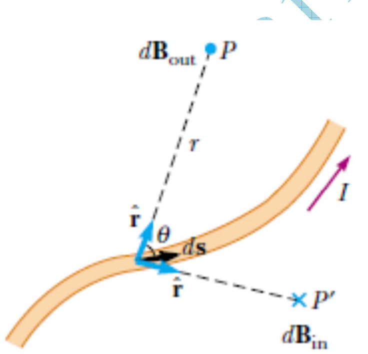
AMPERE'S LAW

35.1 The Biot–Savart Law

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Biot (1774–1862) and Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. The experimental observations of Biot and Savart about the magnetic field produced by a current carrying conductor at some point in space are as follows:

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ toward P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude $|d\mathbf{s}|$ of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.



These observations are summarized in the mathematical expression known today as the Biot–Savart law:

$$|d\mathbf{B}| \;\alpha\; \frac{I d\mathbf{s}\sin\theta}{r^2}$$

$$\Rightarrow |d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2}$$

Where $\frac{\mu_0}{4\pi}$ is the constant of proportionality and $\mu_0 = 4\pi \times 10^{-7}\ \frac{Wb}{A-m}$ is called the permeability of free space.

In vector form:

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2}\,\hat{\mathbf{n}} \qquad \text{------------------------ (1)}$$

Where $\hat{\mathbf{n}}$ is the unit vector showing the direction of field and is determined by right band rule.

If $\hat{\mathbf{i}}$ is a unit vector along current element and $\hat{\mathbf{r}}$ is the unit vector along position vector $\mathbf{r}$ of point P, then

$$\hat{\mathbf{i}} \times \hat{\mathbf{r}} = |\hat{\mathbf{i}}||\hat{\mathbf{r}}|\sin\theta\,\hat{\mathbf{n}} = (1)(1)\sin\theta\,\hat{\mathbf{n}} = \sin\theta\,\hat{\mathbf{n}}$$

The equation (1) will become:

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2}\,\hat{\mathbf{n}} = \frac{\mu_0}{4\pi}\frac{I\,ds\,(\hat{\mathbf{i}}\times\hat{\mathbf{r}})}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,ds\,\hat{\mathbf{i}}\times\hat{\mathbf{r}}}{r^2}$$

As $\hat{\mathbf{i}}$ is a unit vector along current element, therefore, $d\mathbf{s} = ds\,\hat{\mathbf{i}}$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

$$\Rightarrow d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\left(\frac{\mathbf{r}}{r}\right)}{r^2} \qquad \because\ \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$$

$$\Rightarrow d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\mathbf{r}}{r^3} \qquad \text{------------------------ (2)}$$

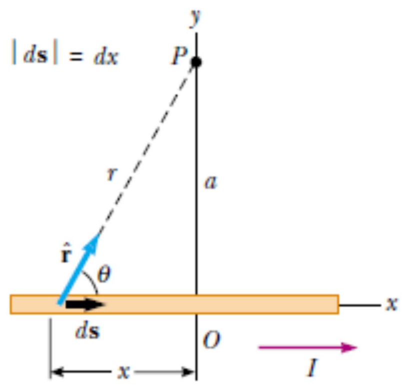
The magnetic induction $\mathbf{B}$ at point P due to whole wire is obtained by integrating eq. (2):

$$\mathbf{B} = \int d\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\mathbf{s}\times\mathbf{r}}{r^3}$$

This is known as Biot–Savart law.

35.2 Application of Biot–Savart law

35.2.1 Magnetic Field due to Current in a Straight Conductor



Consider a long straight conductor carries current I as shown in the figure below:

We want to find out magnetic field strength at point P due to this current carrying conductor. The perpendicular distance of point P from the wire is 'a'.

For this we consider a small length element ds, which is at the distance x from point O (taken as the origin). The magnetic field strength due to length element ds by Biot–Savart law will be:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2} \qquad \text{------------------------} \; (1)$$

$$\because r^2 = x^2 + a^2 \quad and \quad ds = dx$$

Thus equation (1) will become

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dx\sin\theta}{x^2 + a^2} \qquad \text{------------------------} \; (2)$$

From figure:

$$\tan\theta = \frac{a}{-x} \Longrightarrow x = -\frac{a}{\tan\theta}$$

$\because$ negative sign is necessary because ds is located at a negative value of x

$$\Longrightarrow x = -a\cot\theta$$

And $dx = a\csc^2\theta \; d\theta$

The equation (2) will become:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\; d\theta)\sin\theta}{a^2\cot^2\theta + a^2} = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\; d\theta)\sin\theta}{a^2\,(1+\cot^2\theta)} = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\; d\theta)\sin\theta}{a^2\csc^2\theta}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,\sin\theta\, d\theta}{a} \qquad \text{------------------------} \; (3)$$

To find out the field due to the whole wire can be obtained by integrating equation (3):

$$B = \int_{-\infty}^{+\infty} dB = \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty}\frac{I\,\sin\theta\, d\theta}{a} = \frac{\mu_0 I}{4\pi a}\int_{-\infty}^{+\infty}\sin\theta\, d\theta$$

As $x = -a\cot\theta$

$when\ x \to -\infty: \qquad \theta \to 0$

$when\ x \to \infty: \qquad \theta \to \pi$

$$B = \frac{\mu_0 I}{4\pi a}\int_0^{\pi}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi a}\left|-\cos\theta\right|_0^{\pi}$$

$$B = -\frac{\mu_0 I}{4\pi a}(\cos\pi - \cos 0) = -\frac{\mu_0 I}{4\pi a}(-2)$$

$$B = \frac{\mu_0 I}{2\pi a}$$

This is the expression of magnetic field induction due to a current carrying conductor.

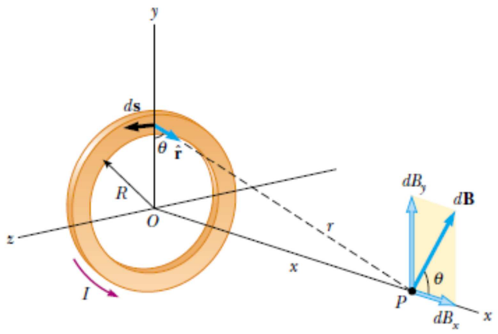
In vector form:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi a}\,\hat{\mathbf{n}}$$

Where $\hat{n}$ is the unit vector which represents the direction of tangent to the circle at point P.

35.2.2 Magnetic Field due to a Circular Current Loop

Consider a circular current loop of radius R carrying current I as shown in the figure below:



We want to find out the value of magnetic field strength at point P on at the distance x from the center of the loop. For this we consider a small element of length $d\mathbf{s}$ such that the angle between $\mathbf{r}$ and $d\mathbf{s}$ is 90°. Therefore,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2}$$

By Biot–Savart law, we find that magnetic induction $d\mathbf{B}$ is perpendicular to $\mathbf{r}$ and $d\mathbf{s}$. So by putting $\theta = 90^\circ$, we get:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{r^2} \qquad \text{------------------------} \ (1)$$

We resolve the $d\mathbf{B}$ into rectangular components dB_x and dB_y. From the symmetry, we find that dB_y has no contribution to the field at point P, because dB_y components of element will cancel out each other. Only the x-components are added up to give the magnetic field strength at point P. Therefore,

$$B = \oint dB_x = \oint dB\cos\theta = \oint \frac{\mu_0}{4\pi}\frac{I\,ds}{r^2}\cos\theta = \frac{\mu_0 I}{4\pi}\oint\frac{ds}{r^2}\cos\theta$$

From figure $r^2 = x^2 + R^2$

And $\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{x^2+R^2}}$

$$B = \frac{\mu_0 I}{4\pi}\oint\frac{ds}{x^2+R^2}\ \frac{R}{\sqrt{x^2+R^2}} = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}\oint ds$$

$$B = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}\oint ds \qquad \because \oint ds = 2\pi R$$

$$B = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}(2\pi R)$$

$$B = \frac{\mu_0 IR^2}{2\,(x^2+R^2)^{3/2}} \qquad \text{------------------------} \ (2)$$

This is the expression of magnetic induction at point P due to circular loop of current.

Special Cases.

Case 1. *Magnetic field induction at center of circular current carrying loop*

At the center of the loop $x = 0$. Putting value in equation (2), we get

$$B = \frac{\mu_0 IR^2}{2\,(R^2)^{3/2}} = \frac{\mu_0 IR^2}{2\,R^3}$$

$$B = \frac{\mu_0 I}{2\,R}$$

Case 2. *Magnetic field induction at very large distance from loop*

At very large distance from the center of the current loop i.e., $x \gg R$, the magnetic field induction will be:

$$B = \frac{\mu_0 IR^2}{2\,(x^2+R^2)^{3/2}} \qquad \because x \gg R; \quad x^2 + R^2 \approx x^2$$

$$B = \frac{\mu_0 IR^2}{2\,(x^2)^{3/2}} = \frac{\mu_0 IR^2}{2\,x^3}$$

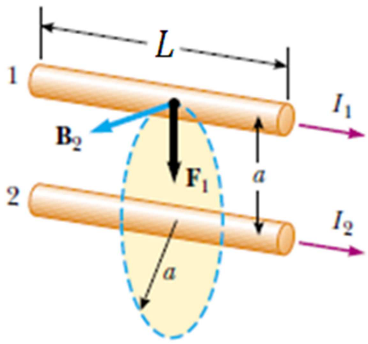
$$B = \frac{\mu_0 I\pi R^2}{2\pi\,x^3} = \frac{\mu_0 IA}{2\pi\,x^3} \qquad \because \textit{Multipying and Dividing by } \pi$$

This is the expression of magnetic induction for a current carrying coil of one loop. For a current carrying loop having N turns:

$$B = \frac{\mu_0 NIA}{2\pi\, x^3} = \frac{\mu_0 \mu}{2\pi\, x^3} \qquad \because \text{Dipole Moment } \mu = NIA$$

35.2.3 Force between Long Parallel Current Carrying Conductor

Consider two long, straight, parallel wires separated by a small distance 'a' carrying currents I_1 and I_2, respectively. The current passing through each wire produces a magnetic field around it and each wire is placed in the magnetic field produced by the other.



The wire 2, which carries a current I_2 creates a magnetic field B_2 at the location of wire 1, which can be find out by using the expression:

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad \text{------------------------} \quad (1)$$

The direction of B_2 is perpendicular to wire 1, as shown in Figure. The magnetic force on wire 1, due to magnetic force of wire 2, will be:

$$\mathbf{F_1} = I_1 \mathbf{L} \times \mathbf{B_2}$$

The magnitude of magnetic force on wire 1 is given by:

$$\Rightarrow F_1 = I_1 L B_2 \sin 90^\circ \qquad \because \mathbf{L} \text{ and } \mathbf{B_2} \text{ are perpendicular to each other}$$

$$\Rightarrow F_1 = I_1 L B_2$$

$$\Rightarrow F_1 = I_1 L \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0\, I_1\, I_2}{2\,\pi\, a} L$$

where L is the length of conductor. The direction of force F_1 is towards wire 2.

Similarly, the force on wire 2, due to the magnetic force of wire 1, carrying current I_2 is expressed as:

$$F_2 = \frac{\mu_0\, I_1\, I_2}{2\,\pi\, a} L$$

The force F_2 is directed towards wire 1. The forces are equal and opposite, so, will attract each other. If the direction of element in one wire is opposite to that in the other, the wires will repel each other.

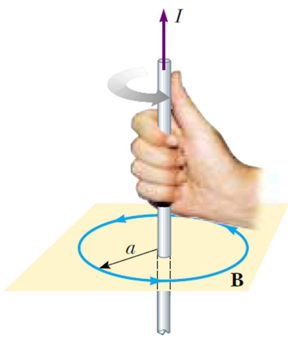
35.3 Ampere's Circuital Law

The ampere law is stated as,

"The line integral of magnetic induction **B** *due to current I around any close loop is $\mu_0$times the current enclosed"*

35.3.1 Integral Form of Ampere's Law

Consider a straight current carrying wire. The magnetic field will produced around the conductor as the current flow through it.



The magnetic field strength due to this current carrying wire at any point at the distance a is expressed as:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}\,\hat{\mathbf{k}} \quad \text{------------------------} \quad (1)$$

The symmetry shows that the magnetic induction is same everywhere on the circumference of circle. The equation (1) can be expressed in more general form as:

$$\oint \mathbf{B}.d\mathbf{l} = \oint \frac{\mu_0 I}{2\pi a}\hat{\mathbf{k}}.d\mathbf{l}$$

$$= \frac{\mu_0 I}{2\pi a}\oint \hat{\mathbf{k}}.d\mathbf{l}$$

$\because$ $\hat{\mathbf{k}}$ is partallel to $d\mathbf{l}$

$$= \frac{\mu_0 I}{2\pi a}\oint dl \qquad \because dl = ad\theta$$

$$\Rightarrow \oint \mathbf{B}.d\mathbf{l} = \frac{\mu_0 I}{2\pi a}\oint ad\theta = \frac{\mu_0 Ia}{2\pi a}\oint d\theta = \frac{\mu_0 I}{2\pi}(2\pi)$$

$$\Rightarrow \oint \mathbf{B}.d\mathbf{l} = \mu_0 I$$

This is called Ampere's circuital law.

As $I = \int \mathbf{J}.d\mathbf{s}$, therefore

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \int \mathbf{J}.d\mathbf{s}$$

This is integral form of Ampere's law.

35.3.2 Differential Form of Ampere's Law

The Ampere's law is expressed in integral form as:

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \int_s \mathbf{J}.d\mathbf{s} \qquad \text{-----------------------} \; (2)$$

Using Stoke's theorem on L.H.S., we get

$$\oint \mathbf{B}.d\mathbf{l} = \int_s curl\,\mathbf{B}.d\mathbf{s}$$

Thus equation (2) will become:

$$\int_s curl\,\mathbf{B}.d\mathbf{s} = \mu_0 \int_s \mathbf{J}.d\mathbf{s}$$

$$\int_s curl\,\mathbf{B}.d\mathbf{s} - \mu_0 \int_s \mathbf{J}.d\mathbf{s} = 0$$

$$\int_s (curl\,\mathbf{B} - \mu_0\mathbf{J}).d\mathbf{s} = 0$$

$$\Rightarrow curl\,\mathbf{B} - \mu_0\mathbf{J} = 0$$

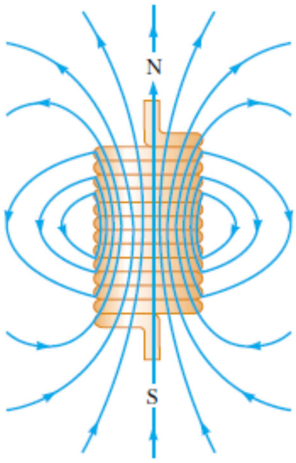
$$\Rightarrow curl\,\mathbf{B} = \mu_0\mathbf{J}$$

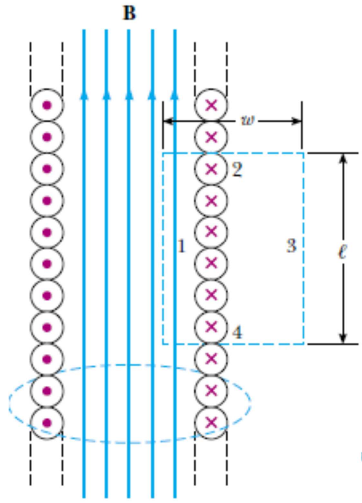
This is differential form of Ampere's law.

35.4 Applications of Ampere's Law

35.4.1 Magnetic Field due to a Solenoid

A solenoid is a cylindrical frame tightly wounded by an insulated wire. The magnetic field produced by the current carrying solenoid is like the field of a bar magnet. The magnetic field strength outside the solenoid is negligible as compared to the field inside it.





We want to find out magnetic field intensity **B** at point P inside a current carrying solenoid. For this we consider an Amperian loop. The symmetry shows that the close path can be divided into four elements1, 2, 3 and 4. By Ampere's law:

$$\oint \mathbf{B}.\,d\mathbf{l} \;=\; \mu_0 \times (Current\ Enclosed)$$

The integral $\oint \mathbf{B}.\,d\mathbf{l}$ can be written as the sum of four integrals:

$$\int_1 \mathbf{B}.\,d\mathbf{l} + \int_2 \mathbf{B}.\,d\mathbf{l} + \int_3 \mathbf{B}.\,d\mathbf{l} + \int_4 \mathbf{B}.\,d\mathbf{l} = \;\mu_0 \times (Current\ Enclosed)$$

Here $\int_2 \mathbf{B}.\,d\mathbf{l} = \int_4 \mathbf{B}.\,d\mathbf{l} =\;0$ because the angle between **B** and d**l** is 90°.

Also $\int_3 \mathbf{B}.\,d\mathbf{l} \;= 0$ as the field outside the solenoid is negligible.

Therefore,

$$\int_1 \mathbf{B}.\,d\mathbf{l} = \;\mu_0 \times (Current\ Enclosed)$$

$$\Longrightarrow \int_1 B\ dl\ \cos 0^\circ \;=\; \mu_0 \times (Current\ Enclosed)$$

$$\Longrightarrow \int_1 B\ dl \;=\; \mu_0 \times (Current\ Enclosed)$$

Since the magnetic field strength **B** for the loop element 1 is constant inside the solenoid. So,

$$B \int_1 dl \;=\; \mu_0 \times (Current\ Enclosed)$$

$$B\ l = \;\mu_0 \times (Current\ Enclosed) \qquad \text{------------------------} \;(1)$$

where l is the length of element 1 of Amperian loop.

To find out the current enclosed by the loop, we consider that there are n turns per unit length of a solenoid. Then

$$Number\ of\ turns\ in\ length\ l\ of\ solinoid = n\ l$$

If I is the current flowing through solenoid, then

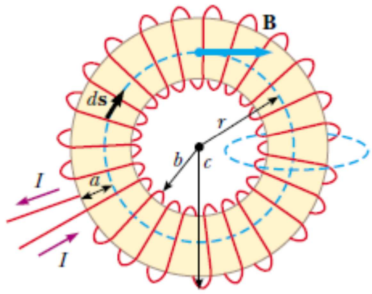
$$Current\ Enclosed\ by\ the\ Amperian\ Loop = n\ l\ I$$

Hence the equation (1) will become:

$$B\ l = \;\mu_0 \times (n\ l\ I\ )$$

$$B = \mu_0 n \ I$$

This is the expression of magnetic field strength due to a current carrying solenoid. This relation shows that the magnetic field inside a solenoid depend only on current I and number of turns per unit length n.



35.4.2 Magnetic field due to a Toroid

A toroid is used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material as shown in the figure.

We want to find out magnetic field strength at any point inside a toroid. For this we consider circular Amperian loop of radius r. By symmetry, we see that the magnitude of the field is constant on this circle and tangent to it, therefore:

$$\mathbf{B}.d\mathbf{s} = B\ ds$$

Where $d\mathbf{s}$ is the small element of amperian loop.

By applying Ampere's law:

$$\oint \mathbf{B}.d\mathbf{s} = \mu_0 \times (Current\ Enclosed)$$

$$\Rightarrow \oint B\ ds = \mu_0 \times (Current\ Enclosed)$$

$$\Rightarrow B \oint ds = \mu_0 \times (Current\ Enclosed)$$

$$\because \oint ds = 2\pi r$$

$$\Rightarrow B \times 2\pi r = \mu_0 \times (Current\ Enclosed) \quad \text{------------------------} \quad (1)$$

If N are the number of turns of the toroid and I is the current in the toroid, then

$$Current\ Enclosed\ by\ the\ Amperian\ Loop = N\ I$$

The equation (1) will become:

$$B \times 2\pi r = \mu_0 \times (N\ I\ )$$

$$B = \frac{\mu_0 N\ I}{2\pi r}$$

This is the expression of magnetic field strength inside a current carrying toroid.