CURRENT AND RESISTANCE

Electric Current

The time rate of flow of charge through a conductor is called current. If a charge 'dq' flows through any cross-section of a conductor in time 'dt', then the current 'I' is given by

$$I = \frac{dq}{dt}$$

The SI unit of current is Ampere, which can be defined as, "when one coulomb charge flows through a cross-section in one second, then the current flowing is one ampere".

When a steady current is flowing through idealized conducting wire, the electric current remains same for all cross-sections, even though the cross-sectional area may be different at different points. The condition of steady current flow is similar to the motion of incompressible fluid. The fluid that flows through any cross-section of the pipe is the same even if the cross-section varies. The fluid flows faster where the cross-section of the pipe is smaller and slower where it is larger, but the volume rate of flow remains constant.

In metals, the charge carriers are electrons. But in electrolytes, the current flow due to motion of negative and positive ions. A positive charge moving in one direction is equivalent in all external effects to a negative charge moving in the opposite the opposite direction. Hence for simplicity and algebraic consistency, we adopt the following convention:

The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative. Thus, the direction of current is taken from the point of higher potential to the point of lower potential.

Even though we assign a direction, current is a scalar quantity, not a vector. The arrow that we draw to indicate the direction of current merely show the sense of charge flow through the wire and is not be taken as a vector. Current does not obey the law of vector addition. Changing the direction of wires does not change the way the currents are added.

Current Density

The current flowing per unit area is called the current density. It is a vector quantity and the SI unit of this quantity is Ampere per square meter ($\frac{A}{m^2}$).

The electric current 'I' can be described in terms of current density as

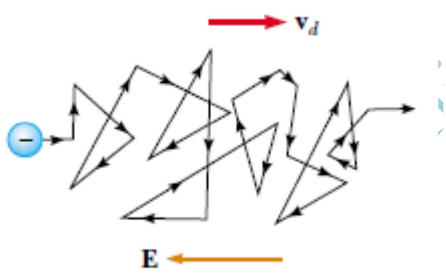
"The scalar product of current density '**J**' and vector area '**A**' is called the electric current". Mathematically,

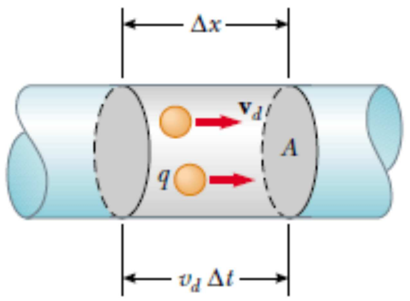
$$I = \mathbf{J}.\mathbf{A}$$

The electric current is macroscopic quantity, while the current density is its corresponding microscopic quantity. In general, if '$d\mathbf{A}$' is the small area element of conductor, then the current flowing through the whole area of conductor is.

$$I = \int \mathbf{J}.d\mathbf{A}$$

Let 'A' is the area of the cross-section of a conductor of length 'L' in which the current 'I' is flowing. The flow of current through a conductor is due to motion of electron in the direction opposite to electric field '**E**'. The force on one electron due to electric field is '$-e\mathbf{E}$'. But this force does not produce any acceleration in the motion of electrons, because the conduction electrons keep on colliding with the lattice ions of conductor. Instead of this, the electrons acquire a constant drift speed 'v_d' in the direction of '$-\mathbf{E}$'.





Let

n = Number of free electrons per unit

AL = Volume of the conductor

nAL = Number of free electrons in the conductor

e = Charge on one electron

$nALe = q$ = Total charge flowing in conductor

If the charge 'q' passes through conductor in time 't', then

$$t = \frac{L}{v_d}$$

So, the current

$$I = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

The current density is

$$J = \frac{I}{A} = \frac{nAev_d}{A}$$

$$J = nev_d$$

The direction of '**J**' is opposite to the direction of flow of electrons.

$$\mathbf{J} = -nev_d$$

The mean drift velocity of electrons is very small i.e., of the order of 'cm/s'. While in random motion, the speed of electrons has a typical value of 10^6 m/s in metals.

Resistance

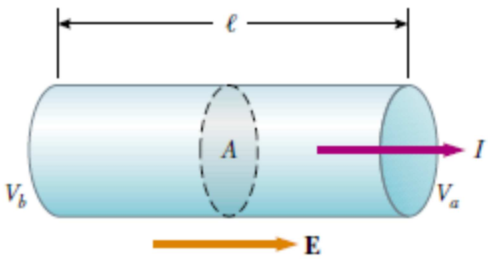
If 'V' is the potential difference between the ends of conductor and 'I' is the current flowing through it, then the resistance of the conductor is

$$R = \frac{V}{I}$$

In system international, its unit is ohm. It is a macroscopic quantity. The corresponding microscopic quantity is resistivity.

Resistivity

The resistance of a conductor of a meter cube of a substance is called resistivity or specific resistance.



As

$$R \propto L \text{ and } R \propto \frac{1}{A}$$

$$R \propto \frac{L}{A}$$

$$R = \rho\frac{L}{A}$$

$$\Rightarrow \rho = \frac{RA}{L}$$

$$\therefore R = \frac{V}{I}$$

$$\rho = \frac{VA}{IL}$$

$$\rho = \frac{V}{\left(\frac{I}{A}\right)L}$$

$$\rho = \frac{V}{\text{J}L}$$

$$\therefore V = EL$$

$$\rho = \frac{EL}{JL}$$

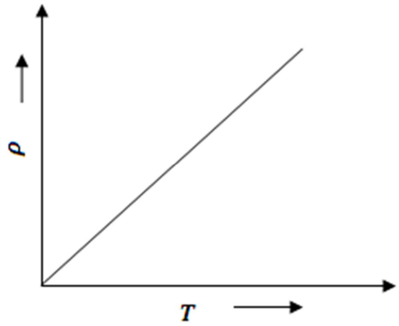
$$\rho = \frac{E}{J}$$

$$E = \rho J$$

As the direction of J and E are the same, so

$$\mathbf{E} = \rho \mathbf{J}$$

This equation is valid only for isotropic materials. The isotropic material are those materials in which the physical properties are same in all direction.



Conductivity

Conductivity 'σ' is reciprocal of resistivity'ρ'. Mathematically

$$\sigma = \frac{1}{\rho}$$

As

$$E = \rho J$$

$$E = \left(\frac{1}{\rho}\right) J$$

$$J = \sigma E$$

$$\mathbf{J} = \sigma \mathbf{E}$$

This is known as microscopic form of ohm's law.

Let 'A' is the area of cross-section of a conductor of length 'L', through which the current 'I' is flowing. Let 'V' is the potential difference between the ends of conductor.

As

$$V = EL \implies E = \frac{V}{L}; \quad J = \frac{I}{A}$$

Therefore

$$\rho = \frac{E}{J} = \frac{V/L}{I/A} = \left(\frac{V}{I}\right)\left(\frac{A}{L}\right) = R\left(\frac{A}{L}\right)$$

$$\implies R = \rho \frac{L}{A}$$

Temperature Variation of Resistivity

The resistivity of the conductor increases with increase in temperature. The temperature dependence of resistivity 'ρ' is shown in the figure.

Let 'ρ_0' and 'ρ' be the values of resistivity at temperature 0˚C and T˚C, respectively. The change of resistivity $(\rho - \rho_0)$ is directly proportional to the resistivity 'ρ_0' and change of temperature $(T - T_0)$. That is

$$(\rho - \rho_0) \propto \rho_0 \ ; \quad (\rho - \rho_0) \propto (T - T_0)$$

So,

$$(\rho - \rho_0) \propto \rho_0 (T - T_0)$$

$$(\rho - \rho_0) = \overline{\alpha}\, \rho_0 (T - T_0)$$

Where $\overline{\alpha}$ is the mean temperature coefficient of resistivity.

$$\overline{\alpha} = \frac{(\rho - \rho_0)}{\rho_0 (T - T_0)}$$

The temperature coefficient of resistivity depends upon the nature of material. The general formula for temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho} \frac{d\rho}{dT}$$

where $\frac{d\rho}{dT}$ is the rate of change of resistivity with respect to temperature. The unit of temperature coefficient of temperature is $°C^{-1}$ or K^{-1}.

Discuss the macroscopic and microscopic forms of Ohm's law.

Ohm's law states that "the potential difference 'V' applied across the ends of a conductor is directly proportional to the current 'I' flowing through it, provided that the temperature of the conductor remains constant". That is

$$V \propto I$$

$$V = RI$$

Where 'R' is constant of proportionality, called the resistance of conductor and it is the measure of opposition against the flow of current. Here V, I and R are the macroscopic quantities. The corresponding microscopic quantities of V, I and R are E, J and ρ, respectively. The macroscopic quantities apply to a particular bodies or extended region, while the microscopic quantities have the particular values at every point in the body. The macroscopic quantities can be found by integrating over microscopic quantities, using the relation given below

$$I = \int \mathbf{J}.d\mathbf{A}$$

$$V_{ab} = -V_{ba} = \int_a^b \mathbf{E}.d\mathbf{s}$$

The current integral is a surface integral, carried out over any cross-section of conductor. The field integral is the line integral carried out along arbitrary line drawn along the conductor, connecting two equipotential surfaces, identified by 'a' and 'b'. For a long wire connected to a battery, equipotential surface 'a' might be chosen as the cross-section of the wire near the positive terminal, and 'b' might be a cross-section near the negative terminal. The resistance of conductor between 'a' and 'b' by the expression

$$R = \frac{V_{ab}}{I} = \frac{\int_a^b \mathbf{E}.d\mathbf{s}}{\int \mathbf{J}.d\mathbf{A}}$$

The microscopic form of ohm's law is

$$E = \rho\, J$$

$$E = \frac{1}{\sigma}\, J$$

$$\therefore \rho = \frac{1}{\sigma}$$

$$J = \sigma E$$

Here 'E' is the electric field intensity, 'J' is the current density,'ρ' is the electrical resistivity and σ is the conductivity of material.

Analogy between the Current and Heat Flow

There is a close resemblance between the flow of current and flow of heat. The current flows due to the difference in the potential and heat flows due to the difference in temperature.

Consider a thin electrically conducting slab of thickness 'Δx' and area 'A'. When the potential difference 'ΔV' is applied at the ends of a conductor, a current 'I' flows through it.

By Ohm's law

$$I = \frac{\Delta V}{R}$$

And $R = \frac{\rho \Delta x}{A}$

$$\Rightarrow I = \frac{\Delta V}{\left(\frac{\rho \Delta x}{A}\right)} = \frac{A}{\rho}\frac{\Delta V}{\Delta x}$$

But

$$I = \frac{dq}{dt}; \frac{\Delta V}{\Delta x}$$

So,

$$\frac{dq}{dt} = -\frac{A}{\rho}\frac{dV}{dx}$$

$$\therefore \frac{1}{\rho} = \sigma$$

$$\frac{dq}{dt} = -A\sigma\frac{dV}{dx} \quad \text{---------------------} \quad (1)$$

The minus sign indicates that the currents flows in the direction of decreasing potential.

If "dQ" is the heat flows through the area 'A' in the small interval of time 'dt', then the rate of flow of heat is

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \quad \text{---------------------} \quad (2)$$

This rate of flow charge is given by eq. (1) and the rate of flow of heat is described in eq. (2). Hence there is a close analogy. Here

$$\frac{dV}{dx} = \text{Potential Gradient}$$

$$\frac{dT}{dx} = \text{Temperature Gradient}$$

Similarly, the electrical conductivity in eq. (1) has the same effect as that of thermal conductivity in eq. (2).

Ohm's Law

In metals, the valance electrons are not attached to the individual atoms but are free to move about within the lattice called conduction electrons. The theory of conduction in metals is usually described on the basis of free electron model. According to this model, the conduction electrons are assumed to move freely throughout the conducting material, somewhat like the molecules of gas in a container. These conduction electron moves randomly like the molecules of gas. The electrons make collisions with atoms and molecules during their random motion. In case of copper, the average speed of electrons in random motion is 1.6×10^6 ms^{-1}.

If the electric field is applied, then the motion of electrons slightly shifted in the direction opposite of that of **E**. Then the force '**F**' acting on the free electron is find out using relation

$$\mathbf{F} = e\mathbf{E}$$

where 'e' is the charge on electron. By using Newton's second law of motion, eq. (1) will become

$$m\mathbf{a} = e\mathbf{E}$$

$$\Rightarrow \mathbf{a} = \frac{e\mathbf{E}}{m}$$

Let λ is the mean free path and τ is the free time between the collisions. If '$\bar{v}$' is the average velocity of the free electrons in random motion, then

$$\lambda = \bar{v}\tau$$

$$\Rightarrow \tau = \frac{\lambda}{\bar{v}}$$

During the collision of free electrons and the atom (or ion core), the tendency of the electron to drift is destroyed. Therefore the average drift speed of the electron will be

$$v_d = a\tau = \frac{eE\tau}{m}$$

Thus the current density J is given by

$$J = nev_d$$

Where n is the number of electrons per unit volume

$$J = ne\left(\frac{eE\tau}{m}\right)$$

$$\frac{E}{J} = \frac{m}{ne^2\tau} \qquad \text{----------------------} \ \textbf{(1)}$$

The resistivity 'ρ' of a conductor is expressed as

$$\rho = \frac{E}{J} \qquad \text{----------------------} \quad \textbf{(2)}$$

Comparing eq. (1) and (2)

$$\rho = \frac{m}{ne^2\tau}$$

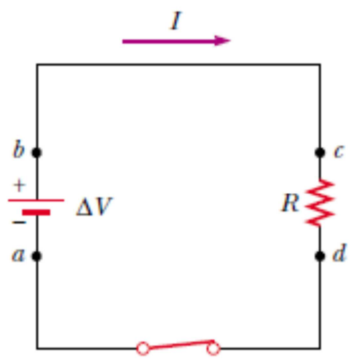
This equation gives the value of electrical resistivity. It is clear from the equation that resistivity of the metal does not depend upon the magnitude of electric field.

Ohmic

A material of a circuit element that obeys the ohm law is called the ohmic. Therefore a conducting device obeys ohm's law, if the resistance between the two points is independent of magnitude and polarity of potential difference.

Energy Transfer in an Electric Circuit and Power Dissipation

Let a battery is connected between the terminals 'a' and 'b' of an electric circuit as shown in the figure.



Let 'ΔV' is the potential difference applied by the battery between the points 'a' and 'b'. As the result the current 'I' flow through the circuit. During this process, energy is transfer from battery to the electrical circuit. Let a small amount of charge 'dq' during the small interval of time 'dt'. Then energy transferred is

$$dU = Vdq$$

For the present case, the power supplied by the battery is dissipated in resistor 'R'. The rate of energy transferred or power dissipated 'P' is

$$P = \frac{dU}{dt} = \frac{Vdq}{dt}$$

$$P = VI$$

$$\Longrightarrow P = (IR)I = I^2R \qquad \because V = IR\ (ohm's\ law)$$

Or

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R} \qquad \because I = \frac{V}{R}$$

So

$$P = VI = I^2R = \frac{V^2}{R} \text{ ---------------------- } \textbf{(1)}$$

This equation is known as Joule's law. Here R is the resistance of the circuit and 'P' is the power dissipated which is equal to the electrical energy per unit time transferred to the circuit of black box.

Joule Heating

The electrical energy consumed in a resistor appears in the form of heat, which is also called 'Joule Heating'. The heat energy produced in t interval of time is given by

Heat Energy = (Power)(Time)

$$= (VI)(t)$$

$$= VIt = I^2R\,t = \frac{V^2}{R}t$$

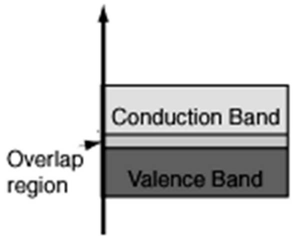
Energy Band Theory and Electrical Behavior of Conductors, Insulators and Semiconductors

Energy band theory

The electrons of an atom have the discrete values of energy which are also called quantized energy levels. The concept of discrete energy level is related to an isolated atoms i.e. the atom that does not interact with other atoms. But if the atom is not alone and is under the influence of its neighboring atoms, then each energy level splits into sub-levels. This group of sub-level is called the energy band. Within the energy band, there are permitted energy states, which are so close together that they are virtually continuous. But there exist an energy gap between these bands, which contains no states that an individual electron may occupy. It is also called forbidden energy gap. The electron may jump from one energy band to another by acquiring energy equal to the energy of forbidden energy gap.

Conductors

All the metals are good conductors of electricity and their resistivity is of the order of $10^{-8}\ \Omega - m$. In case of conductors, there is no forbidden energy gap between the valance and the conduction band. The valance band and conduction band are partially filled at room temperature. So the electrons can easily jump from valance band to the conduction band. Due to this reason, the current can easily pass through conductors.

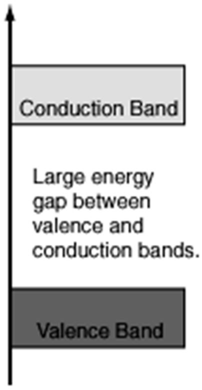


Conductor

The temperature coefficient resistivity is positive. It means that the resistance of conductors increases by increasing the temperature.

Insulators

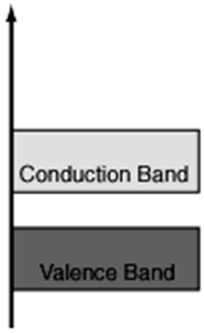
The insulators have the very large value of resistivity which is of the order of $10^{10}\ \Omega - m$. In case of insulators, the valance band is completely filled and the conduction band is empty. The energy gap between the valance and conduction band is very large. Thus, no electron can jump from valence band to conduction band. As there are no free electrons in insulator, hence no current can pass through insulators.



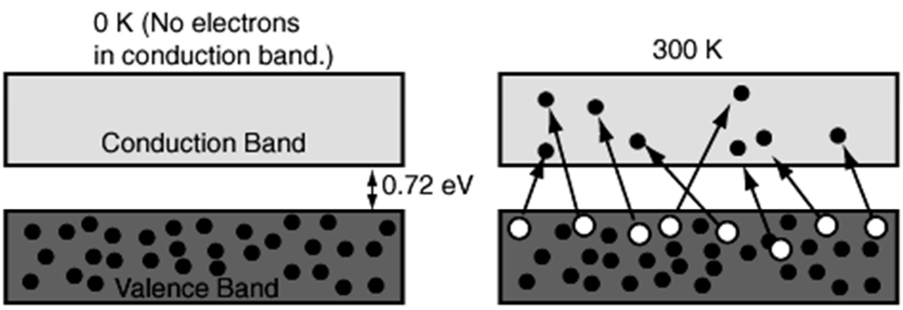
Insulator

Semiconductors

The materials which have intermediate values of resistivity (of the order of $10^{2}\ \Omega - m$) called semiconductor materials. The energy gap between the valance and conduction band is very small. The two most important are germanium and silicon.



Semiconductor



The semiconducting materials have negative temperature coefficient of resistivity. At low temperatures, the valence band is completely filled and conduction band is completely empty. Thus the semiconducting materials behave like insulator at low temperatures.

At comparatively higher temperature, the electrons in valance band acquire sufficient energy to jump in conduction band. As the temperature increases, the probability of the electrons to jump from valance to conduction band increases. Therefore, the conductivity of semiconductors increases with increase in temperature.

The probability of the electrons to jump from valance band to conduction band depends upon the energy distribution factor $e^{\Delta E/kT}$. Here ΔE is the energy gap, k is the boltzman constant and T is the absolute temperature.

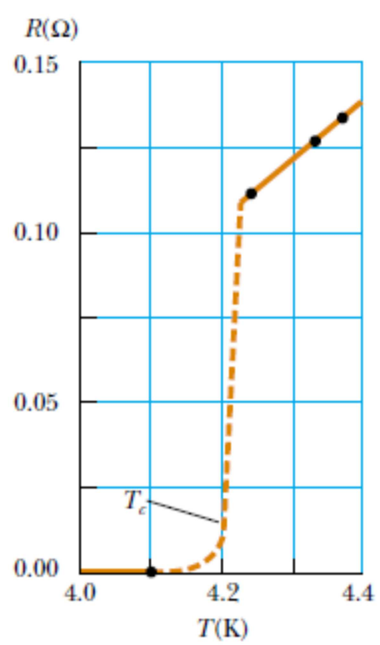
Superconductors

There are some materials whose resistivity becomes zero below a certain critical temperature T_C called critical temperature. below the critical temperature, such materials are called super conductors. Once the resistance of material drops to zero, no energy is dissipated in the material during the flow of current through material. The current established in such materials will continue to exist indefinitely without the source of emf.

In 1911, a Dutch Physicist K. Onnes observed that below 4.2 K, mercury lost its resistivity and became perfect conductor. The resistance versus temperature graph of for a sample of mercury (Hg) is shown in the figure below

The resistance versus temperature graph shows that the electrical resistance of mercury (Hg) drops to zero below the critical temperature $T_C = 4.2\ K$.

In 1986, a series of ceramic materials were discovered which have comparatively high critical temperature of $90\ K$. The research is in progress to discover such materials which show superconducting behavior at room temperature. the interest in the field of superconductivity is due to its following applications:



- The energy can be stored and transported without any resistive loss.
- Superconducting electromagnets can large magnetic field in surrounding space.
- Superconducting components in electronic circuits would generate no joule heating and will permit further miniaturization of the circuits

Important Note:

The best known conductors at room temperature don't show any show any superconductivity at all.