Electrical Charge and Coulomb's Law

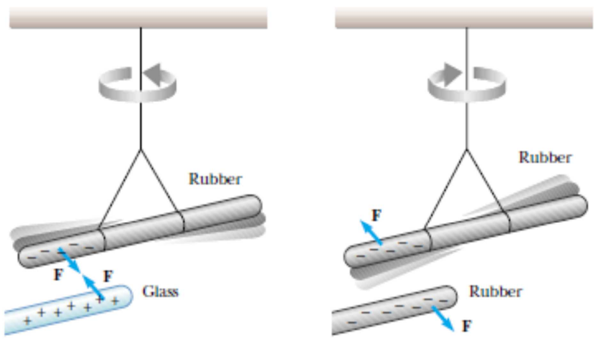
27.1 Electrostatics

The study of properties of charges at rest is called electrostatics.

27.1.1 Electric Charge

A body is said to be electrical neutral if it contains equal number of $+ve$ and $-ve$ charges. When two bodies are rubbed together, their neutrality is distributed due to transfer of electrons from one body to the other. The body which gives electrons becomes electrically $+ve$ and the body which gains electrons becomes negative.

"Charges of the same signs repel each other and
Charges of the oppositely sign attract each other."



These attractive and repulsive forces among the charges are called electrical forces.

27.1.2 Application of Electrical Forces

An electrical force between charged bodies has many industrial applications.

i. Photocopying or xerography
ii. Ink-jet printing
iii. Electrostatic paint spraying
iv. Powder coating etc.

27.1.3 Xerography

The photocopying process is called "Xerography". The main parts of photocopier are (1) Lamp, (2) Rotating drum, (3) Toner, (4) Sheet of paper and (5) Heated roller.

The lamp forms a positively charged image of the document on a rotating drum i.e. Which is an aluminum cylinder coated with a layer of selenium. The negatively charged toner particles stick to the drum at the places of positively charged image. The toner particles are shifted from the drum to a charged sheet of paper, after which they are fused on the paper by heated rollers. This produces a powder print i.e. photocopy.

27.2 Conductors and Insulators

The materials through which charges can flow are called conductors. e.g., metals, human body, tap water etc.

The materials through which charges cannot flow are called insulators. e.g. glass, plastics, wood etc.

If we place charges on an insulator, the charges stay where we put them. But the metallic rod cannot be charged because any charge placed on it easily flow through the rod, through our body and to the ground.

27.2.1 Free Electrons

Hall effect shows in metals, the negative charge (electrons) are free to move. In metals, the atoms are so close to each other, their outermost shell is overlapped. The electrons in the outermost shells are already loosely bound; they are attracted by the neighboring nuclei and become free to move among the lattice atoms. These electrons are called free electrons or conduction electrons. The distinction between conductors and insulators can be made on the basis of number of conduction electrons.

In conductors, each atom contributes one conduction electron. Therefore, there will be on the average about 10^{23} conduction electrons per cm^3.

In insulators, at room temperature, it is very difficult to find even one conduction electrons per cm^3.

Intermediate between conductor and insulators are the semi-conductors e.g. Ge and Si, which might contain $10^{10} - 10^{12}$ conduction electrons per cm^3.The main characteristics of these materials is that the density of conduction electrons can be increased by

i. Adding small quantity impurities (1 atom in $10^6 - 10^9$)
ii. Increase the applied voltage
iii. Increasing the temperature
iv. Increasing the intensity of incident

($\boldsymbol{Definition}$) Point Charges

The charge bodies whose sixes are much smaller than the distance between them are called point charges.

27.3 Quantization of Charges

When the two bodies are rubbed together, transfer of electrons from one body to the other takes place and they are said to be electrified. The magnitude of charge q that can be detected and measured on any object is given by

$$q = ne \qquad \text{---------------------- (1)}$$

where $n = 0, \pm 1, \pm 2, \ldots \ldots \ldots \ldots$ and e is the elementary unit of charge called on unit charge, has the experimentally determined value

$$e = 1.6 \times 10^{-19}\, C$$

When a physical quantity is discrete values, it is called quantized quantity.

Equation (1) shows that charge is also a quantized quantity like matter, energy, angular momentum etc. It means that we can find a body that can have a charge of $10e$ or $-5e$ but it is not possible to find a body with fractional charge such as $+3.57e$ or $-2.35e$.

According to the theory of elementary particles, protons and neutrons are not the elementary particles like electrons. They are consider to be composite particles made up of more fundamental particles called "QUARKS", which have fractional charges of magnitude $+\frac{2}{3}e$ and $-\frac{1}{3}e$.

Protons and neutrons each is made up of three quarks. Proton with the positive charge composed of two $+\frac{2}{3}e$ quarks and one $-\frac{1}{3}e$ quark i.e.,

$$\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = \frac{2e + 2e - e}{3} = \frac{3e}{3} = e$$

Which is the charge of a proton. Neutron with the zero charge is made up of two $-\frac{1}{3}e$ quarks and one $+\frac{2}{3}e$ i.e.,

$$-\frac{1}{3}e - \frac{1}{3}e + \frac{2}{3}e = \frac{-e - e + 2e}{3} = 0$$

Although there is a strong evidence of existence of quarks within the proton and neutrons, but yet it is impossible to create free quark.

27.4 Conservation of Charges

When the two bodies are rubbed together, they are electrified. The process of rubbing does not create charge but only transfer it from one body to the other. Thus the charges can neither be created nor destroyed. This hypothesis is called conservation of charge.

Examples of Charge Conservation

i. **Pair Production**

When a high energy γ-ray photon strike the heavy nucleus, the energy of the photon is converted into an electron-positron pair. This process is called pair production. This process is written as;

$$\gamma \longrightarrow e^- + e^+$$

The net charge is zero on the both sides i.e., the charge is conserved.

ii. **Pair Annihilation**

When an electron of charge $-e$ comes close to a positron of charge $+e$, they disappear and their rest mass energy is converted into radiant energy which appear in the form of two oppositely directed γ-ray photons.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

The net charge is zero on the both sides i.e., the charge is conserved.

iii. **Decay of π^0 −Meson**

The neutral π^0 −Meson decays into two γ-ray photons as

$$\pi^0 \longrightarrow \gamma + \gamma$$

The net charge is zero on the both sides.

In another example of charge conservation, a neutron decays into a proton of charge $+e$ and an electron of charge $-e$ and a neutrino:

$${}_0^1n \longrightarrow {}_1^1p + {}_{-1}^{0}e + \nu$$

The net charge is zero on the both sides i.e., the charge is conserved.

iv. **D-D Reactions**

Two deuterium nuclei fused to tritium and helium as

$${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + {}_1^1H + Energy$$

$${}_1^2H + {}_1^2H \longrightarrow {}_2^3He + {}_0^1n + Energy$$

In these nuclear reactions, charge is constant on both sides.

27.5 Coulomb's Law

Statement: The magnitude of electrical force between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of the distance between their centers.

Mathematical Form

Suppose two point charges q_1 and q_2 separated by distance r. According to the coulomb's law

$$F \quad \alpha \quad q_1 \, q_2$$

$$F \quad \alpha \quad \frac{1}{r^2}$$

$$\Rightarrow F \quad \alpha \quad \frac{q_1 \, q_2}{r^2}$$

$$\Rightarrow F \quad = constant \quad \frac{q_1 \, q_2}{r^2}$$

$$\Rightarrow F \quad = K \quad \frac{q_1 \, q_2}{r^2}$$

where K is called Coulomb's constant. Its value depends upon the system of units and medium between the charges.

For free space and in system international 'K' is expressed as:

$$K = \frac{1}{4\pi \, \varepsilon_0}$$

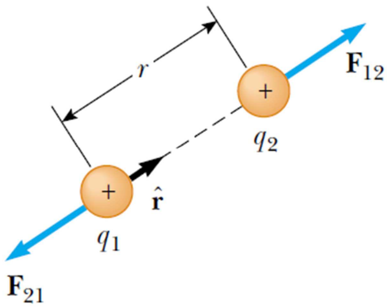
where ε_0 is the permittivity of free space and its value in SI unit is $8.85 \times 10^{-12} \, C^2 N^{-1} \, m^{-2}$.

Question. Show that Coulomb force is a mutual force

Ans. Coulomb's force is a mutual force, it means that if a charge 'q_1' exerts a force on charge 'q_2', then 'q_2' also exerts an equal and opposite force on 'q_1'.

If charge 'q_1' exerts an electrostatic force '$\mathbf{F_{12}}$' on charge 'q_2' and 'q_2' exerts electrical force '$\mathbf{F_{21}}$' on charge 'q_1' and, then

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$



Proof.

If $\hat{\mathbf{r}}_{\mathbf{12}}$ represents the direction of force from charge 'q_1' to 'q_2' and $\hat{\mathbf{r}}_{\mathbf{21}}$ is the unit vector which represent the direction of force from charge 'q_2' to 'q_1', then

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{\mathbf{21}} \quad \text{---------------} \quad (1)$$

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$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \text{----------------} \quad (2)$$

As $\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$, so the eq. (1) becomes

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}(-\hat{\mathbf{r}}_{12})$$

$$\mathbf{F}_{21} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}$$

By eq. (2)

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

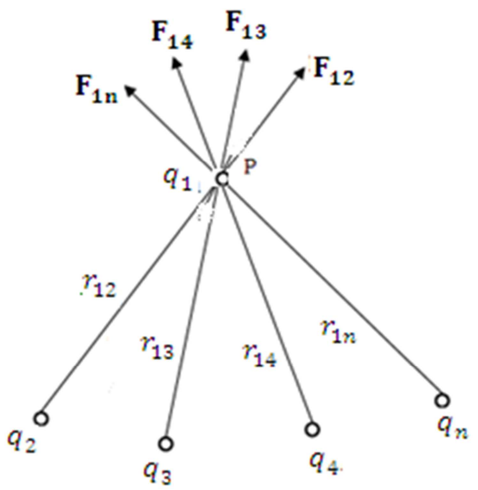
This expression shows that Coulomb force is a mutual force.

27.5.1 Significance of Vector Form of Coulomb's Law

Vector form of coulomb's law has the critical importance, when there is an assembly of point charges. In this case, the resultant force on any one of the charges is the vector sum of the forces due to each of the other forces. This is called principle of superposition.

27.5.2 Coulomb force due to many point charges

Let $q_1, q_2, q_3, \ldots\ldots\ldots, q_n$ are the 'n' point charges as shown in the figure. We want to find out electrostatic force on charge q_1. The point charges $q_2, q_3, q_4, \ldots\ldots\ldots, q_n$ are at the distances $r_{12}, r_{13}, r_{14}\ldots\ldots\ldots, r_{1n}$ from charge q_1, respectively.



Now if $\mathbf{F}_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}\ldots\ldots\ldots, \mathbf{F}_{1n}$ be the electrostatic force on charge q_1 due to the point charges $q_2, q_3, q_4, \ldots\ldots\ldots, q_n$ respectively. Thus, the total electrostatic force $\mathbf{F}_1$ on charge q_1 is given by;

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots\ldots + \mathbf{F}_{1n} \quad \text{---------------------} \quad \mathbf{(1)}$$

where

$\mathbf{F_{12}}$ = Force on charge 'q_1' exerted by 'q_2' = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(r_{12})^2}\ \hat{\mathbf{r}}_{\mathbf{12}}$

$\mathbf{F_{13}}$ = Force on charge 'q_1' exerted by 'q_3' = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{(r_{13})^2}\ \hat{\mathbf{r}}_{\mathbf{13}}$

$\mathbf{F_{14}}$ = Force on charge 'q_1' exerted by 'q_4' = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_4}{(r_{14})^2}\ \hat{\mathbf{r}}_{\mathbf{14}}$

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$\mathbf{F_{1n}}$ = Force on charge 'q_1' exerted by 'q_n' = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_n}{(r_{1n})^2}\ \hat{\mathbf{r}}_{\mathbf{1n}}$

Putting values in equation (1), we get

$$\mathbf{F_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(r_{12})^2}\ \hat{\mathbf{r}}_{\mathbf{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{(r_{13})^2}\ \hat{\mathbf{r}}_{\mathbf{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_4}{(r_{14})^2}\ \hat{\mathbf{r}}_{\mathbf{14}} + \ldots\ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_n}{(r_{1n})^2}\ \hat{\mathbf{r}}_{\mathbf{1n}}$$

$$= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{(r_{12})^2}\ \hat{\mathbf{r}}_{\mathbf{12}} + \frac{q_3}{(r_{13})^2}\ \hat{\mathbf{r}}_{\mathbf{13}} + \frac{q_4}{(r_{14})^2}\ \hat{\mathbf{r}}_{\mathbf{14}} + \ldots\ldots\ldots + \frac{q_n}{(r_{1n})^2}\ \hat{\mathbf{r}}_{\mathbf{1n}}\right)$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\ \frac{q_i}{(r_{1i})^2}\ \hat{\mathbf{r}}_{\mathbf{1i}}$$

27.6 Coulomb's Law and Newton's Law of Gravitation (A Comparison)

The electrical force between two charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them:

$F = k\frac{q_1q_2}{r^2}$ ------------------- **(1)**

The gravitational force between two masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

$F = G\frac{m_1m_2}{r^2}$ ------------------- **(2)**

Similarities among the Electrical and Gravitational Force

(i) Both forces are the conservative forces.

(ii) Both forces obey the inverse square law.

(iii) The charge 'q' plays the same role in the Coulomb's law that the mass 'm' plays in Newton's law of gravitation

Differences among the Electrical and Gravitational Force

(i) Electrical force is might be attractive as well as repulsive, while the gravitational force is only attractive.

(ii) Electrostatic force is medium dependent and can be shielded while the gravitational force lacks this property.

(iii) The value of gravitational constant is very small while the electrical constant is very large. It is because of the fact that gravitational force is very weak as compared to electrical force.

(iv) In using the law of gravitation, we define mass 'm' from $F = ma$, then determine 'G' by applying gravitational law to known masses. While in case of coulomb's law, we define 'K' for a particular value and then determine 'q' by applying coulomb's law.

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