# DC CIRCUITS

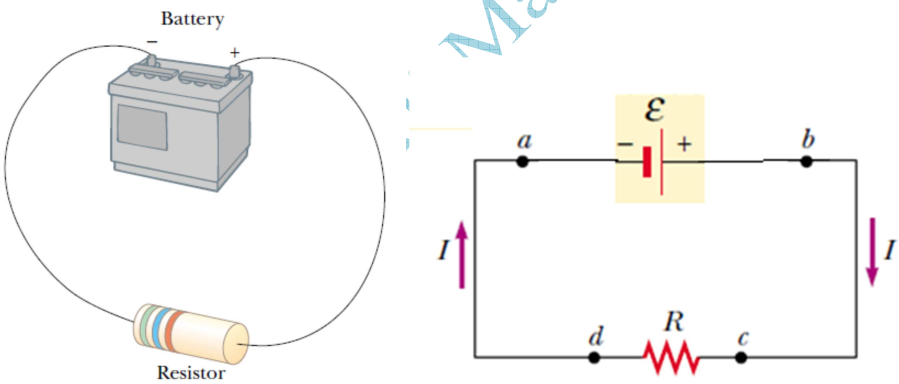
## 33.1 Electromotive Force

The amount of work done per unit charge in moving it in a circuit is called electromotive force. The device that provide electromotive force to charges by maintaining a constant potential difference between the two points of in an electronic circuit is called the source (or seat) of electromotive force. The operation of seat of electromotive force is analogous to the working of a pump which circulates the fluid in a pipe by maintaining a pressure difference between the two points.

The electromotive force is not actually a force: i.e., we don't measure it in newtons. The name originates from the early history of the subject. The unit of emf is joule/coulomb, which is volt:

1 volt = 1 joule/coulomb.

Consider a source of electromotive force $\mathcal{E}$ is connected to a resistor $R$ as shown in the figure below:




The seat of emf maintains its one terminal at a high potential and its other terminal at low potential, as indicated by the + and – signs. Therefore, the emf of the battery would cause positive charge carriers to move in the external circuit as shown by the arrows marked $I$.

In its interior, the seat of emf acts to move positive charges from the point of low potential to the point of high potential. The charges then move through the external circuit, dissipating energy in the process, and return to the negative terminal, from which the emf raises them to the positive terminal again, and the cycle continues.

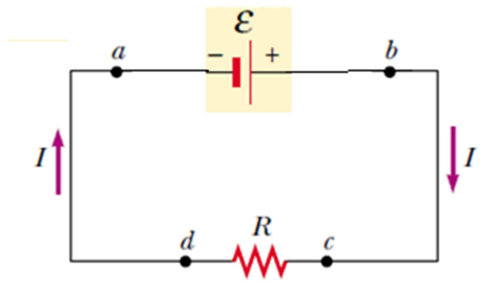
When a steady current has been established in the circuit, a charge $dq$ passes through any cross-section of the circuit in time $dt$. The charge enters the seat of emf at its low-potential end and leaves at its high potential end. In doing so, the seat must do an amount of work $dW$ on charge carriers.

The work done by a seat of emf on charge carriers in its interior must be derived from source of energy within the seat. The energy source may be chemical (battery), mechanical (generator), thermal (thermocouple) or radiant (solar cell). Thus we can describe a seat of emf as a device in which some other forms of energy is change into electrical energy.

The energy provided by the source of emf is stored in the electrical and magnetic fields that surround the circuit. This stored energy does not increase because it is converted into internal energy in the resistor and dissipated as Joule heating, at the same rate at which the energy is supplied. The electric and magnetic fields play the role of intermediary in the energy transferring process.

**33.2 Calculating the Current in a Single Loop using Energy Conservation Principle**

Consider a single loop circuit, as shown in the figure below; containing one seat of emf $\mathcal{E}$ and one resistor R.




In a time $dt$ an amount of energy $I^2R\,dt$ appears in the resistor as internal energy. During this same time a charge $dq(= Idt)$ moves through the seat of emf, and the seat does work on charge given by

$$dW = \mathcal{E}\,dq = \mathcal{E}\,I\,dt$$

From the conservation of energy principle, the work done by the seat must must equal to the internal energy dissipated in the resistor, i.e.,

$$\mathcal{E}\,I\,dt = I^2R\,dt$$

Solving for $I$, we obtain

$$I = \frac{\mathcal{E}}{R}$$

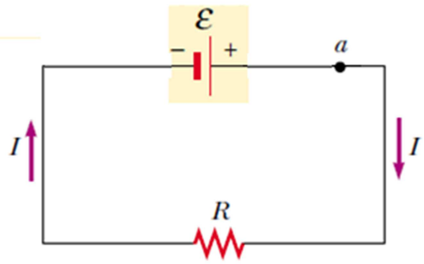
This is the expression of electric current moving through a single loop circuit.

**33.3 Calculating the current in a single loop using Kirchoff Rule**

The Kirchoff's second rule is described as:

*The algebraic sum of the changes in potential encountered in a complete traversal of any close circuit is zero.*

This rule is a particular way of stating the law of conservation of energy for a charge carrier travelling in a closed circuit. Consider the potential at point $a$ of close circuit is $V_a$ as shown in the figure below.




In going through the resistor, there is a change in potential of $-IR$. The minus sign shows that the right side of the resistor is higher in potential than the left side.

Adding the algebric sum of the changes in potential to the initial potential $V_a$ must yeild the identical final value $V_a$, or

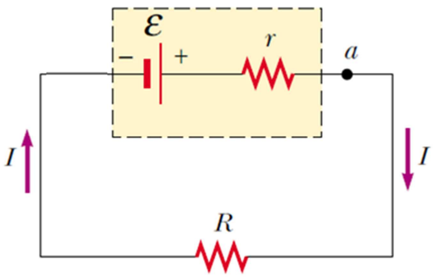
$$-IR + \mathcal{E} = 0$$

$$\Rightarrow I = \frac{\mathcal{E}}{R}$$

**Questions. Write down the expression of electric current in a single loop circuit by considering the internal resistance of a seat of emf.**

**Ans.** All the seats of emf have an intrinsic internal resistance $r$ as shown in the figure.

This resistance can't be removed because it is the inherent part of device. We can apply the Kirchoff's voltage rule (loop rule) starting at any point in the circuit.




Adding the algebric sum of the changes in potential to the initial potential $V_a$ must yeild the identical final value $V_a$, or

$$\Rightarrow -IR + \mathcal{E} - Ir = 0$$

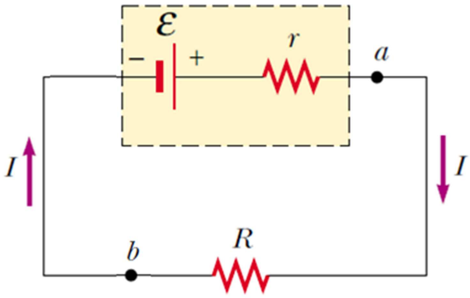
Thus the expression of the current from this single loop of current becomes:

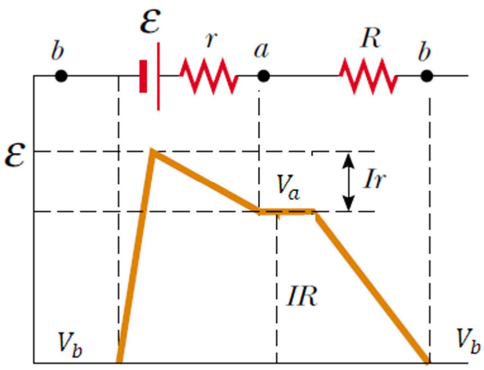
$$\Rightarrow I = \frac{\mathcal{E}}{R+r}$$

Note that the internal resistance $r$ reduces the current that the emf can supply to the external circuit.

## 33.4 Potential Differences between the Points of Circuit

Consider a circuit which consists of a resistor $R$ and a seat of emf $\mathcal{E}$ with internal resistance $r$ as shown in the figure below:




We want to find out the expression of potential difference $V_{ab}$ between the two points $a$ and $b$ of an electrical circuit. If $V_a$and $V_b$ are the potential at points a and b, respectively, than we have

$$V_b + IR = V_a$$

$$\Rightarrow V_{ab} = V_a - V_b = IR \quad \text{-----------------------} \quad (1)$$

As for the single loop, given above, we have

$$I = \frac{\mathcal{E}}{R+r}$$

Thus eq. (1) will become:

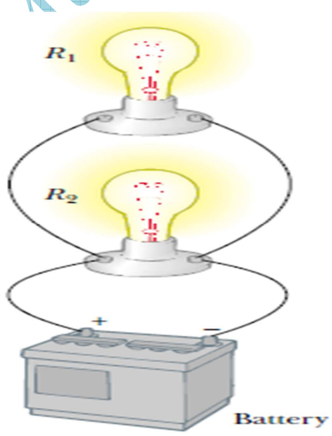
$$V_{ab} = \left(\frac{\mathcal{E}}{R+r}\right) R$$

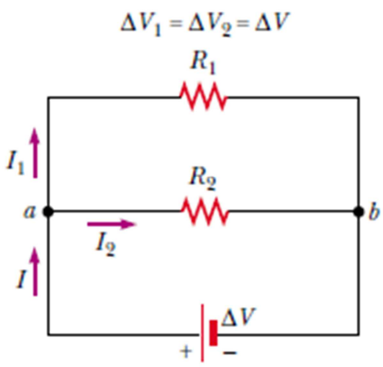
$$\Rightarrow V_{ab} = \mathcal{E}\frac{R}{R+r}$$

This is the expression of electric potential of electric potential difference between the two points of an electrical circuit.

**33.5 Equivalent Resistance Connected in Parallel**

In parallel arrangement a number of resistors are connected side by side with their ends joined together at a common point as shown in the figure below:




According to the properties of a parallel circuit, the total current is shared among the branches, so

$$I = I_1 + I_2 \quad \text{---------------------} \quad (1)$$

In case of parallel combination of resistors, the potential difference across each resistor is $V$, therefore the current through each resistor is:

$$I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2}$$

Thus equation (1) will become:

$$I = \frac{V}{R_1} + \frac{V}{R_2} \quad \text{---------------------} \quad (2)$$

If we replace the parallel combination by a single equivalent resistance $R_{eq}$, the same current $I$ must flow through circuit, i.e.,

$$I = \frac{V}{R_{eq}} \quad \text{---------------------} \quad (3)$$

Comparing eq. (1) and (3), we get

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{V}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

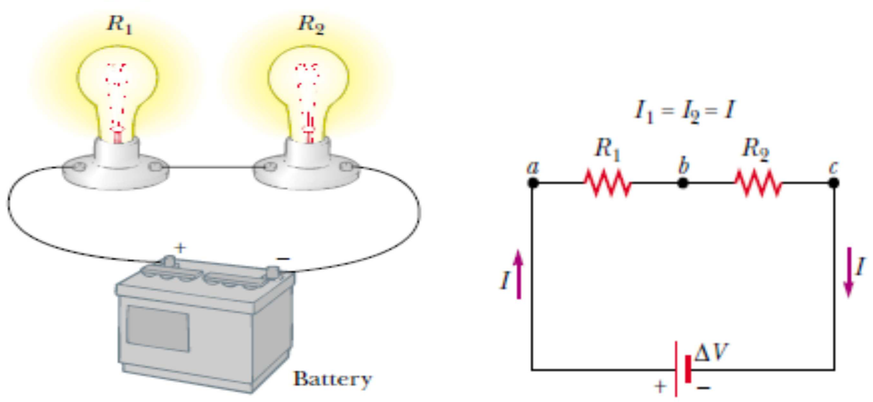
or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

The general expression for the equivalent resistance of a parallel combination of any number of resistor will be:

$$\frac{1}{R_{eq}} = \sum_n \frac{1}{R_n}$$

### 33.6 Resistors Connected in Series

If the resistors are connected end to end such that the same current passes through all of them, they are said to be connected in series as shown in the figure below:




Suppose a battery of potential difference $V$ is connected with the series combination of resistors such that the current $I$ passes through each resistor. The potential difference across each resistor is:

$V_1 = IR_1$ and $V_2 = IR_2$

The sum of potential drop across each resistor must be equal to the potential difference supplied by the battery, i.e.,

$V = V_1 + V_2$ ----------------- (1)

If we replace the combination of resistances with the equivalent resistance, the same current would be established and so

$V = IR_{eq}$

Thus, the equation (1) will become

$IR_{eq} = IR_1 + IR_2$

$R_{eq} = R_1 + R_2$

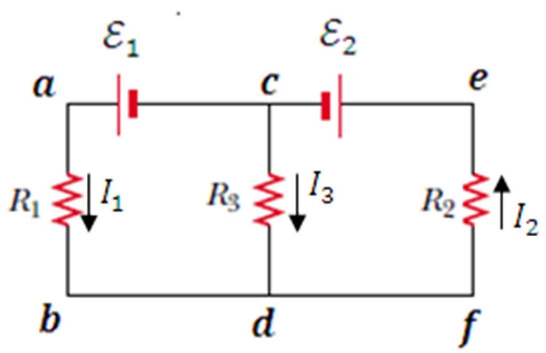
Expending this result to a seires combination of any number of resistors, we obtain:

$R_{eq} = \sum_n R_n$

**33.7 Multiloop Circuit**

The circuits having more than one loop are called multiloop circuit.

Consider a complex network consisting of three resistors circuit consist of $R_1$, $R_2$ and $R_3$ and two batteries of emf $\mathcal{E}_1$ and $\mathcal{E}_2$. We want to find out the unknown currents $I_1$, $I_2$ and $I_3$ move through the circuit, as shown in the figure below:




At junction $d$, the total current entering the junction is $I_1 + I_3$, and the current moving away from junction d is $I_2$. Thus according to the Kirchoff's 1st rule

$I_1 + I_3 = I_2$

$I_1 = I_2 - I_3$ ----------------- (1)

Applying the Kirchoff's 2nd rule on the loop abcd, we get

$\mathcal{E}_1 - I_1R_1 + I_3R_3 = 0$ ----------------- (2)

$\Rightarrow I_3 = \frac{I_1R_1 - \mathcal{E}_1}{R_3}$ ----------------- (3)

Using the kirchoff rule for the loop cdef:

$-\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0$ ----------------- (4)

Putting value of $I_3$ in eqution (3), we get

$-\mathcal{E}_2 - \left(\frac{I_1R_1 - \mathcal{E}_1}{R_3}\right)R_3 - I_2R_2 = 0$

$$-\mathcal{E}_2 - I_1R_1 + \mathcal{E}_1 - I_2R_2 = 0$$

$$\Rightarrow I_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - I_1R_1}{R_2} \qquad \text{-----------------} \ (5)$$

Putting the values of $I_2$ and $I_3$ from equation (4) and (5) in equation (1), we get

$$I_1 = I_2 - I_3$$

$$I_1 = \left(\frac{\mathcal{E}_1 - \mathcal{E}_2 - I_1R_1}{R_2}\right) - \left(\frac{I_1R_1 - \mathcal{E}_1}{R_3}\right)$$

$$I_1 = \left(\frac{\mathcal{E}_1 - \mathcal{E}_2 - I_1R_1}{R_2}\right) - \left(\frac{I_1R_1 - \mathcal{E}_1}{R_3}\right)$$

$$I_1 = \frac{R_3(\mathcal{E}_1 - \mathcal{E}_2 - I_1R_1) - R_2(I_1R_1 - \mathcal{E}_1)}{R_2R_3}$$

$$I_1 = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \text{-----------------} \ (6)$$

Now adding equation (2) and (4), we obtain:

$$\mathcal{E}_1 - I_1R_1 - I_2R_2 - \mathcal{E}_2 = 0$$

$$I_1R_1 = \mathcal{E}_1 - I_2R_2 - \mathcal{E}_2$$

$$I_1 = \frac{\mathcal{E}_1 - I_2R_2 - \mathcal{E}_2}{R_1} \qquad \text{-----------------} \ (7)$$

From equation (4)

$$I_3 = \frac{-\mathcal{E}_2 - I_2R_2}{R_3} \qquad \text{-----------------} \ (8)$$

Putting value of $I_1$ and $I_3$ from equation (7) and (8) in (1), we get

$$I_2 = I_1 + I_3$$

$$I_2 = \left(\frac{\mathcal{E}_1 - I_2R_2 - \mathcal{E}_2}{R_1}\right) + \left(\frac{-\mathcal{E}_2 - I_2R_2}{R_3}\right)$$

Solving this equation, we get

$$I_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2(R_1 + R_3)}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \text{-----------------} \ (9)$$

Putting value of $I_1$ and $I_2$ from equation (6) and (9) in (1), we get

$$I_3 = I_2 - I_1$$

$$I_3 = \left(\frac{\mathcal{E}_1 R_3 - \mathcal{E}_2(R_1 + R_3)}{R_1R_2 + R_2R_3 + R_3R_1}\right) - \left(\frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right)$$

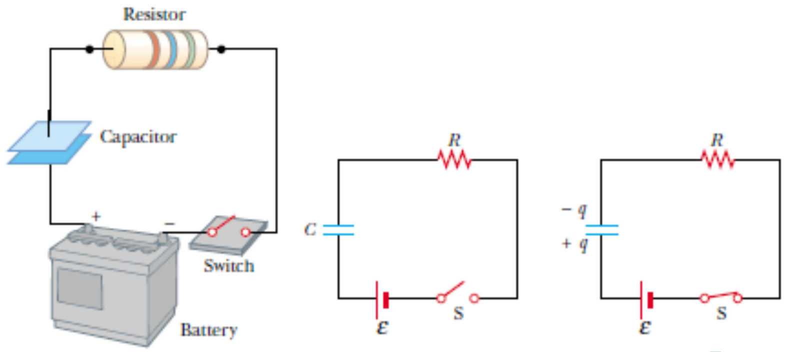
Solving the above equation, we get

$$I_3 = \frac{-\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \text{-----------------} \ (10)$$

Thus equation (6), (9) and (10) are the required expression for unknown currents $I_1$, $I_2$ and $I_3$ in a circuit.

### 33.8 Growth of Charge in an RC Circuit

A circuit containing a series combination of a resistor and a capacitor is called an RC circuit. Consider an RC circuit in series with the battery of emf $\mathcal{E}$ as shown in the figure below:




When the switch S is closed, the growth of charge starts in capacitor. by using the Kirchoff 2nd rule, we get:

$$\mathcal{E} - V_R - V_C = 0$$

$$\Rightarrow \mathcal{E} = V_R + V_C \qquad \text{-----------------(1)}$$

Since R and C are in series, therefore

$$V_R = IR \text{ and } V_C = \frac{q}{C}$$

Putting values in equation (9), we get

$$\mathcal{E} = IR + \frac{q}{C} \qquad \text{----------------- (2)}$$

where $q$ and $I$ denote the current and charge at any time $t$.

Now as $I = \frac{dq}{dt}$, therefore, equation (1) becomes

$$\mathcal{E} = \frac{dq}{dt}R + \frac{q}{C}$$

$$\Rightarrow \mathcal{E}C = \frac{dq}{dt}RC + q$$

$$\Rightarrow \frac{dq}{\mathcal{E}C - q} = \frac{1}{RC}dt$$

$$\Rightarrow -\frac{dq}{\mathcal{E}C - q} = -\frac{1}{RC}dt$$

Integrating, we get

$$\Rightarrow \int \frac{-dq}{\mathcal{E}C - q} = -\frac{1}{RC}\int dt$$

$$\Rightarrow \ln(\mathcal{E}C - q) = -\frac{1}{RC}t + A \qquad \text{----------------- (3)}$$

Where A is the constant of integration. To find $A$, we makes use of initial conditions:

At $t = 0$; $q = 0$, we have

$$\ln(\mathcal{E}C) = A$$

The equation (3) will become:

$$\ln(\mathcal{E}C - q) = -\frac{1}{RC}t + \ln(\mathcal{E}C)$$

$$\Rightarrow \ln(\mathcal{E}C - q) - \ln(\mathcal{E}C) = -\frac{1}{RC}t$$

$$\Rightarrow \ln\left(\frac{\mathcal{E}C - q}{\mathcal{E}C}\right) = -\frac{1}{RC}t$$

$$\Rightarrow \frac{\mathcal{E}C - q}{\mathcal{E}C} = e^{-\frac{1}{RC}t}$$

$$\Rightarrow \mathcal{E}C - q = \mathcal{E}Ce^{-\frac{1}{RC}t}$$

Or

$$q = \mathcal{E}C - \mathcal{E}Ce^{-\frac{1}{RC}t}$$

$$q = \mathcal{E}C - \mathcal{E}Ce^{-\frac{1}{RC}t}$$

$$q = \mathcal{E}C\left(1 - e^{-\frac{1}{RC}t}\right)$$

This equation shows that at $t = \infty$; $q = \mathcal{E}C = q_0$, where $q_0$ is the maximum value of charge on the capacitor. therefore

$$q = q_0\left(1 - e^{-\frac{1}{RC}t}\right) \qquad \text{----------------- (4)}$$

This equation gives the growth of charge in an RC circuit. The equation shows that the charge q goes on increasing and ultimately attains the maximum value $q_0$ after a long time.

**Capacitive time constant**

In the equation (4), the factor RC has the dimensions of time and is called capacitive time constant. It is denoted by $\tau_C$.

The equation (4) is written as:

$$q = q_0\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

**Special Cases:**

- At $t = 0$; $q = 0$
- At $t = \infty$; $q = q_0 = \mathcal{E}C$
- At $t = \tau_C$; $q = q_0\left(1 - e^{-\frac{\tau_C}{\tau_C}}\right)$

$$q = q_0(1 - e^{-1}) = q_0\left(1 - \frac{1}{e}\right) = q_0(1 - 0.37)$$

$$q = 0.63\, q_0$$

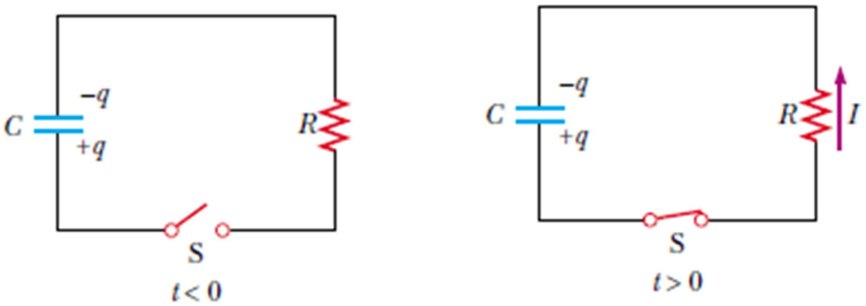
So the capacitance time constant is the time after which the charge on the capacitor grows to 63% of its maximum value.

If we include a resistor with the charging capacitor, the increase of charge of the capacitor towards its maximum value is delayed by a time characterized by the time constant RC. With no resistor present ($RC = 0$), the charge would rise imediately to its limiting value.

Initially, the charge on the capacitor is zero, so the potential difference across the capacitor $V_C = 0$ and potential difference across resistor $V_R = IR$ (maximum) and the maximum current flows through the circuit. As the current flow through the circuit, $V_C$ increases while $V_R$ decreases with time, so the current $I$ through the circuit decreased. The decrease of current will slow down the growth of charge on capacitor with passage of time. When the current I becomes zero, $V_R$ becomes zero and $V_C$ attains the maximum value i.e., the capacitor gets fully charged.

**Decay of Charge in an RC Circuit**

Consider the circuit which consists of a capacitor carrying an initial charge $q$, a resistor $R$, and a switch $S$ as shown in figure below:




When the switch is open, a potential difference $V_C = \frac{q}{C}$ exists across the capacitor and $V_R = 0$ because $I = 0$. If the switch is closed at $t = 0$, the capacitor begins to discharge through the resistor. Applying the loop rule:

$$-V_R - V_C = 0$$

$$\because V_R = IR \text{ and } V_C = \frac{q}{C}$$

$$-IR - \frac{q}{C} = 0$$

$$IR = -\frac{q}{C} \Rightarrow I = -\frac{q}{RC}$$

$$\because I = \frac{dq}{dt}$$

$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$

Integrating both sides, we obtain

$$\int \frac{dq}{q} = -\frac{1}{RC}\int dt$$

$$\ln q = -\frac{1}{RC}t + A \qquad \text{------------------ (1)}$$

By applying the initial conditions, i.e., at $t = 0$; $q = q_0$, we get:

$$A = \ln q_0$$

The equation (1) implies:

$$\ln q = -\frac{1}{RC}t + \ln q_0$$

$$\Rightarrow \ln q - \ln q_0 = -\frac{t}{RC}$$

$$\Rightarrow \ln\frac{q}{q_0} = -\frac{t}{RC}$$

$$\Rightarrow \frac{q}{q_0} = e^{-\frac{t}{RC}}$$

$$\Rightarrow q = q_0 e^{-\frac{t}{RC}} \qquad \text{------------------ (2)}$$

This is the expression for the decay of charge of capacitor in an RC circuit.

**Capacitive Time Constant**

The factor $RC = \tau_C$ is called the capacitive time constant. The equation (2) will become: $q = q_0 e^{-\frac{t}{\tau_C}}$

**Special Cases:**

- At $t = 0$; $q = q_0$
- At $t = \infty$; $q = 0$
- At $t = \tau_C$; $q = q_0 e^{-\frac{\tau_C}{\tau_C}} = q_0 e^{-1} = \frac{q_0}{e} = 0.37\, q_0$

So after time $\tau_C$, the charge of capacitor reduces to 37% of theoratical value.



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