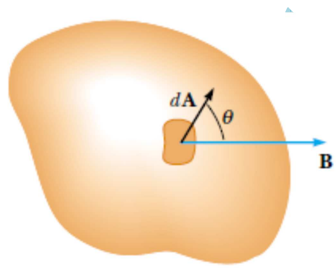
FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

36.1 Magnetic Flux

The number of magnetic lines of force passing normally through certain area is called magnetic flux. It is denoted by Φ_B. It is a scalar quantity and its SI unit is weber (Wb). It is measured by the product of magnetic field strength and the component of vector area parallel to magnetic field.

If d**A** is the vector area element of the surface placed in uniform magnetic field of magnetic field strength **B** as shown in the figure below.



The magnetic flux $d\Phi_B$ through d**A** is given by:

$$d\Phi_B = \mathbf{B}.\mathrm{d}\mathbf{A}$$

$$\Phi_B = \int \mathbf{B}.\mathrm{d}\mathbf{A}$$

$$\Phi_B = \int B\ dA \cos\theta$$

where θ is the angle between magnetic field strength and vector area element.

36.2 Faraday's Law of Electromagnetic Induction

Statement:

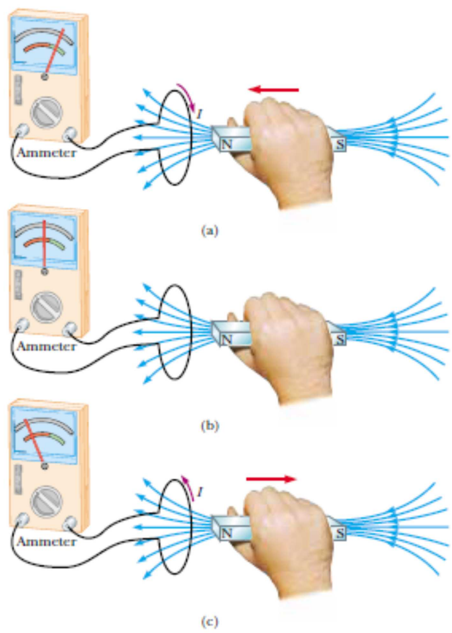
The induced emf in a circuit is equal to the negative of rate at which the magnetic flux through the circuit is changing with time.

Or

The magnitude of induced emf in a circuit is directly proportional to the rate of change of magnetic flux.

Explanation

When a magnet is moved toward the loop, the ammeter needle deflects in one direction, as shown in the figure (a). When the magnet is brought to rest and held stationary relative to the loop figure (b), no deflection is observed. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in figure (c). Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects. From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field.



These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit. We call such a current the induced current which is produced by an induced emf. This phenomenon is called electromagnetic induction.

36.2.1 Integral form of Faraday's Law of Electromagnetic Induction

If ε is the induce emf due to change in flux $d\Phi_B$ in time dt, then we can describe the Faraday's law of electromagnetic induction as:

$$\varepsilon \quad \alpha \quad -\frac{d\Phi_B}{dt}$$

$$\varepsilon \quad = -\text{ constant } \frac{d\Phi_B}{dt}$$

$$\varepsilon \quad = -\frac{Nd\Phi_B}{dt}$$

Where N is constant and called number of turns in a coil.

Now as $\Phi_B = \int_s \mathbf{B}.\mathrm{d}\mathbf{A}$, therefore

$$\varepsilon \quad = -N\frac{d}{dt}\int_s \mathbf{B}.\mathrm{d}\mathbf{A}$$

For a single loop coil, $N = 1$, we have:

$$\varepsilon \quad = -\frac{d}{dt}\int_s \mathbf{B}.\mathrm{d}\mathbf{A} \qquad \text{----------------} \ (1)$$

Also,

$$\varepsilon \quad = \int \mathbf{E}.d\mathbf{r} \qquad \text{----------------} \ (2)$$

Combining equation (1) and (2), we get:

$$\int \mathbf{E}.d\mathbf{r} = -\frac{d}{dt}\int_s \mathbf{B}.\mathrm{d}\mathbf{A} \qquad \text{----------------} \ (3)$$

This is the integral form of Faraday's law of electromagnetic induction.

The negative sign is due to the fact that the direction of induced current is such that it opposes the cause producing it.

36.2.2 Differential form of Faraday's Law

Applying the Stoke's theorem on L.H.S of equation (3),

$$\int_s curl\ \mathbf{E}.d\mathbf{A} = -\frac{d}{dt}\int_s \mathbf{B}.\mathrm{d}\mathbf{A}$$

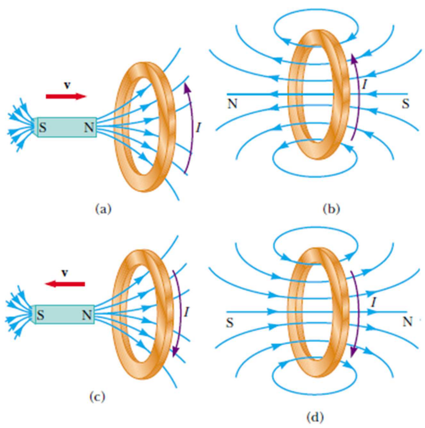
$$\int_s \left(curl\ \mathbf{E} + \frac{d\mathbf{B}}{dt}\right).d\mathbf{A}$$

$$\Rightarrow curl\ \mathbf{E} + \frac{d\mathbf{B}}{dt} = 0$$

$$\Rightarrow curl\ \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

Or $\quad \nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$

This is differential form of Faraday's law of electromagnetism induction.



36.3 Lenz law

Statement:

The direction of induced current is such that it opposes it own cause.

Explanation:

Consider a bar magnet moves toward a stationary metal loop, as in figure (a). As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. As the result, the induced current set up in the loop which produces magnetic field, as illustrated in figure (b). Knowing that like magnetic poles repel each other, we conclude that the left face of the current loop acts like a north pole and that the right face acts like a south pole.

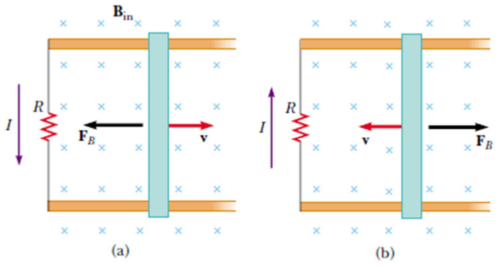
If the magnet moves to the left, as in figure (c), its flux through the area enclosed by the loop decreases in time. Now the induced current in the loop produces the magnetic field as shown in figure (d). In this case, the left face of the loop is a south pole and the right face is a north pole.

36.3.1 Lenz Law and Conservation of Energy

"Lenz law is the statement of law of conservation of energy for the circuit involving induced current"

To understand this statement, consider a conducting bar moving to the right on two parallel rails in the presence of a uniform magnetic field as shown in the figure below. As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases. As the result, the induced current must be directed counterclockwise when the bar moves to the right. Since the current carrying bar is moving in the magnetic field, it will experience a magnetic force $\mathbf{F_B}$. By using right hand rule, the direction of $\mathbf{F_B}$ is opposite to that of **v**, that tends to stop the rod. An external dragging force must be applied to keep the rod moving in the magnetic field.

This dragging force provides the energy for the induced currents to flow. This energy is the source of induced current. Thus the electromagnetic induction is exactly according to the law of conservation of energy.



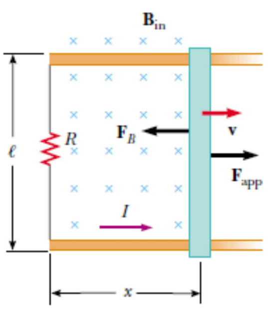
If the bar is moving to the left, as in figure (b), the external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise.

36.4 Motional Induction and Motional emf

The emf induced in a loop by moving it a magnetic field is called motional emf.

Consider a straight conductor of length l is placed in a magnetic field which is directed into the plane of paper. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

As the bar is pulled to the right with a velocity **v** under the influence of an applied force $\mathbf{F_{app}}$, free charges in the bar experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. The magnetic flux passing through the circuit is:



$$\Phi_B = B\,(Area) = B\,(l)\,(x)$$

The induced emf can be determined using Faraday's law of electromagnetic induction:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d\,(B\,l\,x)}{dt} = -B\,l\left(\frac{dx}{dt}\right)$$

$$\varepsilon = -B\,l\,v$$

This is the expression of induced emf in moving conductor. Due to this induced emf, the current induce in the conductor and is given by:

$$I = \frac{\varepsilon}{R} = \frac{-B\,l\,v}{R} \qquad \text{----------------} \quad (1)$$

Where R is the resistance of the loop.

This induced current gives rise to the magnetic forces $\mathbf{F_B} = I\,(\boldsymbol{l} \times \mathbf{B})$ acting on the conductor. The magnitude of magnetic force will be:

$$F_B = I\,l\,B$$

Putting the values of I from equation (1):

$$F_B = \left(\frac{-B\,l\,v}{R}\right) l\,B = \frac{-B^2\,l^2\,v}{R}$$

The negative sign is due to the fact that this magnetic force tends to stop the motion of conductor. In order to move the conductor in this uniform magnetic field with constant velocity, the applied force F_{app} and F_B must be equal and opposite. Therefore,

$$F_{app} = \frac{B^2\,l^2\,v}{R}$$

The power expended in moving the conductor can be find out by the expression:

$$P = F_{app}\,v$$

$$P = \left(\frac{B^2\,l^2\,v}{R}\right) v = \frac{B^2\,l^2\,v^2}{R}$$

This is the expression of power delivered to the conductor t move in magnetic field.

Power Dissipation

The power dissipated due to joule heating can be find out using expression:

$$P_{dissipated} = I^2 R$$

Putting value of I, we get:

$$P_{dissipated} = \left(\frac{-B\,l\,v}{R}\right)^2 R$$

$$P_{dissipated} = \left(\frac{B^2\,l^2\,v^2}{R^2}\right) R$$

$$P_{dissipated} = \frac{B^2\,l^2\,v^2}{R}$$

So in case of motional induction, the power delivered and power dissipation is equal. Therefore the work done by the external agent is dissipated as joule heating.

36.5 Induced Electric Field

A straight current carrying conductor surrounded by a magnetic field. This magnetic field is surrounded by magnetic lines of force which are found to be concentric circles having their centers on the wire.

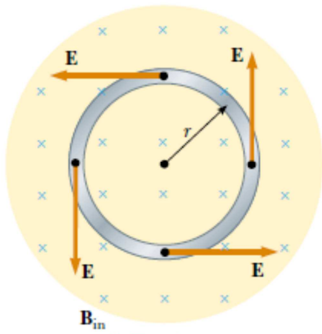
Similarly a changing magnetic field is surrounded by an electric field called induced electric field. This induced electric field is represented by electric lines of force which are found to be concentric circles.

We want to find out the expression of induced electric field due to changing magnetic field. For this, we consider a loop of conducting wire placed in a uniform magnetic field. This magnetic field may be applied by an electromagnet. By varying the current in the electromagnet, we can change the strength of magnetic field.

When **B** is changed, the magnetic flux through the loop changes. So by using the Faraday's law and Lenz law we can find the magnitude and direction of induced emf and induced current.

Consider a circular path of radius r as shown in the figure below.

Since emf is defined as the work done in moving a unit positive charge from one point to another point. For a close path, the work done per unit charge is expressed as:



$$\varepsilon = \oint \mathbf{E} \,.\, d\mathbf{r}$$

If $\mathbf{E} \parallel d\mathbf{r}$,

$$\varepsilon = \oint E \, dr \cos 0°$$

$$\Rightarrow \varepsilon = \oint E \, dr$$

If the rate of change of magnetic flux is constant, then the induced electric field will also be constant. Therefore,

$$\varepsilon = E \oint dr$$

$$\Rightarrow \varepsilon = E \,(\, length\ of\ close\ path \,)$$

$$\Rightarrow \varepsilon = E \,(\, 2\pi r \,) \qquad \text{----------------} \quad (1)$$

By Faraday's law of electromagnetic induction, the induced emf in a coil of single loop will be:

$$\varepsilon = \frac{d\Phi_B}{dt} \qquad \text{----------------} \quad (2)$$

Comparing equation (1) and (2), we have

$$E \,(\, 2\pi r \,) = \frac{d\Phi_B}{dt}$$

$$E = \frac{1}{2\pi r} \frac{d\Phi_B}{dt}$$

This is the expression of magnitude of induced electric field.

36.6 Eddy Currents

When the magnetic flux through a piece of conductor changes, induced currents appears in it. These currents are called eddy currents.

In some cases, the eddy currents produce desirable effects and in some cases they may produce undesirable effects. For example, they increase the internal energy and thus can increase the temperature of material. On the other hand, the principle of eddy currents heating is used in induction furnace. In induction furnace, a sample of material can be heated using the repeatedly changing magnetic field.