CAPACITORS AND DIELECTRICS

31.1 Capacitor

Capacitor is a device winch is used to store charge. A simple capacitor consists of two conductors which are separated a small distance. There may be vacuum or some dielectric medium between the conductors of a capacitor.

When the plates of a capacitor are connected with the terminals of the battery of emf V, then the charge q is stored in the capacitor. This charge stored is directly proportional to the potential difference applied between the plates.

$$q \propto V$$

$$q = CV$$

Here C is constant of proportionality, called the capacitance of a capacitor. The capacitance of a capacitor is its ability to store electrical charge. The SI unit of capacitance is farad which can be defined as "If one coulomb of charge given to the plates to produce a potential difference of one volt, then capacitance of the capacitor is one farad".

31.1.1 Electric Field between the Plates of Capacitor

Let the two plates of a capacitor which are separated by a distance d. Suppose that the length of plate is very large as compared to the distance between the plates. So, E inside the plates of a capacitor is uniform. We want to find out the expression of electric field between the plates of capacitor. For this we consider a box shaped Gaussian surface as shown in the figure. Then by Gauss's law

$$\oint \mathbf{E} . \mathbf{dA} = \frac{q}{\varepsilon_0}$$

Where A area of the side of Gaussian surface through which the flux is passing, while dA is its infinitesimal element.

$$\oint E \, dA \cos 0^\circ = \frac{q}{\varepsilon_0}$$

$$E \oint dA = \frac{q}{\varepsilon_0}$$

$$EA = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \qquad \text{---------------------} \quad \textbf{(1)}$$

$$\therefore \sigma = \frac{q}{A}$$

The equation is expression of electric field intensity inside the plates of capacitor.

31.1.2 Potential Difference

The potential difference between the plates of charged capacitor is

$$V_f - V_i = -\int_i^f \mathbf{E.ds}$$

where the integral is taken over the path which starts from the positive plate and terminate on negative plate. For the present case, E and ds are in same direction

$$V_f - V_i = -\int_i^f E\,ds\,cos0^\circ = -E\int_i^f ds$$

$$= -E\,d \qquad \text{---------------------- } \mathbf{(2)}$$

If the initial plate is positively charged and final plate in negatively charge, then

$$V_f - V_i = -V$$

Putting value in eq. (2), we get

$$V = E\,d$$

The electric field 'E' between the plates of capacitor is the sum of the field due to both plates, therefore

$$E = E_+ + E_-$$

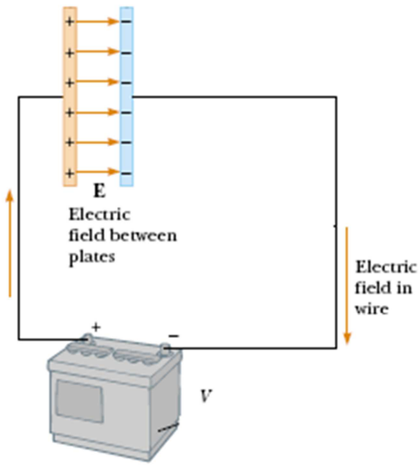
where E_+ is the electric field due to the positive plate and E_- is the electric field due to negative plate. By Gauss's law, E_+ and E_- both are directly proportional to 'q'. so from equation (1), 'V' is also proportional to 'q'. Therefore,

$$\frac{q}{V} = constant$$

This ratio of $\frac{q}{V}$ is called the capacitance.

31.1.3 Capacitance of Parallel Plate Capacitor

Consider a parallel plate capacitor. The size of the plate is very large and the distance between the plates is very small, so the electric field between the plates is uniform.



The electric field 'E' between the parallel plate capacitor is

$$E = \frac{q}{A\varepsilon_0} \qquad \text{----------------- } \mathbf{(4)}$$

Moreover, the potential difference between the parallel plate capacitor is related to the electric field is

$$V = E\,d$$

$$\Longrightarrow E = \frac{V}{d}$$

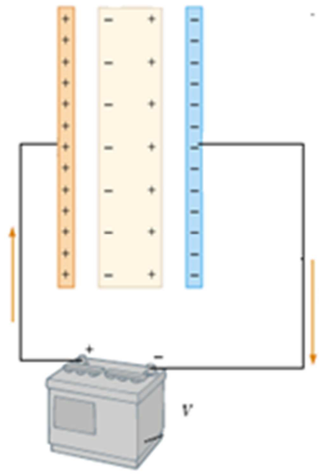
Putting this value in equation (4), we get

$$\frac{V}{d} = \frac{q}{A\varepsilon_0}$$

$$\Longrightarrow \quad \frac{q}{V} = \frac{A\,\varepsilon_0}{d} \qquad\qquad \therefore \frac{q}{V} = C\ (capacitance)$$

$$\Longrightarrow \quad C = \frac{A\,\varepsilon_0}{d}$$

This is the expression for capacitance of a parallel plate capacitor with the free space as the medium between the plates. When any dielectric medium having the dielectric constant 'κ_e' is placed between the plates of the parallel plate capacitor, then the capacitance of parallel plate capacitor will become

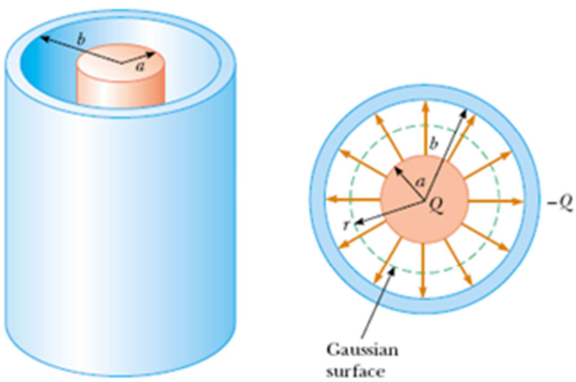


$$C = \frac{A\,\varepsilon_0 \kappa_e}{d} \qquad \text{--------------------} \quad \textbf{(5)}$$

The equation (5) describes that the capacitance of capacitor depends on the geometry of capacitor and medium between the plates.

31.1.4 Capacitance of Cylindrical Capacitor

Consider a cylindrical capacitor of length L, formed by two coaxial cylinders of radii 'a' and 'b'. Suppose $L >> b$, such that there is no fringing field at the ends of cylinders. Let 'q' is the charge stored in the capacitor and 'V' is the potential difference between the plates. The inner cylinder is positively charged while the outer cylinder is negatively charged.



We want to find out the expression of capacitance for the cylindrical capacitor. For this we consider a cylindrical Gaussian surface of radius 'r' such that $a < r < b$.

If 'E' is the electric field intensity on any point of the cylindrical Gaussian surface, then by Gauss's law

$$\oint \mathbf{E}.\,d\mathbf{A} = \frac{q}{\varepsilon_0}$$

$$E\oint dA = \frac{q}{\varepsilon_0}$$

$\therefore$ **E** *is parallel to* **dA**

$$E(2\pi rL) = \frac{q}{\varepsilon_0}$$

$\therefore 2\pi rL = Area\ of\ curved\ part\ of\ Gaussian\ surface$

$$E = \frac{q}{2\pi\varepsilon_0 rL}$$

If 'V' is the potential difference between the plates, then

$$V = \int_{+}^{-} E\,dr = \int_a^b \frac{q}{2\pi\varepsilon_0 rL}\,dr = \frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r}$$

$$V = \frac{q}{2\pi\varepsilon_0 L}\ |\ln r|_a^b$$

$$V = \frac{q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}$$

$$\frac{q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

$\therefore C = \frac{q}{V} = capacitance$

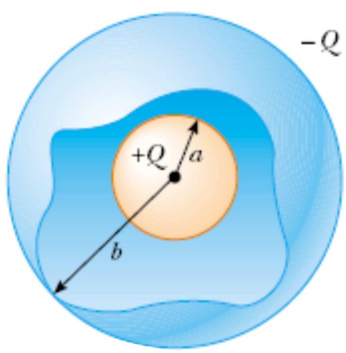
$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

This is the expression for the capacitance of a cylindrical capacitor.

31.1.5 Capacitance of Spherical Capacitor

Consider a spherical capacitor which consist of two concentric spherical shells of radii 'a' and 'b'. Let 'q' is the charge stored in the capacitor and 'V' is the potential difference between the two spherical shells.

We want to find out the expression of capacitance for the spherical capacitor. For this we consider a spherical Gaussian surface of radius 'r' such that $a < r < b$. If 'E' is the electric field intensity on any point of the spherical Gaussian surface, then by Gauss's law



$$\oint \mathbf{E}.\,d\mathbf{A} = \frac{q}{\varepsilon_0}$$

$$E\oint dA = \frac{q}{\varepsilon_0}$$

$$\therefore \mathbf{E}\ and\ \mathbf{dA}\ are\ radially\ outward$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore 4\pi r^2 = surface\ area\ of\ Gaussian\ surface$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

If 'V' is the potential difference between the plates, then

$$V = \int_{+}^{-} E\,dr = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left|-\frac{1}{r}\right|_a^b$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{b} - (-\frac{1}{a})\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\frac{q}{V} = \frac{4\pi\varepsilon_0}{\left(\frac{b-a}{ab}\right)}$$

$$\therefore C = \frac{q}{V} = capacitance$$

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

This is the expression for the capacitance of a spherical capacitor.

31.1.6 Capacitance of an Isolated Sphere

Consider a spherical capacitor which consists of a single isolated sphere of radius 'R'. The other (outer) sphere of this capacitor is the "missing plate" with an infinite radius $= \infty$.

The capacitance of spherical capacitor is

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) = 4\pi\varepsilon_0\left(\frac{a}{\frac{b}{b} - \frac{a}{b}}\right)$$

$$C = 4\pi\varepsilon_0\left(\frac{a}{1-\frac{a}{b}}\right)$$

Here 'a' and 'b' are the inner and outer radii of shells of spherical capacitor, respectively. Putting the values $b = \infty$ and $a = R$, we get

$$C = 4\pi\varepsilon_0 \left(\frac{R}{1 - \frac{R}{\infty}} \right)$$

$$C = 4\pi\varepsilon_0 R$$

This is the expression for the capacitance of an isolate sphere.

31.2 Energy Stored in an Electric Field

Consider a capacitor with the capacitance 'C', which is connected to the battery of emf 'V'. If 'dq' charge is transferred from one plate to other, then the work done 'dW' will be

$$dW = Vdq$$

This work done is stored in the form of electric potential energy 'dU'

$$dU = Vdq$$

When the capacitor is fully charged then the total energy stored is

$$U = \int dU = \int_0^q Vdq$$

$$\therefore V = \frac{q}{C}$$

$$U = \int_0^q \frac{q}{C} dq = \frac{1}{C}\int_0^q q\, dq$$

$$U = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^q = \frac{1}{C}\left(\frac{q^2}{2} - 0\right)$$

$$U = \frac{1}{2}\frac{q^2}{C}$$

$$\therefore q = CV$$

$$U = \frac{1}{2}\frac{C^2V^2}{C}$$

$$U = \frac{1}{2} C V^2$$

The energy stored in the capacitor is the energy store in the electric field between its plates. So, the energy stored can be expressed in terms of electric field strength 'E'.

As $E = Vd$ and $C = \frac{A\,\varepsilon_0}{d}$, therefore

$$U = \frac{1}{2}\left(\frac{A\,\varepsilon_0}{d}\right)(E^2 d^2)$$

$$U = \frac{1}{2}\ \varepsilon_0 E^2 A d$$

This is the expression of energy stored in the electric field between the plates of capacitor.

The energy density 'u' is described as the energy stored 'U' per unit volume 'V'. Mathematically

$$u = \frac{U}{V} = \frac{U}{Ad}$$

$$= \frac{\frac{1}{2}\ \varepsilon_0 E^2 Ad}{Ad}$$

$$u = \frac{1}{2}\ \varepsilon_0 E^2$$

If any dielectric medium having the dielectric constant 'κ_e' is placed between the plates of capacitor, then the expressions of energy stored in the electric field of capacitor 'U' and energy density 'u' will become,

$$U = \frac{1}{2}\ \varepsilon_0 \kappa_e E^2 Ad$$

$$u = \frac{1}{2}\ \varepsilon_0 \kappa_e E^2$$

31.3 Capacitance with Dielectrics

Consider a parallel plate capacitor which is connected with a battery of emf 'V'. Let 'A' is the area of each plate and 'd' is separation between the plates.

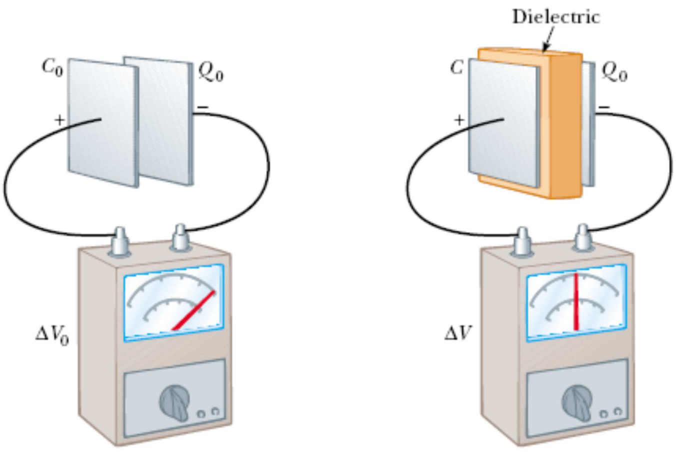
If 'q' charge is stored in the capacitor when there is vacuum or air as medium between the plates, then

$$q = CV$$

Where 'C' is the capacitance of the capacitor, which, for the case of parallel plate capacitor is expressed as

$$C = \frac{A\ \varepsilon_0}{d}$$

Micheal Faraday, in 1937, investigated that if the space between the plates of a parallel plate capacitor is filled with some dielectric medium, then the charge stored in the capacitor increased to 'q''. And, hence, the capacitance of the capacitor also increases to 'C''.



(a) (b)

Therefore, the relation between the stored charge in the capacitor to the capacitance will become

$$q' = C'\,V$$

The factor by which the capacitance of capacitor increases as compared to the capacitance with air as the medium is called the dielectric constant 'κ_e',

$$\kappa_e = \frac{C'}{C}$$

$$\therefore \kappa_e > 1$$

$$C' = \kappa_e C$$

$$C' = \kappa_e\left(\frac{A\,\varepsilon_0}{d}\right)$$

So, $C' > C$ by a factor of 'κ_e'.

The dielectric constant 'κ_e' is a dimensionless quantity. It is also called relative permittivity of the medium.

31.3.1 Effects of Dielectric Medium

- When a dielectric medium, having dielectric constant 'κ_e', is placed between the point charges, then the electrical force between two point charges decreases by a factor of 'κ_e'. The expression of electrical force 'F_e' between two point charges 'q_1' and 'q_2', when the dielectric medium is place between them, is

$$F_e = \frac{1}{4\pi\varepsilon_0\kappa_e}\,\frac{q_1\,q_2}{r^2}$$

- The electrical field intensity 'E' due to a point charge between two point charges decreases, in the presence of a dielectric medium. If 'κ_e' is the dielectric constant of the corresponding dielectric medium, then the electric field intensity at any point due to point chage will be

$$E = \frac{1}{4\pi\varepsilon_0\kappa_e}\frac{q}{r^2}$$

- The electric field near the surface of charged conductor, which is immersed in a dielectric medium of dielectric constant 'κ_e' is

$$E = \frac{\sigma}{\varepsilon_0\kappa_e}$$

where 'σ' is the uniform charge density of conductor.

31.4 Dielectrics: An Atomic View

Dielectrics are the insulating materials through which the electric current cannot pass easily, because these materials have very high value of electrical resistance. For example, paper, pyrex, polystyrence, transformer oil, pure water, silicon etc.

There are two types is dielectrics

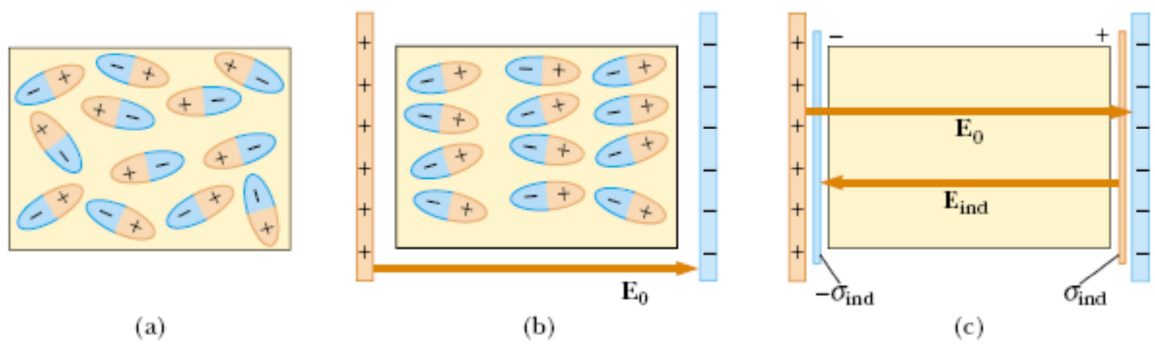
(i) Polar Dielectrics (ii) Non-Polar Dielectrics

31.4.1 Polar Dielectrics

The dielectric materials which have permanent electric dipoles moment '**p**' are called polar dielectrics. These materials consist of molecules which are permanent dipoles.

In the absence of external electric field, the polar molecules are randomly oriented. As the result, these materials have no net dipole moment.

When the external electric field is applied, then all dipoles tend to align themselves with external electric field. But the thermal agitation tends to keep the dipoles randomly oriented. Hence the partial alignment of electric dipoles is produced in a polar dielectric medium for specific electric field strength. However, the alignment of dipoles can be increased by increasing the external electric field and decreasing the temperature.



31.4.2 Non-Polar Dielectrics

The dielectric materials which don't have permanent dipole moments are called non-polar dielectrics. In the absence of external electric field, the atoms are neutral. When the electric field of strength 'E_0' is applied, then it tends to separate to positive and negative charges on the atoms of molecules. As the result, the atoms and molecules of dielectric become dipoles, called induced dipoles and this process is called electric polarization.

If the non-polar dielectric material is placed in a electric field having strength 'E_0', then another electric field 'E'' is produced due to the polarization of medium. The electric field produce due to the polarization of dielectric is always opposite to the direction of external electric field. So, the net electric field in the region is '$E = E_0 - E'$'. Hence, the net electric field is reduced due to polarization of dielectric medium.

31.4.3 Effect of Dielectric Medium on Capacitance of Capacitor

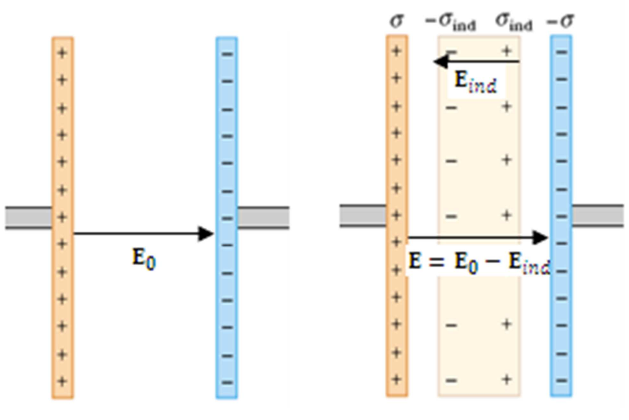
When there is no dielectric medium between the plates of capacitor, the stored charge 'q' in the capacitor can be find out using expression,

$$q = CV$$

But when the dielectric medium is placed between the plates of capacitor, then the net electric field and net potential difference between the plates is reduced due to electric polarization of dielectric medium. Therefore, the battery does work to transfer more charge to increase the potential difference between the plates. Therefore, the more charge is stored in capacitor, when a dielectric medium is placed between its plates and hence the capacitance of capacitor increased.

31.5 Gauss's Law in Dielectrics

Consider a charged capacitor, having '$+q$' charge on one plate and '$-q$' on the other plate as shown in the figure below.



(a) (b)

Let 'E_0' is the electric field intensity without any dielectric medium. So, by applying the Gauss's law,

$$\oint \mathbf{E_0}.\mathbf{dA} = \frac{q}{\varepsilon_0}$$

$$\oint E_0\ dA\ cos0^\circ = \frac{q}{\varepsilon_0}$$

$$E_0 \oint dA = \frac{q}{\varepsilon_0}$$

$$E_0 \mathrm{A} = \frac{q}{\varepsilon_0}$$

$$E_0 = \frac{q}{\mathrm{A}\varepsilon_0}$$

Now, let a dielectric medium of dielectric constant 'κ_e' is placed between the plates of charged capacitor. due to the polarization of dielectric medium, an induced charge '$-q'$' appears near the positively charged plate and '$+q'$' appears near the negatively charged plate as shown in the figure below.

So, the net charge enclosed in the Gaussian surface decreases which remains '$q - q'$'. Again by applying the Gauss's law,

$$\oint \mathbf{E}.\mathbf{dA} = \frac{q-q'}{\varepsilon_0} \qquad \text{---------------------} \quad \textbf{(6)}$$

$$\oint E\ dA\ cos0^\circ = \frac{q}{\varepsilon_0} - \frac{q'}{\varepsilon_0}$$

$$E \oint dA = \frac{q}{\varepsilon_0} - \frac{q'}{\varepsilon_0}$$

$$E\,\mathrm{A} = \frac{q}{\varepsilon_0} - \frac{q'}{\varepsilon_0}$$

$$E = \frac{q}{\mathrm{A}\varepsilon_0} - \frac{q'}{A\varepsilon_0} \qquad \text{---------------------} \quad \textbf{(7)}$$

Hence the electric field intensity decreased when the dielectric medium is placed between the plates of capacitor.

The electric field intensity between two oppositely charged plates, in the presence of a dielectric medium can be find out by the expression,

$$E = \frac{q}{\varepsilon_0 \kappa_e A} \qquad \text{---------------------} \quad \textbf{(8)}$$

Comparing eq. (7) and (8), we get

$$\frac{q}{\varepsilon_0 \kappa_e A} = \frac{q}{\mathrm{A}\varepsilon_0} - \frac{q'}{A\varepsilon_0}$$

$$\frac{q}{\kappa_e} = q - q'$$

$$q' = q - \frac{q}{\kappa_e}$$

$$q' = q\left(1 - \frac{1}{\kappa_e}\right)$$

This shows that the induced surface charge 'q'' is always less than the original free charge 'q'. Considering eq. (6)

$$\varepsilon_0 \oint \mathbf{E}\,.\,\mathbf{dA} = q - q'$$

Putting the value of 'q'', we get

$$\varepsilon_0 \oint \mathbf{E}\,.\,\mathbf{dA} = q - q + \frac{q}{\kappa_e}$$

$$\varepsilon_0 \oint \mathbf{E}\,.\,\mathbf{dA} = \frac{q}{\kappa_e}$$

$$\varepsilon_0 \oint \kappa_e \mathbf{E}\,.\,\mathbf{dA} = q$$

This is the Gauss's law in dielectrics.