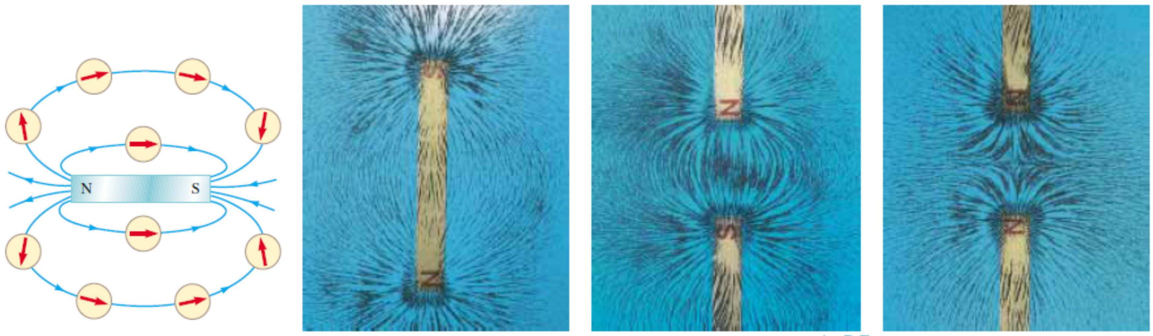
MAGNETIC FIELD EFFECTS

34.1 Magnetic Field

Magnetic field is the region or space around any charge within which its influence can be felt by other magnetic substances.

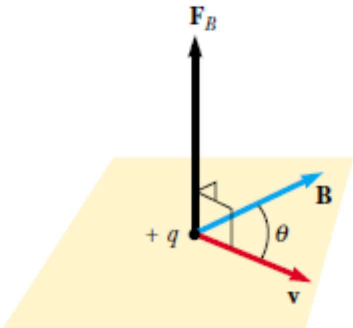
The magnetic field around any magnet is considered as closely spaced magnetic field lines. The magnetic field lines of a bar magnet can be traced with the aid of a compass as shown in the figure below:



In addition to a bar magnet, a moving charge or a current creates a magnetic field in the surrounding space (in addition to its electric field). The magnetic field exerts a force F on any other moving charge or current that is present in the field.

34.2 Magnetic Force on a Charged Particle

Consider a charged object having charge q is projected a uniform magnetic field of flux density **B** with velocity **v**. The magnetic force **F** acting on the object can be expressed as:



$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

$$\mathbf{F} = qvB \sin\theta \,\hat{n} \qquad \text{------------------------} \quad (1)$$

Here $\hat{n}$ is the unit vector that is perpendicular to the plane of **v** and **B**, and is used to describe the direction of magnetic force **F** on the charged object.

It is clear from equation (1) that the maximum magnetic force will act on the charged object when it will be projected perpendicular to the magnetic field. The maximum magnetic force on the charged object will be

$$F = qvB$$

$$\Longrightarrow B = \frac{F}{qv}$$

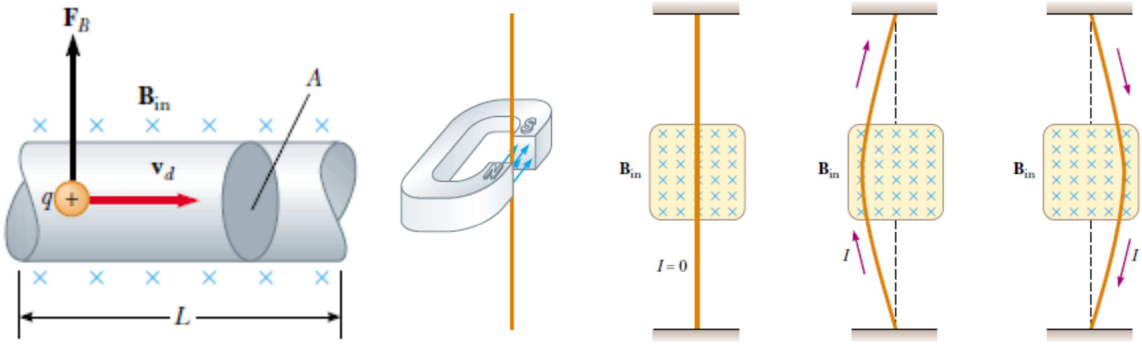
B is also called the magnetic field induction which can be defined as:

"*The force on a unit positive charge moving perpendicular to the magnetic field with uniform velocity*". The SI unit of magnetic field induction **B** is tesla, while in cgs system of units **B** is measured in gauss.

$$1\ tesla \ = \ 10^4\ gauss$$

34.3 Magnetic Force on a Current Carrying Conductor

Consider a current carrying wire of length L and cross-sectional area A, is placed in a uniform magnetic field of flux density **B** as shown in the figure below:



If n is the number of free charges per unit volume of a conductor (each having charge e), then the total charge flowing through the wire is $q = nALe$.

Suppose the charges are moving with drift velocity $\mathbf{v_d}$ and cover length L in t seconds, then

$$t = \frac{L}{v_d}$$

If I is the current flowing through the conductor, then

$$I = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$\Longrightarrow v_d = \frac{I}{nAe}$$

Now

Force on one charge $F' = evB_{\perp}$

Force on nAL charges $F = nALF' = nALevB_{\perp}$

Putting value of v_d, we get

$$F = nALe\left(\frac{I}{nAe}\right)B_{\perp} = ILB_{\perp}$$

In vector form:

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B})$$

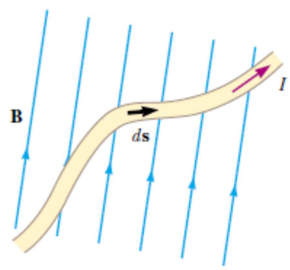
The direction of F will be perpendicular to the plane of **L** and **B**.

If the wire is not straight or field is not uniform, then we divide the wire into small elements of length ‘**ds**’. Then the force on each segment is written as:

$$\mathrm{d}\mathbf{F} = I(\mathrm{d}\mathbf{s} \times \mathbf{B})$$

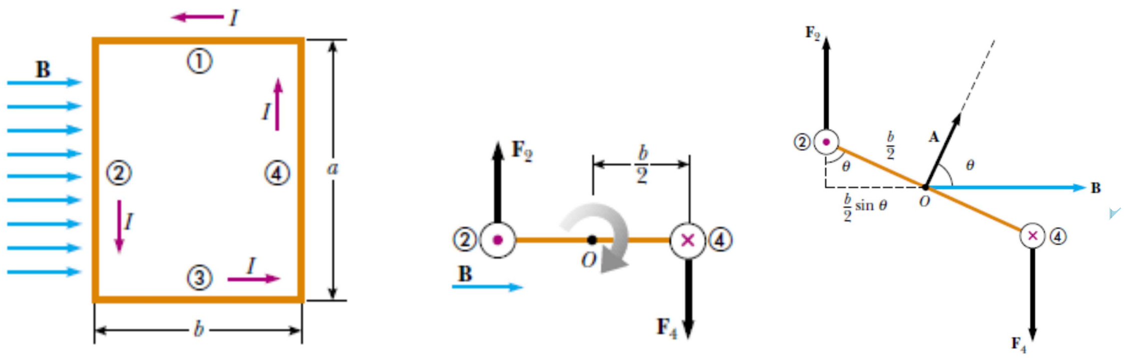
Then the force on whole wire is obtained by integrating over the whole length L.

$$\mathbf{F} = \int \mathrm{d}\mathbf{F} = \int I(\mathrm{d}\mathbf{s} \times \mathbf{B})$$



34.4 Torque on a Current Loop in Magnetic Field

Consider a loop of wire carrying current suspended in a uniform magnetic field of flux density **B** as shown in the figure below:



We want to find out the expression of torque produced in current carrying coil due to the effect of magnetic field.

The side 1 and 3 of coil are oriented parallel, while the sides 2 and 4 are held perpendicular to the magnetic field. No magnetic forces act on sides 1 and 3 because these wires are parallel to the field; hence $\mathbf{L} \times \mathbf{B} = 0$, for these sides. However, magnetic forces do act on sides 2 and 4 because these sides are oriented perpendicular to the field. The magnitude of these forces is

$$F_2 = F_4 = F = ILB \sin\theta = IaB \sin 90^{\circ} = IaB \qquad \because L = a$$

The direction of F_2, the magnetic force exerted on wire 2, is out of the page and that of F_4, the magnetic force exerted on wire 4, is into the page. These two anti parallel forces are separated by a small distance form a couple. The couple tends to rotate the loop about point O. The magnitude of torque of this couple will be:

$$Torque = (Force)\,(Perpendicular\ Distance\ between\ line\ of\ action\ of\ Force)$$

$$\tau = (F)(b\sin\theta) = (IaB)(b\sin\theta) = IabB\sin\theta$$

where θ is the angle between the magnetic field **B** and vector area **A** of loop.

$$\tau = IAB\sin\theta \qquad \because A = ab = area\ of\ loop$$

This is the expression of torque produced in a loop with single turn. For a loop with N turns, the torque will be:

$$\tau = NIAB\sin\theta$$

This result shows that the torque has its maximum value IAB when the field is perpendicular to the normal to the plane of the loop ($\theta = 90°$), and is zero when the field is parallel to the normal to the plane of the loop ($\theta = 0°$).

In vector form:

$$\boldsymbol{\tau} = NI(\mathbf{A} \times \mathbf{B})$$

This is the expression of the torque on a current carrying loop in a magnetic field.

34.5 The Magnetic Dipole

The electric dipole placed in an electric field will experience a torque, which is expressed as:

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

where $p = qd$ is the electric dipole moment. The magnitude of the torque produce in an electric dipole in an electric field is expressed as:

$$\tau = pE\sin\theta$$

where θ is the angle between **p** and **E**.

Similarly, the expression of torque acting on a magnetic dipole placed in a magnetic field of flux density **B** is described as:

$$\boldsymbol{\tau} = NI(\mathbf{A} \times \mathbf{B})$$

The magnitude of the torque is

$$\tau = NIAB\sin\theta \qquad \text{------------------------} \quad (1)$$

By analogy with the electrical case, we define a vector $\boldsymbol{\mu}$, the magnetic dipole moment, to have the magnitude:

$$\mu = NIA \qquad \text{------------------------} \quad (2)$$

Thus, the equation (1) becomes

$$\tau = \mu B\sin\theta$$

In vector form,

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \qquad \text{------------------------} \quad (3)$$

Equation (3) gives the torque on a current carrying loop in a magnetic field of flux density **B**.

The magnetic dipole consist of two opposite magnetic poles (north and south), which are separated by a small distance. And the magnetic dipole moment is a vector associated with a magnet or a current loop, whose cross product with magnetic field strength is equal to the torque exerted on the system by the field.

The work done on a magnetic dipole to change its orientation in the magnetic field is stored as potential energy of magnetic dipole which is given as:

$$P.E = \text{Work Done}$$

$$P.E = \int \tau \, d\theta$$

$$= \int \mu B \sin\theta \, d\theta$$

$$= \mu B \int \sin\theta \, d\theta$$

$$= \mu B(-\cos\theta)$$

$$= -\mu B\cos\theta$$

$$\Rightarrow P.E = -\boldsymbol{\mu}.\mathbf{B} \qquad \text{------------------------} \quad (4)$$

This is the expression for P.E. of magnetic dipole.

The equation (4) tells us that the unit of magnetic dipole moment is obtained by dividing by energy unit by the unit of magnetic induction. Therefore,

$$\text{Unit of Dipole Moment } \mu = \frac{joule}{tesla} = \frac{J}{T} = JT^{-1}$$

The other unit of dipole moment is described by using the equation (2):

$$\mu = NIA$$

$$\text{Unit of Dipole Moment } \mu = (ampere) \times (meter)^2 = A - m^2$$

Question. Prove that $\frac{\boldsymbol{joule}}{\boldsymbol{tesla}} = (\boldsymbol{ampere}) \times (\boldsymbol{meter})^2$

$$\text{L. H. S.} = \frac{joule}{tesla} = \frac{(newton) \times (meter)}{tesla}$$

$$\because work\ W = (force\ F) \times (displacement\ d)$$

$$= \frac{(ampere \times meter \times tesla) \times (meter)}{tesla}$$

$$\because force\ F = (current\ I) \times (length\ L) \times (magnetic\ induction\ B)$$

$$= \frac{ampere \times (meter)^2 \times tesla}{tesla} = ampere \times (meter)^2 = \text{R. H. S.}$$

Hence Proved.

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