ....Now using formula:

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{(T_2+100)}$

or

$0.268 = 1 - \frac{T_2}{(T_2+100)}$

or $\frac{T_2}{T_2+100} = 1 - 0.268 = 0.732$

or $T_2 = 0.732(T_2 + 100) = 0.732\,T_2 + 73.2$

or $T_2 = 0.732\,T_2 + 73.2$

or $T_2 - 0.732\,T_2 = 73.2$

or $0.268\,T_2 = 73.2$, or $T_2 = \frac{73.2}{0.268}$

$\therefore T_2 = 273\,K = 0^\circ C$

$\Rightarrow T_1 = \Delta T + T_2 = 100^\circ C + 0^\circ C = 100^\circ C$

P. 11.7 :- As given: $T_1 = 327^\circ C + 273 = 600\,K$

$T_2 = 27^\circ C + 273 = 300\,K$

To check $\eta = 52\%$ or not.

Sol :- Using formula : $\eta_{(\%)} = (1 - \frac{T_2}{T_1}) \times 100$

or $\eta_{(\%)} = (1 - \frac{300}{600}) \times 100 = (\frac{600-300}{600}) \times 100 = 50\%$

$\therefore \eta_{(\%)} = 50\%$ and not 52%. Hence his claim is wrong.

P. 11.8 :- As given : $W = 100\,J$, $Q_2 = 400\,J$

$\eta = ?$ $Q_1 = ?$

Sol. :- We know that : $W = Q_1 - Q_2$

or $Q_1 = W + Q_2 = 100 + 400 = 500\,J$

$\therefore Q_1 = 500\,J$.

$\therefore \eta = \frac{W}{Q_1} = \frac{100}{500} = 0.2$

or $\eta(\%) = 0.2 \times 100 = 20\%$

$\therefore \eta_{(\%)} = 20\%$.

P. 11.9 :- As given : $T_2 = 7^\circ C + 273 = 280\,K$

$\eta = 50\%$, $\eta' = 70\%$, $T_1 = ?$, $T_1' = ?$

$\Delta T = ?$

Sol. :- Using formula :

$$\eta = 1 - \frac{T_2}{T_1}$$

Putting values :

$$\frac{50}{100} = 1 - \frac{280}{T_1}$$

or $\frac{1}{2} = 1 - \frac{280}{T_1}$

or $\frac{280}{T_1} = 1 - \frac{1}{2} = \frac{1}{2}$

or $\frac{280}{T_1} = \frac{1}{2}$

$\therefore T_1 = 280 \times 2 = 560\,K$

$\therefore T_1 = 560\,K$.

Again using formula :

$$\eta' = 1 - \frac{T_2}{T_1'}$$

$$\frac{70}{100} = 1 - \frac{280}{T_1'}$$

$0.7 = 1 - \frac{280}{T_1'}$

or $\frac{280}{T_1'} = 1 - 0.7 = 0.3$

or $\frac{280}{T_1'} = 0.3$

or $T_1' = \frac{280}{0.3} = 933\,K$

$\therefore T_1' = 933\,K$.

Increase in temp. $= T_1' - T_1$

or $= 933 - 560 = 373\,K$

$\therefore T_1' - T_1 = 373\,K$.

P. 11.10 :- As given : $T_1 = 450K$, $T_2 = 300K$

$\eta = ?$

Sol. :- Using formula : $\eta(\%) = \left(1 - \frac{T_2}{T_1}\right)100$

$\therefore \eta(\%) = \left(1 - \frac{300}{450}\right) \times 100$

$= \left(\frac{450 - 300}{450}\right) \times 100 = \frac{150 \times 100}{450}$

$= 33.3\%$

$\therefore \eta_{(\%)} = 33.3\%$.

P. 11.11 :- As given : Heat of fusion $= L_f = 336 \frac{J}{g}$

$m = 30g = 0.03\,kg$,

$= 336 \frac{J}{10^{-3}kg}$

$T = 0°C + 273 = 273K$, $\Delta S = ?$

$= 336000\, J/kg$

Sol. :- As heat is transferred from water to freeze it into ice, so entropy decreases and is −ve. Hence :

$$\Delta S = -\frac{\Delta Q}{T} = -\frac{mL_f}{T}$$

or $$\Delta S = -\frac{0.03 \times 336000}{273} = -36.92\ J/K$$

$$\therefore \Delta S = -36.92\ JK^{-1}.$$