

ELECTRIC POTENTIAL

"It is defined as the potential energy per unit +ve. charge."

1- Potential due to a Point Charge:

Fig. shows an isolated positive point charge 'q' having radial electric field. We want to calculate the potential at any point in its electric field.

For this purpose we take two points 'a' and 'b' near 'q'. For simplicity we suppose that 'a', 'b' and 'q' are lie on a straight line.

Let us first calculate the potential difference between 'a' and 'b'. We move the test charge q_0 without acceleration from 'a' to 'b' against electric field of charge 'q'.

Now by definition

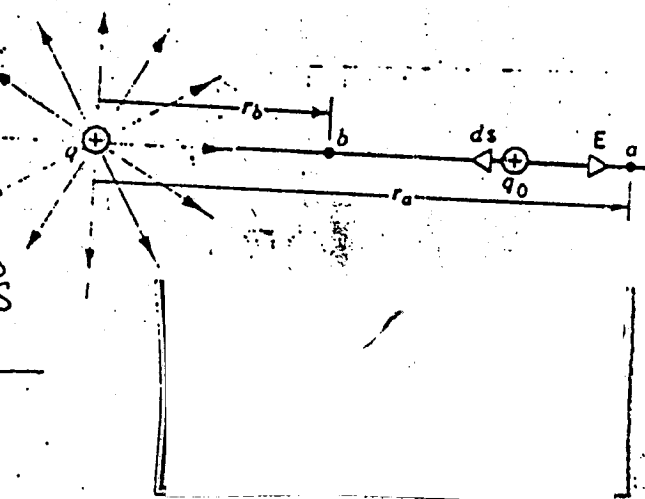
$$V_b - V_a = -\frac{W_{ab}}{q_0}$$

$$= -\int_a^b \vec{F} \cdot d\vec{s}$$

$$= -q_0 \int_a^b \vec{E} \cdot d\vec{s}$$

$$= -\int_a^b \vec{E} \cdot d\vec{s}$$

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$



In the fig. electric intensity $\vec{E}$ is towards right while the displacement $d\vec{s}$ is towards left. So angle between $\vec{E}$ and $d\vec{s}$ is 180° .

Thus $V_b - V_a = -\int_a^b E ds$

2

If r_a and r_b denote the distances of points a & b from q, then above expression can be written as,

$$V_b - V_a = \int_{r_a}^{r_b} E ds.$$

As distance ds is moved to the left so in moving from a to b distance from q is decreased. So ds means decrease in 'r'.

$$\therefore ds = -dr.$$

$$\therefore V_b - V_a = - \int_{r_a}^{r_b} E dr.$$

$$\text{As } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\therefore V_b - V_a = - \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr.$$

$$= - \frac{q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r^2}.$$

$$= - \frac{q}{4\pi\epsilon_0} \int_{r_a}^{r_b} r^{-2} dr.$$

$$= - \frac{q}{4\pi\epsilon_0} \left| \frac{r^{-2+1}}{-2+1} \right|_{r_a}^{r_b}.$$

$$= - \frac{q}{4\pi\epsilon_0} \left| \frac{r^{-1}}{-1} \right|_{r_a}^{r_b}.$$

$$= - \frac{q}{4\pi\epsilon_0} \left| -\frac{1}{r} \right|_{r_a}^{r_b}$$

$$V_b - V_a = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right) \quad \text{--- (1)}.$$

This equation give potential difference b/w a and b.

To calculate potential at point 'b' we suppose that

3.

point 'a' is at ∞ So $r_a = \infty$ and $V_a = 0$

So equation (1) becomes,

$$V_b - V_a = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{\infty} \right)$$

$$V_b - 0 = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - 0 \right)$$

$$V_b = \frac{1}{4\pi\epsilon_0} \frac{q}{r_b}$$

The general expression for potential at any point distant 'r' from q is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Sample Problem-1

Sample Problem 1 Two protons in the nucleus of a ^{238}U atom are 6.0 fm apart. What is the potential energy associated with the electric force that acts between these two particles?

Sol: $q_1 = 1.6 \times 10^{-19} \text{ C}$, $q_2 = 1.6 \times 10^{-19} \text{ C}$, $r = 6.0 \text{ fm} = 6 \times 10^{-15} \text{ m}$.

P.E = ?

AS $U = F \cdot r$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \cdot r$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6 \times 10^{-15}}$$

$$= \frac{9 \times 1.6 \times 1.6}{6} \times 10^{-19-19+15+9}$$

$$U = 3.84 \times 10^{-14} \text{ Joule}$$

$$= \frac{3.84 \times 10^{-14}}{1.6 \times 10^{-19}} \text{ eV}$$

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$$U = 2.4 \times 10^5 \text{ eV}$$

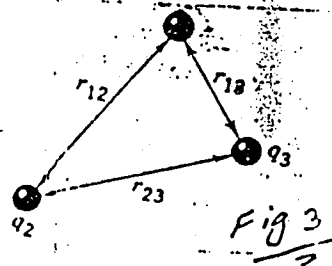
$$U = 240 \text{ KeV} \quad \text{Ans.}$$

Sample Problem 2.

Sample Problem 2 In the system shown in Fig. 3, assume that $r_{12} = r_{13} = r_{23} = d = 12 \text{ cm}$, and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

where $q = 150 \text{ nC}$. What is the potential energy of the system?



Sol.:- $r_{12} = r_{13} = r_{23} = d = 12 \text{ cm} = 0.12 \text{ m}$

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q$$

$$q = 150 \text{ nC} = 150 \times 10^{-9} \text{ C}$$

P.E of the system $U = ?$

As

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}}$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q \times -4q}{d} + \frac{q \times 2q}{d} + \frac{-4q \times 2q}{d} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{1}{d} (-4q^2 + 2q^2 - 8q^2)$$

$$= \frac{-10q^2}{4\pi\epsilon_0 d}$$

$$= \frac{-10 \times (150 \times 10^{-9})^2 \times 9 \times 10^9}{0.12}$$

$$= \frac{-10 \times 150 \times 150 \times 10^{-18} \times 9 \times 10^9}{0.12}$$

$$= - \frac{2025000}{0.12} \times 10^{-9} = -16775000 \times 10^{-9}$$

$$= -16775 \times 10^{-6}$$

$$= -1.6775 \times 10^{-2}$$

$$U = -1.68 \times 10^{-2}$$

7.

$$= 120 \times 0.15 \times 4 \times 3.14 \times 8.15 \times 10^{-12}$$

$$= 2000 \times 10^{-12}$$

$$= 2.0 \times 10^{-9} \text{ C}$$

$$q = 2nC \quad \text{Ans}$$

Sample Problem-6

Sample Problem 6 What is the electric potential at the surface of a gold nucleus? The radius is $7.0 \times 10^{-15} \text{ m}$, and the atomic number Z is 79.

Sol. $V = ?$, $r = 7.0 \times 10^{-15} \text{ m}$, $Z = 79$.

As $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ $\therefore q = Ze$
 $= 79e$

$$V = \frac{1}{4\pi\epsilon_0} \frac{79e}{r}$$

$$= \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{7.0 \times 10^{-15}}$$

$$= \frac{9 \times 79 \times 1.6}{7} \times 10^{9-19+15}$$

$$= 162.5 \times 10^5$$

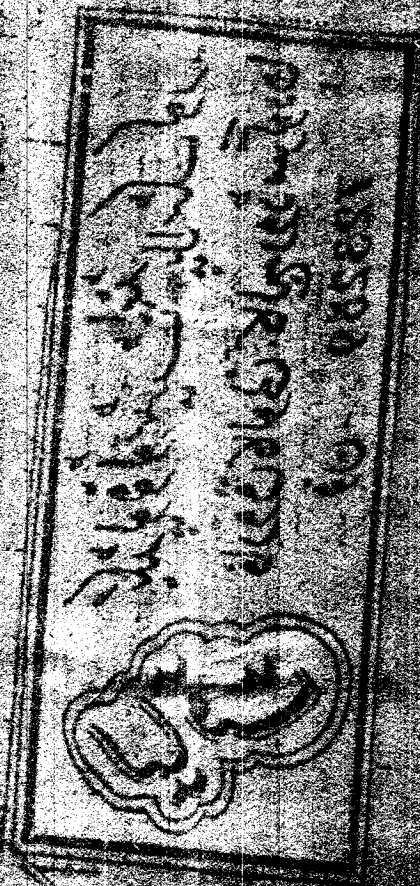
$$= 1.625 \times 10^7$$

$$V = 1.6 \times 10^7 \text{ Volts} \quad \text{Ans.}$$

2. Potential due to a Collection of Point Charges:

The potential at any point, due to a group of N point charges can be found by applying principle of Superposition.

Consider many charges $q_1, q_2, q_3, \dots, q_N$ lying at distances $r_1, r_2, r_3, \dots, r_N$ from the field point. Then



130 x 10 cm
200 x 10 cm
200 x 10 cm

Sample Problem 6

Some Problems...
in a...
...

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$
$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Potential due to a Collection of Point Charges

The potential at any point due to a group of point charges can be found by adding the potentials of the individual charges at that point. This is because the electric potential is a scalar quantity.

8.

total potential at P the field point is found by calculating the potential due to each charge by considering other charges as absent.

iii) Then we add all the potentials scalarly.

So the total potential at a point due to N point charges is equal to the algebraic sum of potentials due to individual charges i.e

$$V = V_1 + V_2 + V_3 + \dots + V_N$$

$$\text{or } V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2} + \dots + \frac{1}{4\pi\epsilon_0} \frac{q_N}{r_N}$$

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots + \frac{q_N}{r_N} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_i}$$

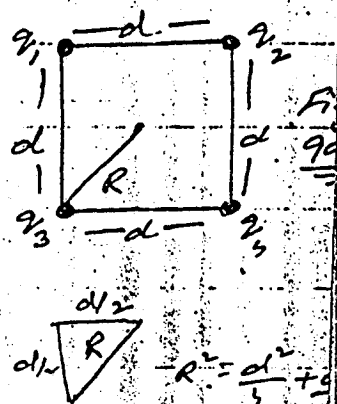
where q_i is the value of i th charge and r_i is the distance of the i th charge from field point.

Sample Problem-7

Sample Problem 7 Calculate the potential at point P, located at the center of the square of point charges shown in Fig. 9a. Assume that $d = 1.3$ m and that the charges are

$$q_1 = +12 \text{ nC}, \quad q_3 = +31 \text{ nC},$$

$$q_2 = -24 \text{ nC}, \quad q_4 = +17 \text{ nC}.$$



Sol. Potential V_P at point P = ?

$$q_1 = 12 \text{ nC} = 12 \times 10^{-9} \text{ C}, \quad q_2 = -24 \text{ nC} = -24 \times 10^{-9} \text{ C}$$

$$q_3 = +31 \text{ nC} = +31 \times 10^{-9} \text{ C}, \quad q_4 = +17 \text{ nC} = +17 \times 10^{-9} \text{ C}$$

$$d = 1.3 \text{ m}.$$

$$\text{Now } V_P = \sum_{i=1}^4 V_i$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{R} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{R} + \frac{1}{4\pi\epsilon_0} \frac{q_3}{R} + \frac{1}{4\pi\epsilon_0} \frac{q_4}{R}$$

Ans. in m.u.

9.

$$V_P = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 + q_2 + q_3 + q_4}{R} \right) \quad \text{--- (1)}$$

Now from fig.

$$R^2 = \left(\frac{d}{2} \right)^2 + \left(\frac{d}{2} \right)^2$$

$$= 2 \left(\frac{d}{2} \right)^2$$

$$= \frac{2d^2}{4}$$

$$R^2 = \frac{d^2}{2}$$

$$\therefore R = \frac{d}{\sqrt{2}}$$

So eq. (1) becomes

$$V_P = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 + q_2 + q_3 + q_4}{d} \right) \sqrt{2}$$

$$= \frac{\sqrt{2}}{4\pi\epsilon_0} \left(\frac{q_1 + q_2 + q_3 + q_4}{d} \right)$$

$$= 9 \times 10^9 \times 1.414 \left(\frac{12 \times 10^{-9} - 24 \times 10^{-9} + 31 \times 10^{-9} + 17 \times 10^{-9}}{1.3} \right)$$

$$= 9 \times 1.414 \times \frac{(12 - 24 + 31 + 17) \times 10^{-9}}{1.3}$$

$$= 9.7892 \times 3.6$$

$$= 352 \text{ Volts.}$$

$$\boxed{V_P = 3.52 \times 10^2 \text{ volt}} \quad \text{Ans.}$$

3. Potential due to a Dipole:

An electric dipole is defined as,

"Two equal and opposite charges separated by a small distance."

The magnitude of electric dipole moment vector $\vec{P}$ is $q \cdot d$ and its direction is from $-q$ to $+q$.

We want to find the electric potential V at point 'P' due to dipole.

10.

Let d be the distance between $+q$ and $-q$ and r_1, r_2 be the distances of point 'P' from $+q$ to $-q$ respectively as shown.

Applying principle of superposition;
the total potential at point 'P' due to dipole 'x'

$$V_P = \sum_{i=1}^2 V_i.$$

$$V_P = V_1 + V_2.$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{-q}{r_2}$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$V_P = \frac{q}{4\pi\epsilon_0} \left(\frac{r_2 - r_1}{r_1 r_2} \right) \quad \text{--- (1)}$$

Now for $r \gg d$.

$$\therefore \frac{r_2 - r_1}{d} \approx \cos \theta.$$

$$r_2 - r_1 \approx d \cos \theta.$$

$$\text{and } r_1 r_2 \approx r^2.$$

$\therefore$ expression (1) becomes

$$V_P \approx \frac{q}{4\pi\epsilon_0} \left(\frac{d \cos \theta}{r^2} \right)$$

$$\approx \frac{1}{4\pi\epsilon_0} \frac{q d \cos \theta}{r^2}$$

But $q d = p$ the magnitude of the dipole moment.

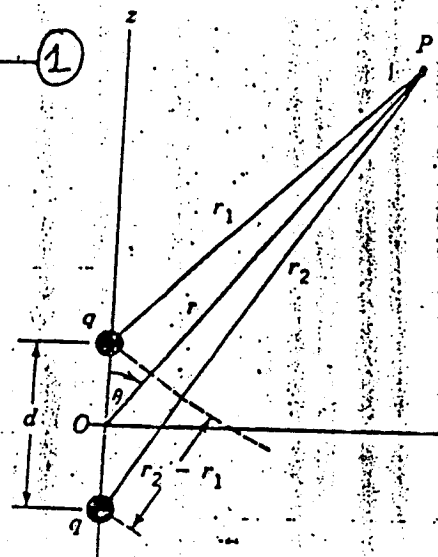
$$\therefore V_P \approx \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad \text{--- (2)}$$

This is the expression for electric potential at pt. 'P' due to dipole.

From equation (2) we see that

i.e. V_P is zero for $\theta = 90^\circ$ i.e. when

the distance of pt. 'P' is



11

(ii) V_p has greatest +ve value for $\theta = 0^\circ$

(iii) V_p has greatest -ve value for $\theta = 180^\circ$

It should be noted that V_p does not depend separately on q and d but only on their product P .

Applications (uses) of electric dipole.

Radio and TV antennas are in the form of rod or metal wire in which electrons move back and forth periodically.

At certain time end of rod is -ve and the other end is +ve. After half cycle the polarity of the ends is reversed. So it is an oscillating electric dipole. In the dipole the dipole moment changes periodically.

Sample Problem-8

Sample Problem 8 An electric quadrupole consists of two electric dipoles so arranged that they almost, but not quite, cancel each other in their electric effects at distant points (see Fig. 12). Calculate $V(r)$ for points on the axis of this quadrupole.

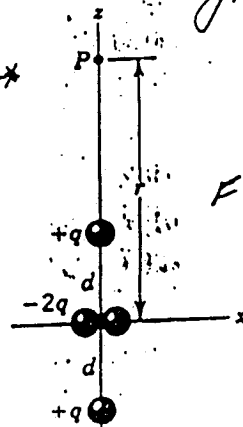


Fig 12

Quadrupole - The combination of two oppositely directed electric dipoles is called Quadrupole.

Sol.:- $V_{(r)} = ?$; $q_1 = +q$; $q_2 = -2q$, $q_3 = -q$.

By the principle of superposition.

$$V = \sum_{i=1}^3 V_i$$

$$= V_1 + V_2 + V_3$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{(r-d)} + \frac{1}{4\pi\epsilon_0} \frac{(-2q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(+q)}{r+d}$$

12.

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r-d} - \frac{2q}{r} + \frac{q}{r+d} \right] \\
 &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r-d} - \frac{2}{r} + \frac{1}{r+d} \right] \\
 &= \frac{q}{4\pi\epsilon_0} \left[\frac{r(r+d) - 2(r-d)(r+d) + r(r-d)}{r(r-d)(r+d)} \right] \\
 &= \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 + rd - 2(r^2 - d^2) + r^2 - rd}{r(r^2 - d^2)} \right] \\
 &= \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 + rd - 2r^2 + 2d^2 + r^2 - rd}{r(r^2 - d^2)} \right] \\
 &= \frac{q}{4\pi\epsilon_0} \left[\frac{2d^2}{r(r^2 - d^2)} \right]
 \end{aligned}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{2qd^2}{r^3(1 - \frac{d^2}{r^2})} \quad \text{Putting } 2qd^2 = Q$$

where Q is called electric quadrupole moment.

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^3(1 - \frac{d^2}{r^2})}$$

Because $r \gg d$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^3}$$

NOTE:- (i) $V \propto \frac{1}{r}$ for point charge because $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
 (ii) $V \propto \frac{1}{r^2}$ for a dipole because $V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$
 (iii) $V \propto \frac{1}{r^3}$ for a quadrupole because $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^3}$

4. Electric Potential of Continuous Charge Distribution

As we know that electric potential at a point due to a number of point charges is equal to the algebraic sum of potentials due to individual charges is

$$V = \sum_{i=1}^n V_i$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

If we have a continuous charge distribution then sigma is replaced by integral.

Then the above expression becomes,

$$V = \int dv$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

The potential due to small element of continuous element of continuous charge distribution dq at a distance ' r ' is given as,

$$dv = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

i) When charge is uniformly distributed along a line, we have linear charge density λ defined as

"Charge per unit length".

$$\text{i.e. } \lambda = \frac{dq}{ds}$$

$$dq = \lambda ds$$

$$\text{So } dv = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r}$$

The potential due to whole distribution is given as,

$$V = \frac{\lambda}{4\pi\epsilon_0 r} \int ds$$

14

(ii) When charge is uniformly distributed over a surface,

we have surface charge density σ defined as

"Charge per unit area."

$$\text{i.e. } \sigma = \frac{dq}{dA}$$

$$dq = \sigma dA$$

$$\text{So } dV = \frac{1}{4\pi\epsilon_0 r^2} \sigma dA$$

The potential due to whole distribution is given as,

$$V = \frac{\sigma}{4\pi\epsilon_0} \int \frac{dA}{r}$$

(iii) When charge is distributed uniformly into a volume,

we have volume charge density ρ defined as

"Charge per unit volume."

$$\text{i.e. } \rho = \frac{dq}{dV}$$

$$dq = \rho dV$$

$$\text{So } dV = \frac{1}{4\pi\epsilon_0 r^2} \rho dV$$

The potential due to whole distribution is given as,

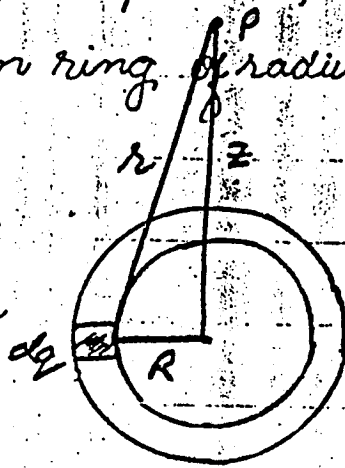
$$V = \frac{\rho}{4\pi\epsilon_0} \int \frac{dV}{r}$$

5. Potential due to a Ring of Charge:

Example:- Find the electric potential at a point P, a distance z along the axis of a uniform ring of radius R and total charge q .

Sol:- Consider a charge element dq on the ring. The potential at point P due to the element is given as,

$$dV = \frac{1}{4\pi\epsilon_0 r^2} dq$$



15

By symmetry all such elements are at the same distance 'r' from point 'P'.

$$V = \int_0^R dv.$$

$$= \frac{1}{4\pi\epsilon_0 r} \int dq.$$

$$V = \frac{1}{4\pi\epsilon_0 r} \times q.$$

Now $r^2 = R^2 + z^2$

$$r = \sqrt{R^2 + z^2}.$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{R^2 + z^2}}$$

This is the potential at a point P due to ring of charge.

Sample Problem-9

Sample Problem 9 Calculate the potential at a point on the axis of a circular plastic disk of radius R, one surface of which carries a uniform charge density σ .

Sol. Consider a charge element dq in the form of a ring of radius 'w' and width dw.

Then $\sigma = \frac{dq}{dA}$

Now $dA = (2\pi w)dw$.

$$\therefore \sigma = \frac{dq}{2\pi w dw}$$

$$dq = \sigma (2\pi w)(dw)$$

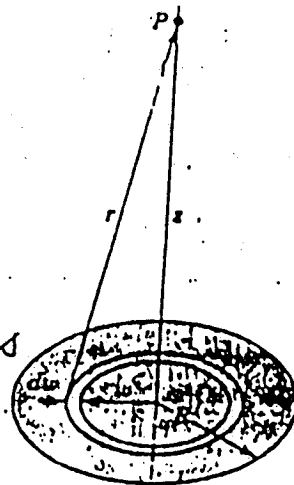
The potential at point 'P' due to the ring is

$$dv = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

$$dv = \frac{1}{4\pi\epsilon_0} \frac{\sigma (2\pi w) dw}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi w dw}{\sqrt{w^2 + z^2}}$$

$$dv = \frac{\sigma}{4\epsilon_0} \frac{2w dw}{\sqrt{w^2 + z^2}}$$



Now from fig

$$r^2 = z^2 + w^2$$

$$r = \sqrt{w^2 + z^2}$$

16

Total potential at point P due to whole distribution is given as $V = \int dv$.

$$= \frac{\sigma}{4\epsilon_0} \int_0^R \frac{2\omega d\omega}{\sqrt{\omega^2 + z^2}} = \frac{\sigma}{4\epsilon_0} \int_0^R (\omega^2 + z^2)^{-1/2} (2\omega d\omega)$$

$$V = \frac{\sigma}{4\epsilon_0} \left| \frac{(\omega^2 + z^2)^{-1/2 + 1}}{-1/2 + 1} \right|_0^R = \frac{\sigma}{4\epsilon_0} \times 2 \left| (\omega^2 + z^2)^{1/2} \right|_0^R$$

$$V = \frac{\sigma}{2\epsilon_0} \left[(R^2 + z^2)^{1/2} - z \right]$$

$$V = \frac{\sigma}{2\epsilon_0} \left[\sqrt{R^2 + z^2} - z \right] \quad \text{--- (1)}$$

Special Case:- If $z \gg R$.

$$\text{Then } \sqrt{R^2 + z^2} = (z^2 + R^2)^{1/2} = z \left(1 + \frac{R^2}{z^2} \right)^{1/2}$$

$$= z \left(1 + \frac{1}{2} \frac{R^2}{z^2} + \dots \right)$$

$$= z + \frac{1}{2} z \frac{R^2}{z^2}$$

$$= z + \frac{R^2}{2z}$$

$\therefore$ Expression (1) becomes

$$V = \frac{\sigma}{2\epsilon_0} \left[z + \frac{R^2}{2z} - z \right]$$

$$= \frac{\sigma}{2\epsilon_0} \frac{R^2}{2z}$$

$$= \frac{\sigma R^2}{4\epsilon_0 z}$$

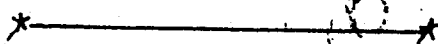
$$\therefore V = \frac{q/\pi R^2}{4\epsilon_0 z} \frac{R^2}{z}$$

$$= \frac{q}{4\pi\epsilon_0 z}$$

$$\therefore \sigma = \frac{q}{\pi R^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{z}$$

This is the potential at point P for $z \gg R$ due to disk of charge.



potential at point P due to

17

6. Equipotential Surfaces:

"An equipotential surface is that whose every point is at the same potential."

Fig (i) shows a uniform electric field $\vec{E}$.

Now potential difference between points A & B, is EL .

i.e. work done by the electric field in moving a test +ve charge q_0 from A to B, is $FL = q_0 EL$.

However, if we move the test charge q_0 to field from B_1 to B_3 , then no work is done because the points B_1 , B_2 and B_3 have same potential.

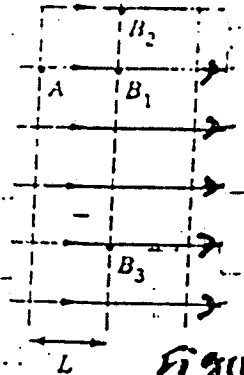


Fig (i)

In the fig. solid lines are electric lines of force while dotted lines are equipotential planes.

The difference of potential between any point on one plane and any point on the adjacent plane is EL where L is the spacing between two adjacent planes.

Fig (ii) shows radial electric field due to a positive point charge.

The concentric circles represent equipotential lines.

If we move the test +ve charge q_0 from A to B, work will be done. But no work will be done if we move the test +ve charge on the same circle. Because all points on the same circle are equidistant from the point charge.

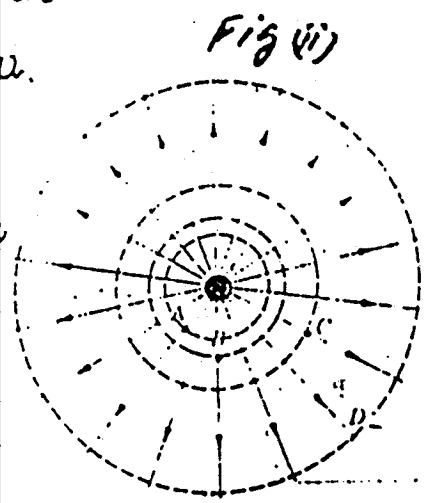


Fig (ii)

So when a test charge moves along an equipotential

3

surface, no work is done on it by the electric field

because $\Delta V = 0$

Fig (iii) shows a family of equipotential surfaces.

The work done by the field when a charge moves along path 1 or 2 is zero because both these paths start

and end on the same equipotential surface.

But a long path 3 and 4 a work is not

zero because these paths start and end on

different equipotential surfaces. The work done along path

3 and 4 are equal because initial and final potentials are

same. Paths 3 and 4 connect the same pair of

equipotential surfaces.

From figs (i) and (iii) we find that equipotential surface are $\perp$ to electric field $\vec{E}$.

If $\vec{E}$ is not $\perp$ to equipotential surface then $\vec{E}$ would have a component along the surface and so the component would exert a force on a test charge. This would have to be done on a test charge as it moves on an equipotential

surface. But for equipotential surface $\Delta V = 0$ for any two points on that surface. So $\vec{E}$ must be $\perp$ to the equipotential surface.

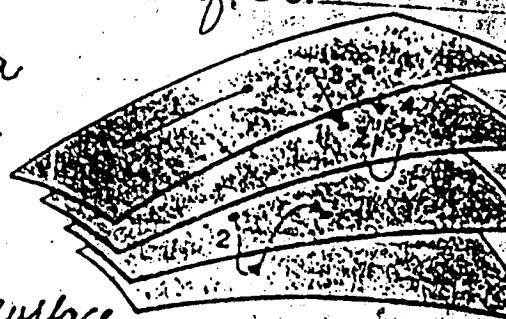


Fig (iii)

Calculating the Field from Potential:

(Elec Field as the Gradient of Potential)

The potential V and electric field E can be calculated from each other.

By the equation $V = -\int \vec{E} \cdot d\vec{s}$ we can find V from the knowledge of $\vec{E}$.

Consider a family of equipotential surfaces having potential difference dV between any pair of surfaces.

Let us move a test charge q_0 from point 'P' through a displacement $d\vec{s}$ towards surface having potential V .

The work done by the electric field is given by,

$$\begin{aligned} dW &= \vec{F} \cdot d\vec{s} \\ &= q_0 \vec{E} \cdot d\vec{s} \\ dW &= q_0 E ds \cos \theta \quad \text{--- (i)} \end{aligned}$$

Now work done by the electric field on the +ve test charge q_0 is -ve. So by the definition

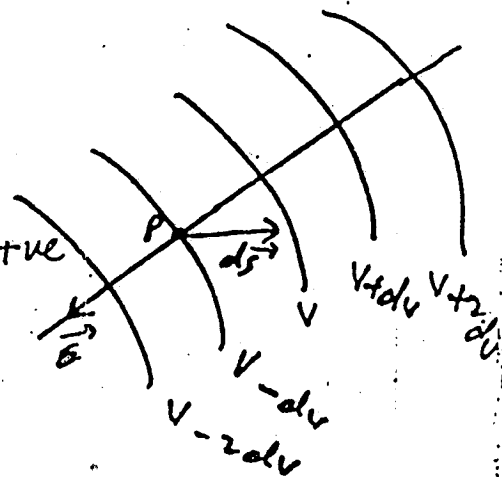
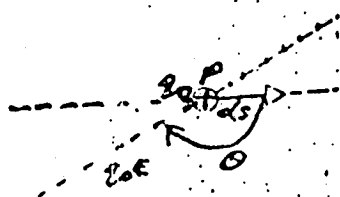
$$\begin{aligned} dV &= \frac{-dW}{q_0} \\ \therefore dW &= -q_0 dV \quad \text{--- (ii)} \end{aligned}$$

From (i) and (ii)

$$\begin{aligned} -q_0 dV &= q_0 E ds \cos \theta \\ -dV &= E ds \cos \theta \\ E \cos \theta &= -\frac{dV}{ds} \end{aligned}$$

Now $E \cos \theta$ is the component of $\vec{E}$ along $d\vec{s}$ and is denoted by E_s .

$$\therefore E_s = -\frac{dV}{ds} \quad \text{--- (1)}$$



20

This equation shows that the negative of rate of change of potential V position in any direction is equal to the component of $\vec{E}$ in that direction. The -ve sign shows that $\vec{E}$ is in the direction of decreasing V . It is clear that $\vec{E}$ is only one direction $\vec{ds}$ for which $-\frac{dV}{ds}$ is maximum. E_s will be maximum for the direction $\vec{E}$.

$$\therefore E = -\left(\frac{dV}{ds}\right)_{\max} \quad \text{--- (2)}$$

The maximum value of $\frac{dV}{ds}$ at any point is called potential gradient at that point. The direction $\vec{ds}$ for which $\frac{dV}{ds}$ has maximum value is always $\perp$ to the equipotential surface.

In component form

$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

So (2) can be written as

$$dV = -\vec{E} \cdot \vec{ds}$$

$$dV = -E_x dx - E_y dy - E_z dz \quad \text{--- (3)}$$

Now potential V is a function of position co-ordinate.

$$\therefore V = V(x, y, z)$$

$$\therefore dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \quad \text{--- (4)}$$

From (3) & (4) we have

$$-E_x dx - E_y dy - E_z dz = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

21

$$E_z = -\frac{\partial V}{\partial z}$$

$$\text{As } \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\therefore \vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$$

$$= -\left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}\right) V$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = -\text{grad } V$$

-ve sign shows that V increases in the direction of $-\vec{E}$.

So electric intensity is equal to -ve of potential gradient.

So if V is known at any point then E at that point can be calculated by taking derivative of V at that point.

Sample Problem-10

Sample Problem 10 Using Eq. 26 for the potential on the axis of a uniformly charged disk, derive an expression for the electric field at axial points.

Sol. $E_z = ?$

By the equation

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + z^2} - z)$$

$$\text{As } E_z = -\frac{\partial V}{\partial z}$$

$$= -\frac{\partial}{\partial z} \left[\frac{\sigma}{2\epsilon_0} \{(\sqrt{R^2 + z^2}) - z\} \right]$$

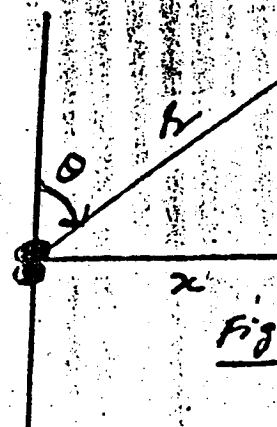
$$= \frac{\sigma}{2\epsilon_0} \frac{\partial}{\partial z} [z - (\sqrt{R^2 + z^2})]$$

$$= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{2} (R^2 + z^2)^{-1/2} (2z) \right]$$

$$\boxed{E_z = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right]} \quad \text{Ans.}$$

Sample Problem-11.

Sample Problem 11 Figure 18 shows a (distant) point P in the field of a dipole located at the origin of an xz coordinate system. Calculate E as a function of position.



Sol: $E_z = ?$ (on the z axis)

$E_x = ?$ (on the z axis)

From the fig. $r^2 = x^2 + z^2$.

As the potential due dipole is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{P \cos \theta}{r^2}$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{P \cos \theta}{(x^2 + z^2)}$$

From the fig. $\frac{z}{r} = \cos \theta$.

$$V = \frac{1}{4\pi\epsilon_0} \frac{Pz}{(x^2 + z^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Pz}{(x^2 + z^2)^{3/2}}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Pz}{(x^2 + z^2)^{3/2}} \quad \text{--- (1)}$$

Now

$$E_z = - \frac{\partial V}{\partial z}$$

$$= - \frac{\partial}{\partial z} \left[\frac{P}{4\pi\epsilon_0} - \frac{z}{(x^2 + z^2)^{3/2}} \right]$$

$$= - \frac{P}{4\pi\epsilon_0} \frac{\partial}{\partial z} \left[\frac{z}{(x^2 + z^2)^{3/2}} \right]$$

$$= - \frac{P}{4\pi\epsilon_0} \left[\frac{(x^2 + z^2)^{3/2} \cdot 1 - z \cdot \frac{3}{2} (x^2 + z^2)^{1/2} (2z)}{(x^2 + z^2)^3} \right]$$

$$= - \frac{P}{4\pi\epsilon_0} \left[\frac{(x^2 + z^2)^{1/2} \{ (x^2 + z^2) - 3z^2 \}}{(x^2 + z^2)^3} \right]$$

$$= - \frac{P}{4\pi\epsilon_0} \left[\frac{x^2 + z^2 - 3z^2}{(x^2 + z^2)^{5/2}} \right]$$

Figure 18 shows a (distant) point P in the field of a dipole located at the origin of an xz coordinate system. Calculate E as a function of position.

23

$$= \frac{-P}{4\pi\epsilon_0} \left(\frac{x^2 - 2z^2}{(x^2 + z^2)^{5/2}} \right)$$

For E_z on the z-axis put $x=0$.

$$\begin{aligned} \therefore E_z &= \frac{-P}{4\pi\epsilon_0} \frac{-2z^2}{z^5} \\ &= \frac{2P}{4\pi\epsilon_0} \frac{1}{z^3} \end{aligned}$$

$$\boxed{E_z = \frac{1}{4\pi\epsilon_0} \frac{2P}{z^3}} \quad \text{Ans.}$$

Now from (1)

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ &= -\frac{\partial}{\partial x} \left[\frac{1}{4\pi\epsilon_0} \frac{Pz}{(x^2 + z^2)^{3/2}} \right] \\ &= \frac{-P}{4\pi\epsilon_0} \frac{\partial}{\partial x} \left[\frac{z}{(x^2 + z^2)^{3/2}} \right] \\ &= \frac{-P}{4\pi\epsilon_0} \left[\frac{(x^2 + z^2)^{3/2} \cdot 0 - z \cdot \frac{3}{2} (x^2 + z^2)^{1/2} (2x)}{(x^2 + z^2)^3} \right] \\ &= \frac{-P}{4\pi\epsilon_0} \left[\frac{-3xz(x^2 + z^2)^{1/2}}{(x^2 + z^2)^3} \right] \\ &= \frac{P}{4\pi\epsilon_0} \left[\frac{3xz}{(x^2 + z^2)^{5/2}} \right] \end{aligned}$$

$$E_x = \frac{3P}{4\pi\epsilon_0} \frac{xz}{(x^2 + z^2)^{5/2}}$$

For E_x on the z-axis potential $x=0$.

$$\therefore E_x = 0.$$

$\therefore E_x$ is zero on the z-axis.

E_x is also at $z=0$ in medium plane.

—————

24

Potential and Field inside and outside an Isolated Conductor

We know that an excess charge placed on an isolated conductor moves entirely to the outer surface of the conductor.

This property of isolated conductors can also be obtained in terms of potential as

"An excess charge placed on an isolated conductor distributes itself on the surface so that all points of the conductor - whether on the surface or inside - come to the same potential."

This statement can be proved as follows.

- (i) We know from experiment that in equilibrium internal currents do not exist in the conductor. If two points within a conductor are having different potentials then free electrons will move from low potential to high potential which violates the experimental observation of no internal currents in equilibrium. So internal points cannot have different potentials.
- (ii) The above statement can also be proved as follows. We know that electric field $E=0$ in a conductor, then $\int \vec{E} \cdot d\vec{s} = 0$ between any two points a and b in the conductor.

24

 So $V_b - V_a = 0$ for all pairs of points and so potential has a constant value.

25

Shows the variation of potential
with radial distance for an isolated
spherical conducting shell of radius 1.0m
carrying a charge of 1.0mc.

For all external points V_r can be calculated
by the equation $V_r = -\int \vec{E} \cdot d\vec{s}$ because for
all external points charge q behaves as if
it were concentrated at the centre of the
sphere. The equation $V_r = -\int \vec{E} \cdot d\vec{s}$ gives
the potential as we move from outside up
to the surface of sphere.

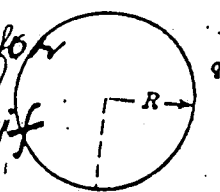
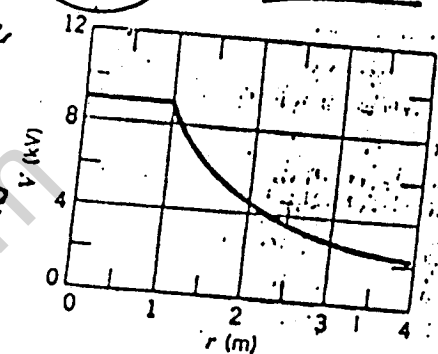


Fig
(1-a)



If we make a hole in the surface so
that we can move a test charge into the interior the
potential does not change because no force acts on the test
charge from inside.

From fig. (1-a) it is clear that potential everywhere inside
is equal to that on the surface.

Fig (1-b) shows variation of E with r . Because $E=0$ everywhere
inside. So $V = \text{constant}$ everywhere inside. Because

$$E = -\frac{dV}{dr} \Rightarrow \frac{dV}{dr} = 0 \Rightarrow V = \text{constant},$$

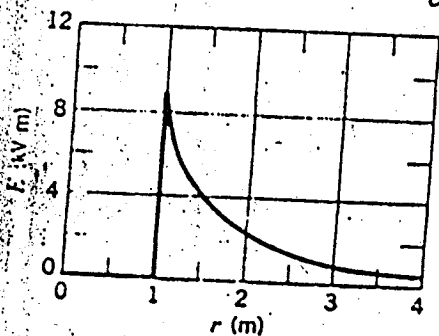


Fig (1-b)