LIGHT AND QUANTUM PHYSICS

49.1 Thermal Radiations

The radiations emitted by a body due to its temperature are called thermal radiations.

All bodies not only emit the thermal radiations, but also absorb these radiations from surroundings. If the rate of emission of radiation is equal to the rate of absorption for a body, then the body is said to be in thermal equilibrium.

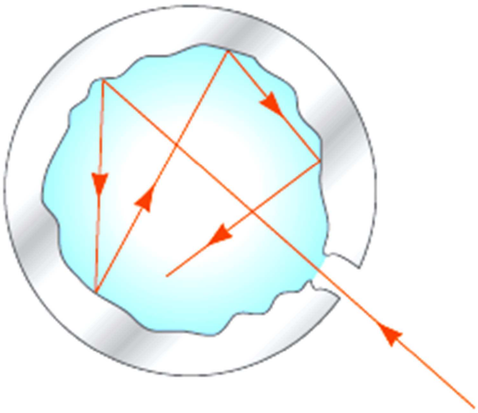
The radiations emitted by a hot body depend not only on the temperature but also on the material of which the body is made, the shape and the nature of the surface.

49.1.1 Cavity Radiator or the Black Body

A black body or cavity radiator is that which absorbs approximately all radiations falling on it. So a black body has maximum rate of emission and absorption of radiations.

A perfect black body does not exist. However a small hole in a cavity whose inner wall are lamped black is the nearest approach to a perfect black body.

For an ideal radiator (black body), the spectrum of the emitted thermal radiation depends only on the temperature of radiating body and not on the material, nature of the surface, size or shape of the body.



An opening in the cavity of a body is a good approximation of a blackbody. As light enters the cavity through the small opening, part is reflected and part is absorbed on each reflection from the interior walls. After many reflections, essentially all of the incident energy is absorbed.

49.1.2 Radiant Intensity

Energy emitted per unit area per unit time over all the wavelengths, is called radiant intensity. It is denoted by $I(t)$. Or

Power radiated per unit area over all the wavelengths is called radiant intensity.

49.1.3 Stephen-Boltzmann Law

The radiant intensity $I(t)$ is directly proportional to the forth power of absolute temperature. Mathematically,

$$\text{Radiant Intensity} \propto (\text{Absolute Temperature})^4$$

$$I(t) \propto (T)^4$$

$$I(t) = \sigma (T)^4$$

where σ is a universal constant, called Stephen-Boltzmann constant. Its value is $5.67 \times 10^{-8} \frac{W}{m^2K^4}$.

49.1.4 Spectral Radiancy

Spectral radiancy of the black body is defined as the radiant intensity per unit wavelength at given temperature. it is denoted by $R(\lambda)$. Mathematically, it is described as:

$$R(\lambda) = \frac{dI(t)}{d\lambda}$$

$$dI(t) = R(\lambda)d\lambda$$

This shows that the product of $R(\lambda)d\lambda$ gives energy emitted per unit area per unit time over all the wavelength lies in the range from λ to $\lambda + d\lambda$.

The energy emitted per unit area per unit time over all the wavelengths at a particular temperature is obtained by integrating the equation (1), i.e.,

$$I(t) = \int_0^\infty R(\lambda)d\lambda$$

49.1.5 The Wien Displacement Law

Statement. The wavelength for which the spectral radiancy becomes is inversely proportional to the absolute temperature of the black body.

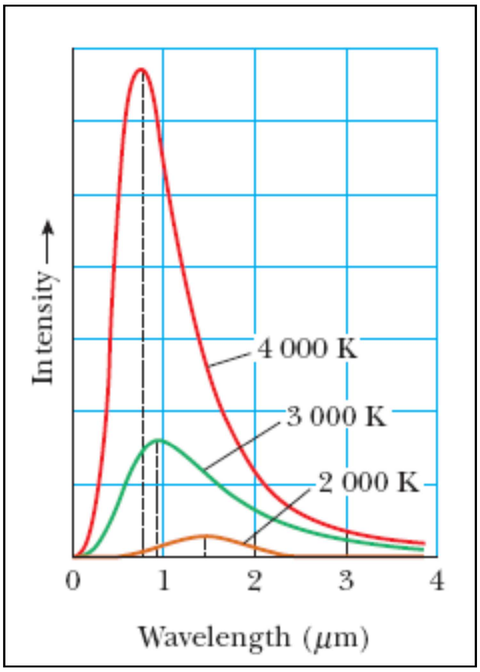
If λ_{max} is the wavelength corresponding to maximum spectral radiancy for temperature T, then the Wien displacement is described mathematically as:

$$\lambda_{max} \propto \frac{1}{T}$$

$$\lambda_{max} = Constant\ \frac{1}{T}$$

$$\lambda_{max}T = Constant$$

The constant has the value of $2.89 \times 10^{-3}mK$



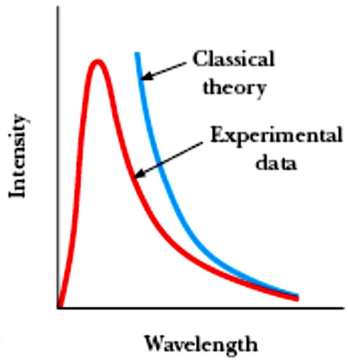
Question: Discuss the failure of classical physics and success of quantum physics to solve the problem of energy distribution along the curve of black body radiation.

Ans. The derivation of theoretical formula for distribution of spectral radiancy among various wavelengths has remained an unsolved problem over a long period of time. On the basis classical physics, following two formulas were derived to solve the problem of the energy-distribution along the curve of the black body radiation.

- Rayleigh-Jeans formula
- Wien's formula

Rayleigh-Jeans Formula

Rayleigh-Jeans derived a theoretical formula for the distribution of spectral radiancy among various wavelengths on the basis of classical physics, which is given by:



$$R(\lambda) = \frac{2\pi CkT}{\lambda^4}$$

Where k is the Boltzmann constant.

This formula was excellent for longer wavelengths but not good for shorter wavelengths as shown in the figure.

Wien's Formula

On the basis of analogy between the spectral radiancy curve and Maxwell speed distribution curve, the Wien's formula about spectral radiancy is described as

$$R(\lambda) = \frac{a}{\lambda^5} e^{-b/\lambda t}$$

Where a and b are constant.

Wien's formula was in good agreement with the experimental curve for shorter wavelength but not good for longer wavelengths.

49.1.6 Max-Plank's Radiation Law

Rayleigh-Jeans and Wien's formulae were failed to solve the problem of distribution of spectral radiancy among various wavelengths in cavity radiations because these formulae were based upon the classical theory.

Max-Plank's Quantum theory of Radiation

In 1900, Max-Planks gave the new concept about the nature of radiation, called Quantum theory of radiation in which Planks assumed the discrete nature of radiation. He assumed the atoms of the cavity emit and absorb radiation in the form of packet of energy, called quanta. The energy of each quanta is directly proportional to the frequency:

$$E \propto f$$

$$E = hf$$

Where h is the plank's constant having the numerical value of $6.625 \times 10^{-34}\ \mathrm{J-s}$.

In addition to this concept Max-Planks made the following assumption to derive his radiation law:

- The atoms of the cavity behave like tiny harmonic oscillators.
- The oscillators radiate and absorb energy only in the form of packets or bundles of electromagnetic waves.
- An oscillator can emit or absorb any amount of energy which is the integral multiple of hf. Mathematically,

$$E = nhf$$

Where n is an integer.

Using these assumptions, Plank's derived his radiation law given by:

$$R(\lambda) = \frac{a}{\lambda^5}\frac{1}{e^{b/\lambda t} - 1}$$

Where a and b are the constant whose values can be chosen from the best fit of experimental curve. Within two months, Plank's succeeded to reform his law as:

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kt} - 1}$$

49.1.7 Derivation of Rayleigh-Jeans Formula from Max-Plank's Radiation Law

Plank's radiation law approaches to Rayleigh-Jeans law at very long wavelengths. The Plank's radiation law is given by:

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kt} - 1} \quad \text{--------------} \quad (1)$$

Putting $\frac{hc}{\lambda kt} = x$

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^x - 1}$$

For a very long wavelengths i.e.,

$$\lambda \to \infty$$

$$x \to 0$$

And $e^x = 1 + x$

Putting these values in equation (1), we get:

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{1 + x - 1}$$

$$\Rightarrow R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{x}$$

Back substituting the value of x, we have

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{\lambda kt}{hc}$$

$$R(\lambda) = \frac{2\pi ckt}{\lambda^4}$$

This is the Rayleigh-Jeans formula.

49.1.8 Derivation of Wien's Formula from Max-Plank's Law

Plank's radiation law approaches to Wien's formula at very short wavelength. The Plank's radiation law is given by:

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kt} - 1} \quad \text{--------------} \quad (1)$$

Putting $2\pi c^2 h = a$ and $\frac{hc}{k} = b$

$$R(\lambda) = \frac{a}{\lambda^5} \frac{1}{e^{b/\lambda t} - 1} \quad \text{--------------} \quad (2)$$

For very short wavelengths, i.e.,

$$\lambda \to 0$$

$$\frac{b}{\lambda t} \to \infty$$

$$e^{b/\lambda t} \gg 1$$

So $e^{b/\lambda t} - 1 \cong e^{b/\lambda t}$

Putting these values in equation (2), we have:

$$R(\lambda) = \frac{a}{\lambda^5} \frac{1}{e^{b/\lambda t}}$$

$$R(\lambda) = \frac{a}{\lambda^5} e^{-b/\lambda t}$$

This is the Wien's formula for spectral radiancy.

49.2 Photo Electric Effect

The interaction between radiation and atoms of the cavity lead to the idea of quantization of energy. It means that energy can be emitted and radiated in the form of packets. Photo electric effect is another example of interaction between radiation and matter.

Def.

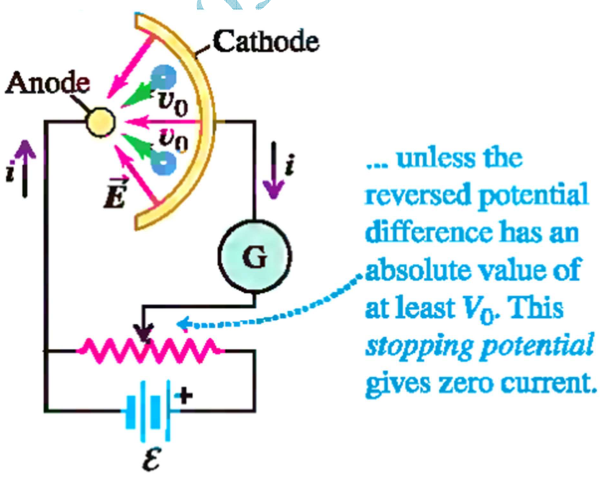
When a light of suitable frequency falls on a metal surface, the electrons are emitted out. These electrons are called photo electrons and this phenomenon is called photo electric effect.

49.2.1 Experimental Set-up

The apparatus used to study the photo electric effect is shown in the figure. The light of a suitable frequency falls on the metal surface, which is connected to the negative terminal of variable voltage source. If the frequency is high enough, the electrons are emitted out from metal plate and accelerate towards anode. These electrons are called photo electrons and current flows through circuit due to photo electrons is called photo electric current. This phenomenon is called photo electric effect.

49.2.2 Maximum K.E of Photo Electrons

The maximum K.E of photo electrons can be measured by reversing the polarity of the battery. Now the photo electric current will be reduced. The photo electric current does not drop to zero immediately because the photo electrons emit from metal plate with different speeds. Some will reach the cathode even though the potential difference opposes their motion. However if we make the reversed potential difference large enough V_0 (called stopping potential) at which the photo electric current drops to zero. This potential difference multiplied by the electronic charge e gives the maximum kinetic energy of photo electrons.



Mathematically, the maximum kinetic energy $K.E_{max}$ of photo electrons is described as:

$$K.E_{max} = e\,V_0$$

where V_0 is the stopping potential and e is the charge of an electron.

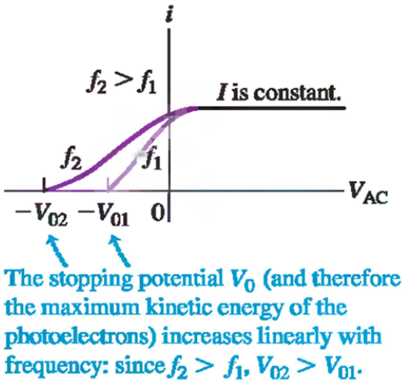
49.2.2 Experimental Results

- Photo electrons are emitted out from the given metal surface when the frequency f of incident light is equal to or greater than a critical value f_0, called threshold frequency, whatever the intensity of light may be.
- Photo electric emission will not take place from a given metal surface if the frequency of the incident light is less than the threshold frequency f_0 whatever the intensity of light may be.
- Threshold frequency depends upon the nature of the metal surface.

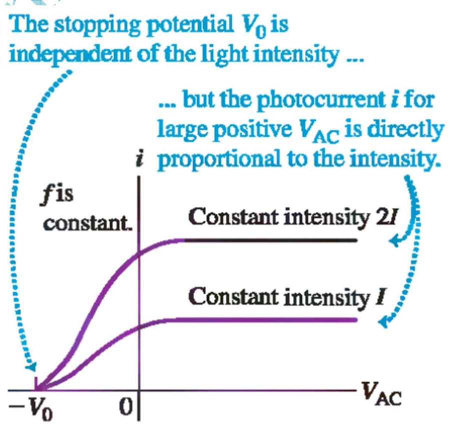
Work Functions of Selected Metals

Metal	ϕ (eV)
Na	2.46
Al	4.08
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14
Fe	4.50

- The energy of photo electrons depends upon the frequency of incident light and independent of the intensity of light.



- The number of photo electrons emitted per second is directly proportional to the intensity of light provided that the frequency of light is equal to or greater than the threshold frequency.



49.2.3 Threshold Frequency

The frequency of the incident light required to remove least tightly bound electron from the metal surface, is called threshold frequency.

49.2.4 Photo Electric Effect on the Basis of Classical Wave Theory

According to classical wave theory, the light consists of electromagnetic waves and their function is to transfer energy from one place to another.

When light falls on metal surface, it transfer energy to the electrons continuously. When an electron acquires sufficient energy, it escapes out the metal surface.

This theory successfully explains the emission electrons apparently, but this theory can't explain the three major features of photo electric effect.

- The Intensity Problem
- The Frequency Problem
- Time Delay Problem

The Intensity Problem

According to classical wave theory, the light consists of oscillating electric and magnetic vector, which increases in amplitude as the intensity of light beam is increased. Since the force applied to the electrons is 'eE_0', therefore the kinetic energy of the electrons should also increased with the intensity of light. However, the experimental results suggest that the K.E of electrons is independent of intensity of light.

The Frequency Problem

According to classical wave theory, the photo electric effect should occur for any frequency of light provided that the incident light is intense enough to supply the energy needed to eject the photo electrons. However, the experimental results show that there exists a critical frequency f_0 for each material. If the frequency of the incident light is less than f_0 the photo electric effect does not occur, mo matter how much the intensity of light is.

Time Delay Problem

The classical wave theory predicts that there must be a time interval between the incidence of light on the metal surface and the emission of photo electrons. During this time, the electron should be absorbing energy from the beam until it has accumulated enough energy to escape the metal surface. However the experimental results show that there is no detectable time interval between the incidence of light and emission of photo electrons provided that frequency of light is equal to or greater than the threshold frequency.

These three major features could not be explained on the basis of wave theory of light. However Quantum light theory has successfully explained the photo electric effect which was proposed by the Einstein in 1905.

49.2.5 Photons

In 1905, Einstein proposed quantum theory to explain the photo electric effect, according to which the light consist of bundles or packets of energy, called photons. The energy E of a single photon is

$$E = h\upsilon$$

where υ is the frequency of light and h is the planks constant $h = 6.626 \times 10^{-34}\ \mathrm{J-s}$.

According to quantum theory, on photon of energy is absorbed by a single electron. If this energy is greater than or equal to a specific amount of energy (called work function), then the electrons will be ejected, otherwise not.

49.2.6 Work Function

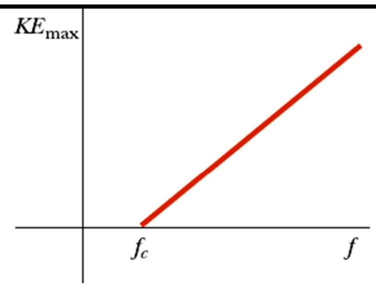
The minimum amount of energy required to eject the electrons out of metal surface, is called work function Φ, which can be described as:

$$\Phi = h\,\upsilon_0 = \frac{hc}{\lambda_0}$$

Here υ_0 is the threshold or cur off frequency and λ_0 is known as cut off wavelength.

49.2.7 Threshold Frequency or Cut Off Frequency

It is the minimum frequency of incident light at which the photoelectric effect takes place.



A sketch of KE_{max} versus the frequency of incident light for photoelectrons in a typical photoelectric effect experiment. Photons with frequency less than f_c don't have sufficient energy to eject an electron from the metal.

Einstein's Photo Theory or Quantum Theory of Photoelectric Effect

When a incident light of suitable energy is exposed to the metal surface, then a part of this energy Φ is utilized in ejecting electron from metal surface and the excess energy $(h\upsilon - \Phi)$ becomes the kinetic energy of photoelectrons. If the electrons does not lose any energy by the internal collision as it escapes from the metal, then its energy will be maximum $K.E_{max}$ which can be described by the formula:

$$K.E_{max} = h\upsilon - \Phi$$

As $K.E_{max} = e\,V_0$ and $\Phi = h\,\upsilon_0$

$$e\,V_0 = h\upsilon - h\,\upsilon_0 \qquad \text{------------} \quad (1)$$

$$\Rightarrow e\,V_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

From equation (1), it is clear that the stopping potential V_0 and the frequency of light υ has a linear relationship and graph between these quantities is a straight line.

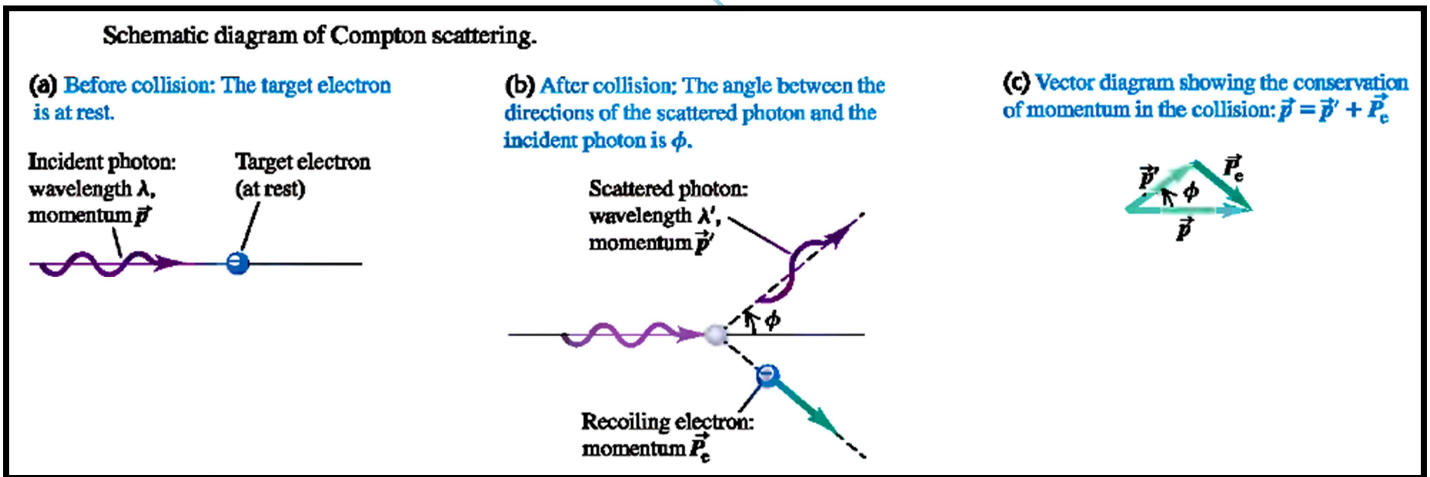
In 1916, Millikan showed that Einstein's equation agreed with experiments in every detail. Hence the photon theory explains that, if the frequency of light is less than the threshold frequency υ_0, then no photoelectrons will be emitted, how much the intensity of light may be.

If the frequency of light is equal to greater than cut off value υ_0, then weakest possible beam of light can produce photo electric effect. Because the energy of one photon depends upon the frequency of light i.e., $E = h\upsilon$ and not on the intensity of light.

49.3 The Compton Effect

In 1923, Compton performed an experiment and he observed that the wavelength of X-rays changes after scattering from a graphite target. Compton explained his experimental results postulating that the incident X-rays beam consists of photons, and these photons experienced Billiard-Ball like collision with the free electrons in the scattering target.

The experimental set-up for observing the Compton effect is shown in the figure below.



Compton effect can be explained by considering the elastic collision between X-ray photon and electrons.

Consider an incident photon having energy and momentum $E = h\upsilon = \frac{hc}{\lambda}$ and $p = \frac{h}{\lambda}$ respectively. The photon is scattered by a stationary electron along an angle Φ with its regional direction, as shown in the figure.

The energy and momentum of the scattered photon are $E' = h\upsilon' = \frac{hc}{\lambda'}$ and $p' = \frac{h}{\lambda'}$ respectively. The rest mass energy of electron is m_0c^2, which is recoiled making an angle θ with the original direction of incident photon.

During the elastic collision, both the energy and momentum remains conserved. So the energy equation in this case is:

$$h\upsilon + m_0c^2 = h\upsilon' + m\,c^2$$

$$\frac{hc}{\lambda} + m_0c^2 = \frac{hc}{\lambda'} + m\,c^2$$

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_0c^2 = m\,c^2$$

Dividing equation by 'c', we get:

$$\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0c = m\,c$$

Squaring both sides of the equation,

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0c\right)^2 = (m\,c)^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + {m_0}^2c^2 - 2\frac{h^2}{\lambda\lambda'} - \frac{2m_0c\,h}{\lambda'} + \frac{2m_0c\,h}{\lambda} = m^{\,2}c^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'} + 2m_0c\,h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = m^{\,2}c^2 - {m_0}^2c^2 \quad \text{---------} \quad (1)$$

The conservation of momentum along the original direction of incident photon (along x-axis) is

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\Phi + mv\cos\theta$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\Phi = mv\cos\theta$$

Squaring both sides, we get:

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2}\cos^2\Phi - 2\frac{h^2}{\lambda\lambda'}\cos\Phi = m^2v^2\cos^2\theta \quad \text{---------} \quad (2)$$

The conservation of momentum along the direction perpendicular to the original direction of photon (y-axis) is:

$$0 + 0 = \frac{h}{\lambda'}\sin\Phi - mv\sin\theta$$

$$\frac{h}{\lambda'}\sin\Phi = mv\sin\theta$$

Squaring both sides, we get:

$$\frac{h}{\lambda'}\sin\Phi = mv\sin\theta$$

$$\frac{h^2}{\lambda'^2}\sin^2\Phi = m^2v^2\sin^2\theta \qquad \text{---------} \quad (3)$$

Adding equation (2) and (3)

$$\frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}(\cos^2\Phi-\sin^2\Phi)-2\frac{h^2}{\lambda\lambda'}\cos\Phi = m^2v^2(\cos^2\theta+\sin^2\theta)$$

$$\frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-2\frac{h^2}{\lambda\lambda'}\cos\Phi = m^2v^2 \qquad \text{---------} \quad (4)$$

Subtracting equation (4) from equation (1), we get:

$$2\frac{h^2}{\lambda\lambda'}\cos\Phi-2\frac{h^2}{\lambda\lambda'}+2m_0c\,h\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) = m^2c^2-m^2v^2-m_0^2c^2$$

$$= m^2(c^2-v^2)-m_0^2c^2$$

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$=\frac{m_0^2(c^2-v^2)}{\left(1-\frac{v^2}{c^2}\right)}-m_0^2c^2$$

$$=\frac{m_0^2(c^2-v^2)}{\left(\frac{c^2-v^2}{c^2}\right)}-m_0^2c^2$$

$$=m_0^2c^2-m_0^2c^2$$

$$=0$$

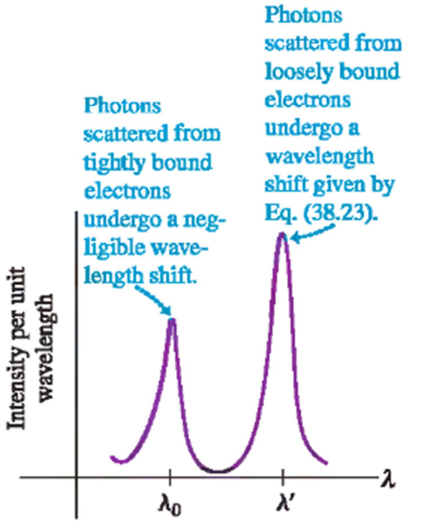
$$\Rightarrow 2\frac{h^2}{\lambda\lambda'}\cos\Phi-2\frac{h^2}{\lambda\lambda'}+2m_0c\,h\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=0$$

$$\Rightarrow 2m_0c\,h\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=2\frac{h^2}{\lambda\lambda'}-2\frac{h^2}{\lambda\lambda'}\cos\Phi$$

$$\Rightarrow 2m_0c\,h\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right)=2\frac{h^2}{\lambda\lambda'}(1-\cos\Phi)$$

$$\Rightarrow (\lambda'-\lambda)=\frac{h}{m_0c}(1-\cos\Phi)$$

Intensity as a function of wavelength for photons scattered at an angle of 135° in a Compton scattering experiment.



Govt. Degree College Naushera
District Khushab