

# ELECTROSTATICS

**ELECTROSTATICS** :- It is the branch of physics which deals with the study of electric charges at rest under the action of electric forces.

**KINDS OF CHARGES** :- There are two kinds of charges, named positive and negative charges.

The charge on an electron is assumed to be negative and charge on proton is positive.

Therefore a positively charged body is one which loses electrons and a negatively charged body is one which gains electrons.

**ELECTRIC FORCE** :- It is the force due to which charged particles attract or repel each other.

This force is responsible for the stability of the atoms and the molecules of the matter.

The human body is composed entirely of atoms and molecules, thus we owe our existence to the electric force.

## **QUALITATIVE NATURE OF ELECTRIC FORCE**

- Like charges repel each other and unlike charges attract each other.

**Point Charge** :- A charge whose special dimensions (size) is very small just like a point.

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## **COULOMB'S LAW**

1. **Introduction** :- Charles Coulomb, a French army engineer made a lot of research work about the quantitative nature of electric force. On the basis of the measurements, he formulated his law in 1784 AD.

2. **Statement** :- The force between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.

### 3. MATHEMATICAL FORM:-

According to the statement

$$F \propto \frac{q_1 q_2}{r^2}$$

or

$$\boxed{F = k \frac{q_1 q_2}{r^2}} \quad \text{--- (1)}$$

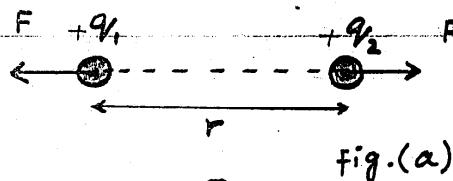


fig.(a)

Where  $F$  is the magnitude of the mutual force that acts each of the two charges  $q_1, q_2$  and ' $r$ ' is the distance between them. The force  $F$  always acts along the line joining the point charges.

$k$  is the constant of proportionality. Its value depends upon the nature of the medium between two charges and the system of units in which  $F, q$  and  $r$  are measured.

**For Free Space :-** If the medium between the two point charges is free space, then

$$k = \frac{1}{4\pi\epsilon_0}$$

where  $\epsilon_0$  is an electrical constant, known as permittivity of free space. In SI units, its value is  $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .

$$\therefore k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

Thus Coulomb's force in free space is

$$\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}} \quad \text{--- (2)}$$

### 4. Vector Form :-

As Coulomb's force is mutual force, it means that if ' $q_1$ ' exerts a force on  $q_2$ , then  $q_2$  also exerts an equal and opposite force on  $q_1$ .

If we denote the force exerted on  $q_2$  by  $q_1$  as  $\vec{F}_{21}$ , and that on charge  $q_1$  due to  $q_2$  as  $\vec{F}_{12}$  as shown in figure (b).

The magnitude of both these forces is the same i.e

$$|\vec{F}_{12}| = |\vec{F}_{21}|$$

To represent the direction of these forces, we introduce unit vectors.

If  $\hat{r}_{21}$  is the unit vector directed from  $q_1$  to  $q_2$

and  $\hat{r}_{12}$  is the unit vector directed from  $q_2$  to  $q_1$ , then

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

and

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

It can be seen from figure that  $\hat{r}_{21} = -\hat{r}_{12}$   
So

$$\vec{F}_{21} = -\vec{F}_{12}$$

The -ve sign shows that both the forces are in opposite direction.

### 5. Effect Of The Medium :-

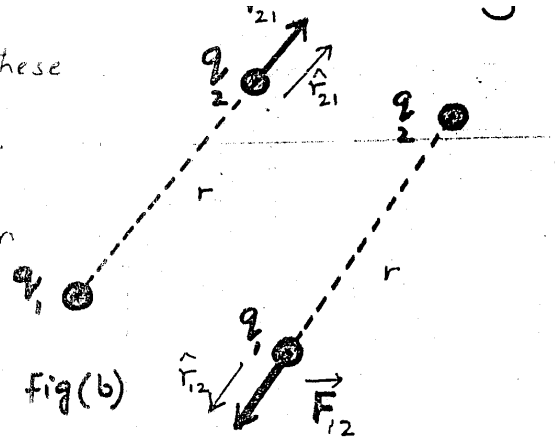
If a medium between the charges is an insulator, which is

also known as dielectric, then experimentally it is observed that, the Coulomb's force between them decreases by a constant factor  $\epsilon_r$  known as dielectric constant or relative permittivity, as compared with that in free space.

$$F' = \frac{1}{\epsilon_r} F$$

Thus Coulomb's force in a medium of relative permittivity  $\epsilon_r$  is given by

$$F' = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$



Material	$\epsilon_r$
Vacuum	1
Air (1 atm)	1.0006
Ammonia (liquid)	22-25
Bakelite	5-18
Benzene	2.284
Germanium	16
Glass	4.8-10
Mica	3-7.5
Paraffined paper	2
Plexiglas	3.40
Rubber	2.94
Teflon	2.1
Transformer oil	2.1
Water (distilled)	78.5

The relative permittivity for air is 1.0006 i.e.

$\epsilon_r \approx 1$  for air, then electric force is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$


**NOTE :-** The ratio of the force between two charges placed in air or vacuum to the force when they are placed in the insulating medium is called relative permittivity or dielectric constt. of the medium.

$$\epsilon_r = \frac{F}{F'}$$

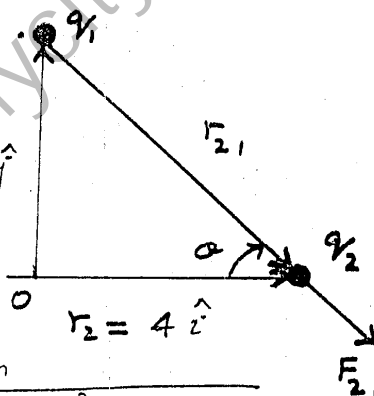
**EXAMPLE 12.1 :-** Charges  $q_1 = 100 \mu\text{C}$  and  $q_2 = 50 \mu\text{C}$  are located in  $xy$ -plane at positions  $\mathbf{r}_1 = 3.0 \hat{j}$  and  $\mathbf{r}_2 = 4.0 \hat{i}$  respectively, where the distances are measured in metres. Calculate the force on  $q_2$ .

**SOLUTION :-**

$$q_1 = 100 \mu\text{C} = 100 \times 10^{-6} \text{C}$$

$$q_2 = 50 \mu\text{C} = 50 \times 10^{-6} \text{C}$$

Position vector of  $q_2$  relative to  $q_1$



$$\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1 = (4\hat{i} - 3\hat{j}) \text{ m}$$

$$r = \text{magnitude of } \mathbf{r}_{21} = \sqrt{(4\text{ m})^2 + (-3\text{ m})^2}$$

$$r = 5 \text{ m}$$

$$\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_{21}}{r} = \frac{4\hat{i} - 3\hat{j}}{5}$$

As we have

$$\mathbf{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{21}$$

$$\mathbf{F}_{21} = \frac{9 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \times 100 \times 10^{-6} \text{ C} \times 50 \times 10^{-6} \text{ C}}{(5 \text{ m})^2} \times \frac{4\hat{i} - 3\hat{j}}{5}$$

$$\mathbf{F}_{21} = 1.44 \hat{i} - 1.08 \hat{j}$$

$$\text{Magnitude of } \mathbf{F}_{21} = F = \sqrt{(1.44)^2 + (-1.08)^2}$$

$$F = 1.8 \text{ N}$$

Direction of  $\mathbf{F}_{21}$

$$\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{-1.08}{1.44} \right) = -37^\circ \text{ with } x\text{-axis}$$

## 12.2 FIELDS OF FORCE

1. The magnitude and direction of electric force and gravitational force can be found with the help of Coulomb's law and Newton's gravitational law respectively. Now question arises that

(a) What are the origins of these forces?

(b) How are these forces transmitted from one mass to another or from one charge to another?

The answer to (a) is still unknown and these forces are accepted as basic forces of the nature due to their existence.

To describe the mechanism by which electric force is transmitted, Michael Faraday (1791-1867) introduced the concept of an electric field.

2. Electric Field:- It is an intrinsic property of the nature that an electric field exists in the space around an electric charge. This electric field is considered to be a force field that exerts a force on other charges placed in that field.

3. Field Theory:- The field theory concept as introduced by Michael Faraday has the experimental support. This theory can be explained in two steps.

step-1:- A charge  $q$  produces an electric field in the space surrounding it as shown in figure (a).



Fig (a)

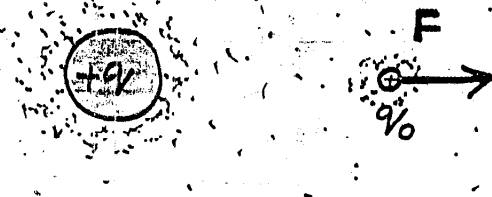


Fig (b)

Step-II: This field interacts with a test charge  $q_0$  which is placed in it and produces an electric force  $F$  on  $q_0$  as shown in figure (b). In the figures (a) and (b) the density of dots is proportional to the strength of the electric field at the various points.

#### 4. ELECTRIC FIELD INTENSITY

It is also known as electric field strength.

(a). Definition:- The Coulomb's force per unit charge is called Electric Field Intensity.

→ It is a vector quantity represented by  $E$  pointing in the direction of  $F$ .

(b). Mathematical Form:-

If a positive test charge  $q_0$  (which is very small so that it may not distort the field which it has to measure) experiences an electric force  $F$  at any point  $P$  within the field, then electric intensity is given as

$$E = \frac{F}{q_0}$$

(c). Unit :- The SI unit of electric intensity is  $N C^{-1}$ .

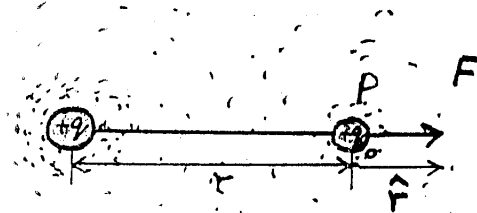
(d). Electric Intensity due to a point charge

Consider a point charge  $q$  due to which electric field is produced. To evaluate electric intensity at

point 'P', place a test charge  $q_0$  at that pt. P. Then the force experienced by  $q_0$  in the field of charge  $q$  in vacuum is given as

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r}$$

where  $\hat{r}$  is the unit vector directed from the point charge  $q$  to the test charge  $q_0$ .



Therefore electric intensity can be calculated:

$$\text{as } E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{r^2} \hat{r} \times \frac{1}{q_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

**NOTE :-** The effect of medium on the electric intensity will be the same as on electric force. i.e.

$$E' = \frac{E}{\epsilon_r}$$

This shows that electric intensity decreases by a factor of  $\epsilon_r$  in the presence of a dielectric (insulator).

**EXAMPLE 12.2** Two positive point charges  $q_1 = 16 \mu\text{C}$  and  $q_2 = 4 \mu\text{C}$  are separated by a distance of 3 m as shown in figure. Find the spot of the line joining the two charges where electric field is zero.

**SOLUTION :-**

In order to determine the electric intensity at a point, a unit +ve test charge is placed at that point to find the direction of electric intensity. So electric intensity at point 'P' due to charge  $q_1$  is  $E_1$  and due to charge  $q_2$  is  $E_2$ . Both having opposite directions. Therefore for resultant electric intensity zero both should have the same magnitudes

$$\text{i.e. } E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{(3-d)^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{d^2}$$

$$\frac{16 \times 10^{-6} \text{ C}}{9 + d^2 - 6d} = \frac{4 \times 10^{-6} \text{ C}}{d^2}$$

$$d^2 + 2d - 3 = 0$$

$$d = \frac{-2 \pm \sqrt{4 - 4(1)(-3)}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}$$

$$d = 1 \text{ m } \dots -3 \text{ m}$$

There are two possible values of 'd'. The -ve value corresponds to a location off to the right of both the charges where magnitudes of  $\vec{E}_1$  and  $\vec{E}_2$  are equal but directions are same. In this case  $\vec{E}_1$  and  $\vec{E}_2$  do not cancel out at this spot. The +ve value corresponds to the location shown in figure and is zero field location. Hence

$$\boxed{d = +1.0 \text{ m}} \quad \text{Answer}$$

### 12.3 ELECTRIC FIELD LINES

- 1. Introduction:** The concept of electric field lines or electric lines of forces was introduced by Michael Faraday. These give the visual representation of the electric field. Electric field lines can be thought of a "map" that provides information about the direction and strength of the electric field at various places. As electric field lines provides information about the electric force exerted on a charge, the lines are so called lines of force.
- 2. Definition:** An electric line of force is considered to be the path followed by a unit positive charge in an electric field.
- 3. Explanation:** To introduce electric field lines, we place +ve test charges each of magnitude  $q_0$  at the different places but at equal distances from

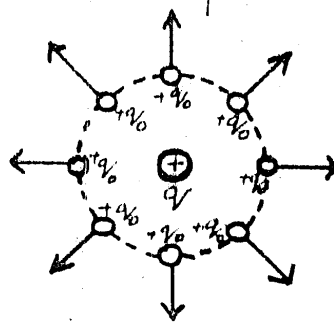


Fig (a)



a positive charge  $+q$  as shown in figure (a).

This situation gives two informations about field.

**(a). Information About Direction of Electric Field**

In the above case each test charge will experience a repulsive force as indicated by arrows in figure (a). Therefore, the electric field created by the charge  $+q$  is directly radially outward. Figure (b) shows the corresponding field lines which show the electric field direction. Figure (c) shows the electric field lines in the vicinity of a negative charge  $q$ . In this case the lines are directed radially "inward" because the force on a positive test charge is now of attraction, indicating the electric field points inward.

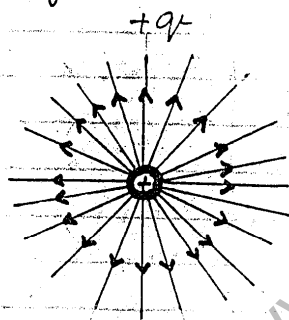


Fig. (b)

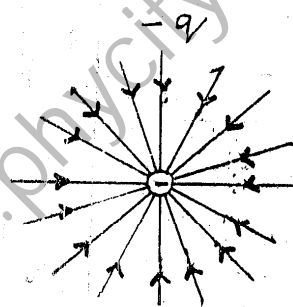


Fig. (c)

The figures (b) and (c) represent two dimensional pictures of the field lines. However, the electric field lines emerge from the charges in three dimensions, and an infinite number of the lines could be drawn.

**(b). Information About Strength of Electric Field**

The electric field lines "map" also provides information about the strength of the electric field. As it can be seen from the figures (b) and (c) that the field lines are closer to each other near the charges where the field is strong while they continuously spread out indicating

a continuous decrease in field strength.

"The number of lines per unit area passing perpendicularly through an area is proportional to the magnitude of the electric field."

#### 4. Some Typical Patterns

##### (a). For identical spherical charges

In case of two identical positive point charges of equal magnitude, the electric field lines are curved as shown in figure (d). It can be seen from figure that the lines in the region between two charges seem to repel each other.

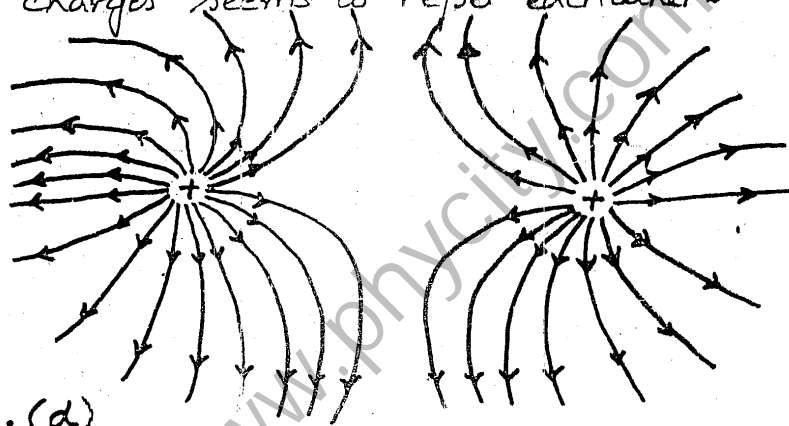


Fig. (d)

The behaviour of two identical negatively charges will be exactly the same, only direction of electric field lines will be inward.

Neutral Zone: The region where resultant electric field is zero is known as Neutral Zone as in the middle region of Fig (d). It is also known as Zero field spot.

##### (b). For two opposite charges.

###### (i). Spherical Charges

The electric field pattern of two opposite charges of same magnitude is shown in figure (e). The field lines start from positive charge and end on a negative charge. The electric field

at points such as 1, 2, 3 is the resultant of fields created the two charges at these points. The directions of the resultant intensities is given by drawing the tangents at that points as shown in figure (e).

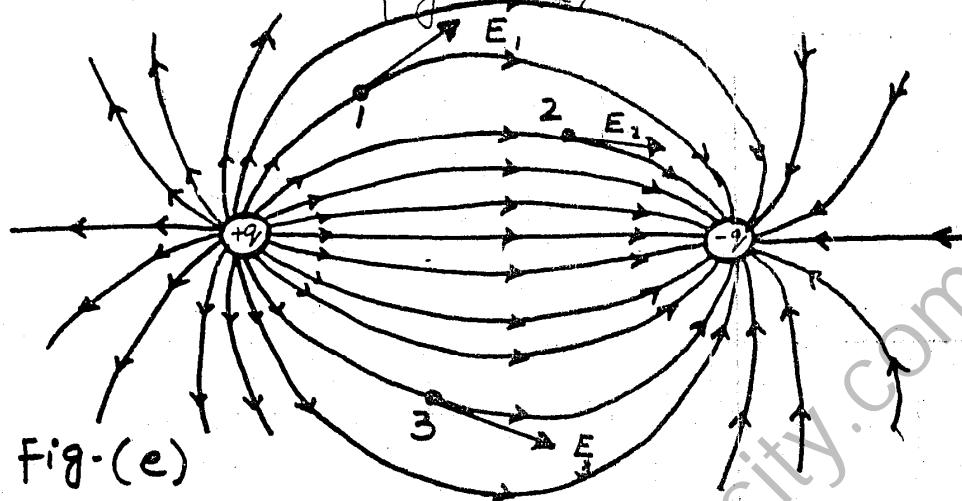


Fig. (e)

### (ii). Oppositely Charged Sheets

The pattern of electric field lines in case of two oppositely charged plates of parallel plate capacitor is shown in figure (f).

It can be seen that the electric field lines are equally spaced in the middle region, while is unequally spaced towards the end.

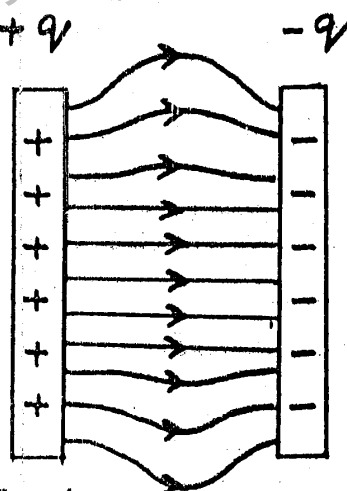


Fig (f).

### (5) (a) Uniform Electric Field

The region where the field lines are parallel and equally spaced, the same number of lines pass per unit area, in that region field is said to be the uniform field.

i.e field has the same magnitude and direction.

### (b) Non-Uniform Field or Fringing Field. 12

The region where electric field lines are not equally spaced i.e. curved as in fig. (c) and fig (d) or towards the ends of parallel plate capacitor. Such field in the region is said to be non-uniform. In this case electric intensity has different magnitude and direction.

### (6). Properties of Electric Field Lines

- The electric field lines originate from positive charges and end on negative charges.
- The tangent to a field line at any point gives the direction of the electric field intensity at that point.
- The lines are closer where the field is strong, the lines are farther apart where the field is weak.
- No two lines cross each other. This is because  $E$  has only one direction at any given point. If the lines cross,  $E$  could have more than one direction, which is impossible for a vector.
- The lines of force start from the surface of a conductor normally.

## 12.4 APPLICATIONS OF ELECTROSTATICS

### 1- XEROGRAPHY (PHOTOCOPIER)

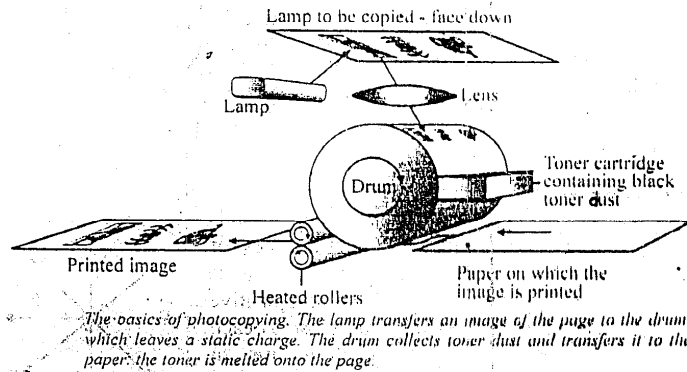
#### (a). Introduction :-

Xerography is taken from the Greek word "xeros" and "graphos", which mean "dry writing".

#### (b). Definition :-

The copying process is called Xerography.

(c). Diagram:- The principle diagram of photocopy machine is given below.



### (d). Principle Of Functioning

The heart of machine is a drum which is an aluminium cylinder coated with a layer of selenium. Aluminium is an excellent conductor. On the other hand, selenium is an insulator in the dark and becomes a conductor, when exposed to light, it is a photoconductor. As a result, if a positive charge is sprinkled over the selenium, it will remain there as long as it remains in dark. If the drum is exposed to light, the electrons from aluminium pass through the conducting selenium and neutralize the positive charge.

If the drum is exposed to an image of the document to be copied, the dark and light areas of the document produces same dark and light areas on the drum. The dark areas retain the positive charge, but light areas become conducting, i.e. lose their positive charge and become neutral.

In this way a positive charge image of the document remains on the selenium surface. Then a special dry, black powder called "Toner" is given a negative charge

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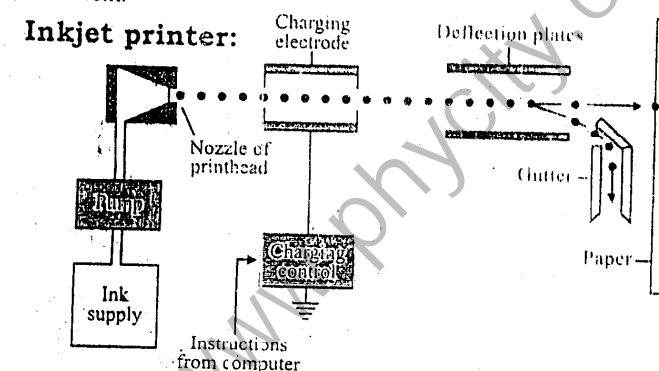
and spread over the drum, where it sticks to the positive charged areas.

The toner from the drum is transferred on to a sheet of paper on which the document is to be copied. Heated pressure rollers then melt the toner into the paper to produce the permanent impression of the document.

## 2:- INKJET PRINTERS

An inkjet printer is a type of printer which uses electric charge in its operation.

Schematic Diagram :-



An inkjet printhead ejects a steady flow of ink droplets. The charging electrodes are used to charge the droplets that are not needed on the paper. Charged droplets are deflected into a gutter by the deflection plates, while uncharged droplets fly straight onto the paper.

Working :- An inkjet printer ejects a small stream of ink when shuttling back and forth on the paper. The ink is forced out of a small nozzle by pump and breaks up into extremely small droplets. During their flight, the droplets pass through two electrical components one is "charging electrode" and other is "deflection plates" (a parallel plate capacitor). When the printhead moves over regions of the paper which are not to be inked, the charging electrode is left on and gives the ink droplets a net charge. The deflection plates divert such charged droplets into a

gutter and in this way such drops are not able to reach the paper. Whenever ink is to be placed on the paper, the charging control, responding to computer, turns off the charging electrode. The uncharged droplets fly straight through the deflection plates and strike the paper. Inkjet printers can also produce coloured copies.

## 12.5 ELECTRIC FLUX

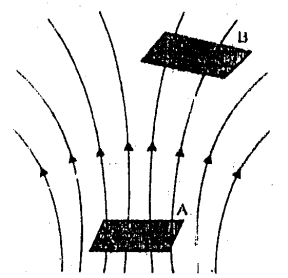
1- Introduction :- The term 'flux' is a Latin word which means "Flow".

When we place an element of area in an electric field, some of the lines of force pass through it.

2- Definition :- The number of electric field lines passing through a certain element of area perpendicularly is known as electric flux through that area.

3- Explanation :- It is usually denoted by Greek letter  $\phi$ .

(a). Example :- If a certain area A is held perpendicular to the electric field lines then 4 lines are passing through it while through another area B at another position 2 lines are passing. So the flux through area A is 4 while flux through area B is 2 in spite of the fact that area A and area B are equal but their positions are different as shown in fig (a).



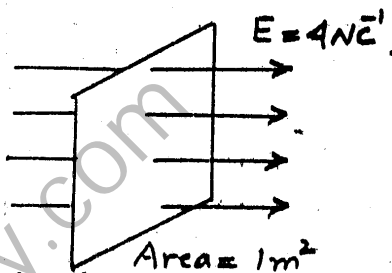
Electric flux through a surface normal to E.

fig (a)

(b). Quantitative Analysis :-

In order to give the quantitative meaning to flux we first define electric intensity in terms of field lines. "The number of electric field lines passing through a unit area held perpendicular to the field lines at a point represent the electric intensity  $\vec{E}$  of the field at that point."

Suppose at a given point the value of  $\vec{E}$  is  $4 \text{ N C}^{-1}$ . This means that if  $1 \text{ m}^2$  is area which is held perpendicular to the field lines at that point, then 4 field lines are passing through it as shown in fig (b).

(i). Maximum Flux

If a certain area 'A' is held perpendicular to the electric field lines, then maximum electric field lines will pass through it.

As we know

No. of field lines pass through unit area

$$\text{normally} = E$$

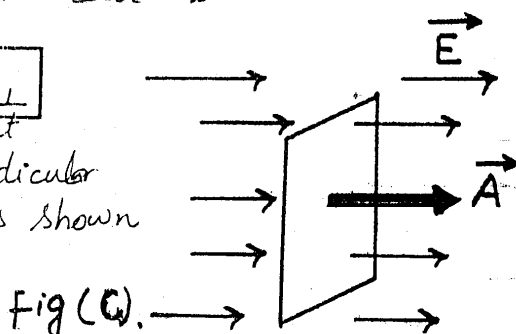
No. of field lines pass through certain area

$$\text{normally} = EA_{\perp}$$

Then flux  $\Phi_e$  in this case is

$$\Phi_e = EA_{\perp}$$

where  $A_{\perp}$  denotes that area is held perpendicular to the field lines as shown in figure (c).

(ii) Minimum Flux

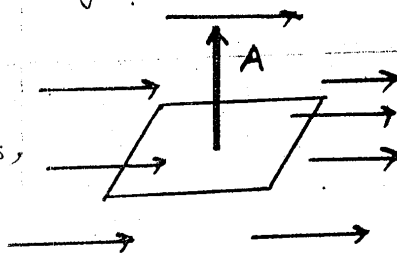
In this case area A is held parallel to the field lines as shown in figure (d). No field line cross this area, so flux  $\Phi_e$  in this



case will be minimum and is given as

$$\Phi_e = EA_{\parallel} = 0$$

where  $A_{\parallel}$  indicates that  $A$  is held parallel to the field lines, as shown in figure (d)



(iii). For Any Orientation

Fig (d)

In this case area is placed in such a way that it makes an angle  $\alpha$  with the field lines as shown in fig. (e). In this case we have to find the projection of the area which is perpendicular to the field lines.

Projection of  $A$  perpendicular to  $E = A \cos \alpha$ .

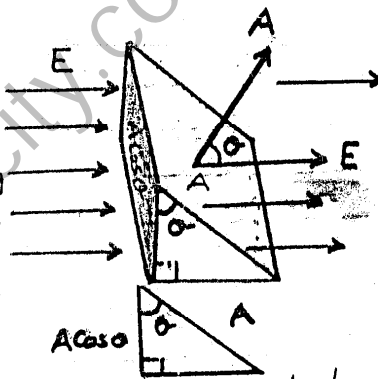
As we know

No. of electric field lines passing normally through unit area =  $E$

So no. of electric field lines passing through  $A \cos \alpha$  normally =  $E A \cos \alpha$

$\therefore$  The flux  $\Phi_e$  in this case

$$\Phi_e = EA \cos \alpha$$



Usually the element of area is represented by a vector area  $\vec{A}$  whose magnitude is equal to the surface area  $A$  of the element and whose direction is along normal to the plane area  $A$ . Therefore electric flux  $\Phi_e$  through a patch of flat surface in terms of  $\vec{E}$  and  $\vec{A}$  is

$$\Phi_e = EA \cos \alpha = \vec{E} \cdot \vec{A}$$

where  $\alpha$  is the angle between the field lines and the normal to the area.

Electric flux being a scalar product, is a scalar quantity.

(c) Unit Its SI unit is  $Nm^2C^{-1}$ .

## 12.6 ELECTRIC FLUX THROUGH A SURFACE ENCLOSING A CHARGE

Consider a closed surface, in shape of a sphere of radius 'r'. Let a point charge  $q$  is placed at the centre of sphere as shown in figure.

To calculate the electric flux

the formula  $\Phi_e = \mathbf{E} \cdot \mathbf{A}$  is

applied when surface area is flat. For this reason, the total surface area of the sphere is divided into 'n' small patches with areas of magnitudes

$\Delta A_1, \Delta A_2, \Delta A_3, \dots, \Delta A_n$ .

If 'n' is very large, each patch would be a flat element of area. In vector form vector areas are  $\Delta A_1, \Delta A_2, \Delta A_3, \dots, \Delta A_n$  by drawing normal unit vectors on the patches. Therefore the direction of each vector area is along perpendicular to the corresponding patch.

The electric intensities at the centres of vector areas  $\Delta A_1, \Delta A_2, \Delta A_3, \dots, \Delta A_n$  are

$E_1, E_2, E_3, \dots, E_n$  respectively.

So by applying formula  $\Phi = \mathbf{E} \cdot \mathbf{\Delta A}$ , the total flux passing through the closed surface (sphere)

$$\Phi_e = \Phi_1 + \Phi_2 + \Phi_3 + \dots + \Phi_n$$

$$\Phi_e = E_1 \cdot \Delta A_1 + E_2 \cdot \Delta A_2 + E_3 \cdot \Delta A_3 + \dots + E_n \cdot \Delta A_n$$

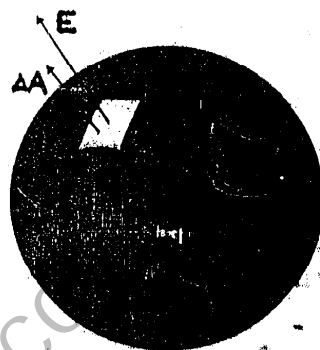
It can be seen from the figure that electric intensity and area vector are in the same direction.

Thus angle between intensity with the corresponding area vector is zero.

$$\Phi_e = E_1 \Delta A_1 \cos 0^\circ + E_2 \Delta A_2 \cos 0^\circ + E_3 \Delta A_3 \cos 0^\circ + \dots + E_n \Delta A_n \cos 0^\circ$$

$$\Phi_e = E_1 \Delta A_1 + E_2 \Delta A_2 + \dots + E_n \Delta A_n \quad \text{--- ①}$$

Due to spherical symmetry, at the surface of



sphere the magnitude of electric intensity will be

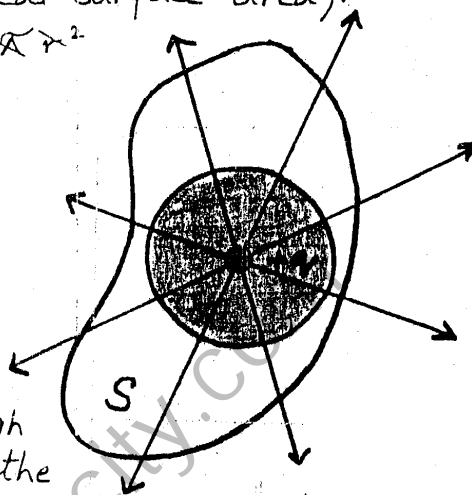
same i-e  $|\mathbf{E}_1| = |\mathbf{E}_2| = |\mathbf{E}_3| = \dots = |\mathbf{E}| = E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

putting these values in eq ①.

$$\begin{aligned}\phi_e &= E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \dots = E\Delta A_n \\ &= E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \dots + \Delta A_n) \\ &= E \times (\text{Total spherical surface area}).\end{aligned}$$

$$\phi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \times 4\pi r^2$$

$$\boxed{\phi_e = \frac{q}{\epsilon_0}}$$



**Factors** Now imagine that a closed surface  $S$  is enclosing this sphere.

It can be seen from figure that the flux through the closed surface ' $S$ ' is the same as that through the sphere. So it is concluded that total flux through a closed surface does not depend upon the shape or geometry of closed surface.

It depends upon the medium and the charge enclosed.

## 12.7 GAUSS'S LAW

1- **Statement** :- "The flux through any closed surface is  $\frac{1}{\epsilon_0}$  times the total charge enclosed in it."

2- **Explanation** :- Suppose point charges  $q_1, q_2, q_3, \dots, q_n$  are arbitrarily distributed in an arbitrary shaped closed surface as shown in figure. Draw sphere around each and every point charge, such that the surface of sphere lies wholly into 'Surface  $S$ '. Since electric flux through a closed surface

L(U)

is independent of its shape, so each point charge will act as independent source of flux.

The electric flux through sphere  $S_1$  is given by

$$\phi_1 = q_1 / \epsilon_0$$

Similarly for other spheres

$$\phi_2 = q_2 / \epsilon_0$$

$$\phi_3 = q_3 / \epsilon_0$$

⋮

$$\phi_n = q_n / \epsilon_0$$

The total electric flux passing through the closed surface is

$$\phi_e = \phi_1 + \phi_2 + \phi_3 + \dots + \phi_n$$

$$= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \dots + \frac{q_n}{\epsilon_0}$$

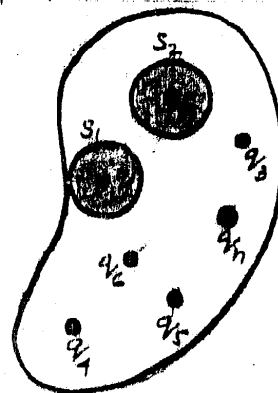
$$= \frac{1}{\epsilon_0} \times (q_1 + q_2 + q_3 + \dots + q_n)$$

$$= \frac{1}{\epsilon_0} \times (\text{Total charge enclosed by the closed surface.})$$

$$\phi_e = \frac{1}{\epsilon_0} \times Q$$

$$\text{where } Q = q_1 + q_2 + q_3 + \dots + q_n$$

$$= \text{Total charge enclosed by closed surface.}$$



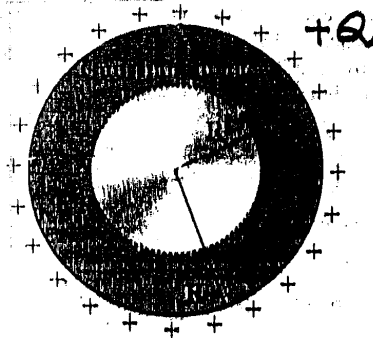
## 12.8 APPLICATIONS OF GAUSS'S LAW

Gauss's law is applied to calculate the electric intensity due to different charge configurations. In all such cases an imaginary closed surface is considered which passes through the point at which the electric intensity is to be evaluated. This closed surface is known as Gaussian surface.

a) Intensity of Field Inside a Hollow Charged Sphere. Suppose that a hollow conducting sphere of radius  $R$  is given a positive charge  $+Q$ . We wish to calculate the field intensity first at

a point inside the sphere.

Now imagine a sphere of radius  $R' < R$  to be inscribed within the hollow charged sphere as shown in figure. The surface of this sphere is the Gaussian surface. Let  $\phi$  be flux through this closed surface.



It can be seen in the figure that the charge enclosed by the Gaussian surface is zero.

i.e.  $q = 0$

Applying Gauss's law

$$\phi_e = \frac{q}{\epsilon_0} = 0$$

As we have

$$\phi_e = E \cdot A = 0$$

Since Area of Gaussian surface  $A \neq 0$

$$\therefore E = 0$$

Thus the interior of a hollow charged metal sphere is a field free region. As a result, any apparatus placed within a metal enclosure is "shielded" from electric fields.

### b) Electric Intensity Due to an Infinite Sheet of Charge.

Consider a plane sheet of infinite extent on which positive charges are uniformly distributed.

Let the uniform surface charge density is  $\sigma$ .

$\sigma$  = Charge per unit area

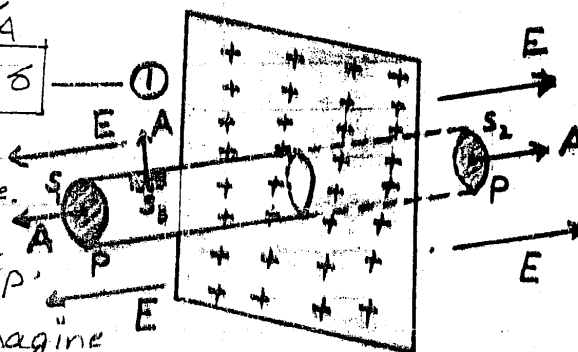
$$\sigma = \frac{Q}{A}$$

or

$$Q = A\sigma$$

A finite part of this sheet is shown in figure.

To calculate the electric intensity  $E$  at a point 'P' close to the sheet, imagine



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a closed Gaussian surface in the form of a cylinder <sup>normally</sup> passing through the sheet, whose one flat face contains point 'P'. The electric lines of force leave the sheet normally on both the sides indicating the direction of  $\vec{E}$ . The electric intensity  $\vec{E}$  acts parallel to the curved surface of the cylinder but at right angles to the end faces directed away from the sheet.

For calculating electric flux, we consider the end faces  $S_1, S_2$  having area  $A$  and the curved surface of cylinder  $S_3$ . Then total flux is

$$\begin{aligned}\Phi_e &= (\Phi_e)_{S_1} + (\Phi_e)_{S_2} + (\Phi_e)_{S_3} \\ &= (E \cdot A)_{S_1} + (E \cdot A)_{S_2} + (E \cdot A)_{S_3} \\ &= EA \cos 0^\circ + EA \cos 0^\circ + EA \cos 90^\circ \\ &= EA + EA\end{aligned}$$

$$\Phi_e = 2EA \quad \text{--- (2)}$$

where  $A$  is the surface area of each flat surface.

According to Gauss's law

$$\Phi_e = \frac{1}{\epsilon_0} \times \text{charge enclosed by the closed surface.}$$

$$= \frac{1}{\epsilon_0} \times Q$$

Putting value of  $Q$  from eq (1) we get

$$\Phi_e = \frac{1}{\epsilon_0} \times \sigma A \quad \text{--- (3)}$$

Comparing eq (2) and (3)

$$2EA = \frac{1}{\epsilon_0} \times \sigma A$$

$$E = \frac{\sigma}{2\epsilon_0}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{r}$$

where  $\hat{r}$  is a unit vector normal to the sheet directed away from it.

Note:-

For a negatively charged sheet

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{r}$$

$\hat{r}$  is directed towards the sheet.

### (C). ELECTRIC INTENSITY BETWEEN TWO OPPOSITELY CHARGED PARALLEL PLATES

Suppose that two parallel and closely spaced metal plates of infinite extent separated by vacuum are given opposite charges,  $+q$  and  $-q$ .

Under these conditions the charges are essentially concentrated on the inner surfaces of the plates. The field lines which originate on positive charges on the inner face of one plate, terminate on negative charges on the inner face of the other plate as shown in fig. (a).

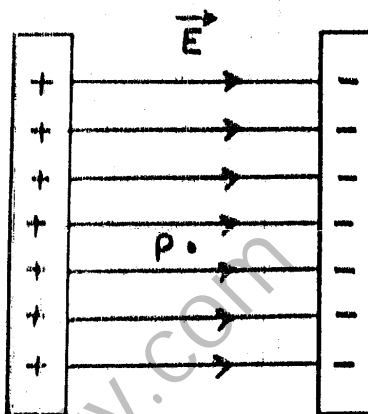


fig. (a)

Thus charges are uniformly distributed on the inner surface of the plate in a form of sheet of charges of surface density  $\sigma$

$$\text{i.e. } \sigma = \frac{q}{A}$$

or  $q = \sigma A$  (1)  
where 'A' is the area of the plate and 'q' is the charge on either of the plates.

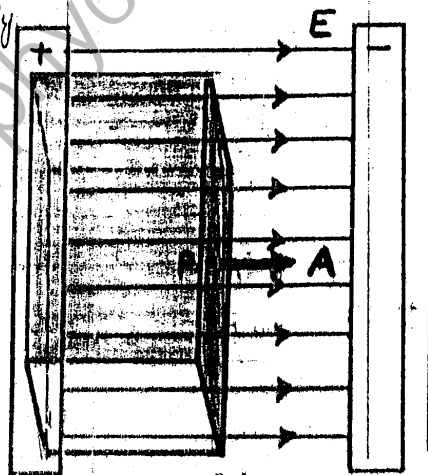


fig. (b)

Imagine now, a Gaussian surface in the form of a hollow box with its top inside the left metal plate and its bottom in space between the plates as shown in figure (b).

As the field lines are parallel to the sides of the box except the left and right sides.

There is no flux through the left side of the box because there is no field inside the metal plate.

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Thus the total electric flux through the Gaussian surface will be actually through the right face of the box. i.e.

$$\Phi_e = \mathbf{E} \cdot \mathbf{A} = EA \cos 0^\circ = EA \quad \text{--- (2)}$$

The charge enclosed by Gaussian surface is  $\sigma A$ , by applying Gauss's law

$$\Phi_e = \frac{1}{\epsilon_0} \times \text{Total charge enclosed}$$

$$\Phi_e = \frac{1}{\epsilon_0} \times \sigma A \quad \text{--- (3)}$$

By comparing eq (2) and (3)

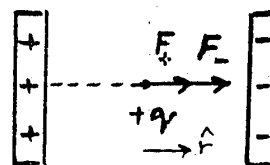
$$EA = \frac{\sigma A}{\epsilon_0}$$

$$\text{or } E = \sigma / \epsilon_0$$

The field intensity is the same at all points between the plates. The direction of the field is from +ve to -ve plates because a unit +ve charge anywhere between the plates would be repelled from +ve and attracted to negative plate and these forces are in the same direction.

In vector form

$$\mathbf{E} = \frac{\sigma}{\epsilon_0} \hat{r}$$



where  $\hat{r}$  is a unit vector directed from positive to negative plate.



## 12.9 ELECTRIC POTENTIAL

### 1- POTENTIAL DIFFERENCE

(a). Definition : The potential difference between two points is defined as the work done in moving a unit positive charge from one point to the other against electric field while keeping the charge in equilibrium.

Or The change in potential energy per unit positive charge between the two points is called potential difference.

#### (b). Explanation:-

Consider a positive charge  $q_0$  which is allowed to move in an electric field produced between two oppositely charged parallel plates as shown in figure (a).

The positive charge will move from plate B to A and will gain K.E.

If it is to be moved from A to B, an external force is needed to make the charge move

against the electric field and gain P.E. When the charge is moved from A to B, let it be moved by keeping electrostatic equilibrium i.e. it moves with uniform velocity. This condition could be achieved by applying a force  $F$  equal and opposite to  $q_0 E$  at every point along its path as shown in fig (b).

As we know that the work done against the natural trend is known as P.E, so the work done by the external force against the electric field increases electrical potential energy of the charge that is moved.

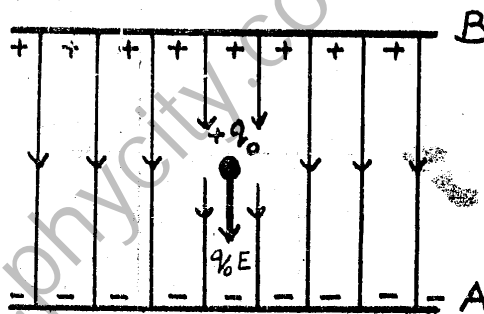


Fig (a).

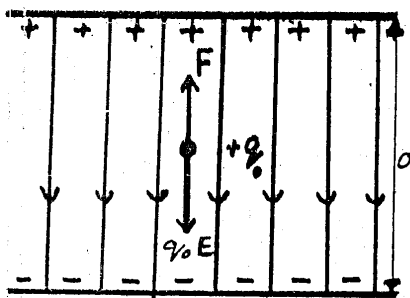


Fig (b)

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Let  $W_{AB}$  be the work done by the force in carrying the positive charge  $q_0$  from A to B while keeping the charge in equilibrium. The change in its potential energy

$$\Delta U = W_{AB}$$

or  $U_B - U_A = W_{AB}$  ————— ①

where  $U_A$  and  $U_B$  are the potential energies at points A and B respectively.

According to definition it implies that

$$\frac{U_B - U_A}{q_0} = \frac{\Delta U}{q_0} = \frac{W_{AB}}{q_0}$$

$$\frac{U_B}{q_0} - \frac{U_A}{q_0} = \frac{\Delta U}{q_0} = \frac{W_{AB}}{q_0}$$

$$\Delta V = V_B - V_A = \frac{\Delta U}{q_0} = \frac{W_{AB}}{q_0}$$

where  $V_A$  and  $V_B$  are defined electric potentials at point A and B respectively.

i.e

$$\Delta V = \frac{W_{AB}}{q_0} = \frac{\Delta U}{q_0}$$
 ————— ②

So,

Electric potential energy difference and electric potential difference between the points A and B are related as

$$\Delta U = q_0 \Delta V = W_{AB}$$
 ————— ③

Change in P.E = Charge x Potential difference

### (c) Unit of Potential Difference :-

The SI unit of potential difference is volt.

From eq. ②  $1 \text{ volt} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$  or  $1V = \frac{1J}{1C}$

**VOLT:** "A potential difference of one volt exists between two points if work done in moving a unit positive charge from one point to other by keeping the charge in its electrostatic equilibrium is one joule."

## 2. ELECTRIC POTENTIAL OR ABSOLUTE ELECTRIC POTENTIAL

To define electric potential at a point in an electric field, we select a reference point to which we

assign zero electric potential. This point is usually taken at infinity. (Outside the field).

(a). **Definition** :- "The work done in moving a unit positive charge from infinity to that point against the electric field keeping the electrostatic equilibrium."

(b). **Mathematically**:- From eq. ②

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$$

If we take 'A' at infinity,

$$V_B - V_\infty = \frac{W_{\infty B}}{q_0}$$

$$\text{As } V_\infty = 0 \Rightarrow V_B = \frac{W_B}{q_0}$$

In general

$$\text{Note } V = \frac{W}{q_0} \quad \text{④}$$

It is to be noted that potential at a point is still potential difference between the potential at that point and potential at infinity.

Both potential and potential differences are scalar quantities because both work and charges are scalars.

### 3. ELECTRIC FIELD AS POTENTIAL GRADIENT

Let us consider two oppositely charged plates.

The electric intensity  $\vec{E}$  is constant between the plates. The potential difference between A and B is given by

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$$

$$\text{As } W_{A,B} = \vec{F} \cdot \vec{d}$$

$$= \vec{F} \cdot \vec{d} = Fd \cos 130^\circ$$

$$W_{AB} = -q_0 E d$$

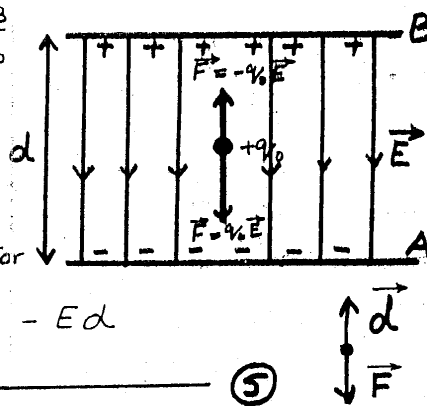
The -ve sign is needed because

F must be applied opposite to  $q_0 E$  for equilibrium.  $\therefore \Delta V = -\frac{q_0 E d}{q_0} = -E d$

or

$$E = -\frac{\Delta V}{d} \quad \text{⑤}$$

If the plates A and B are separated by infinitesimally small distance  $\Delta r$ , then eq ⑤ can be written as



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$$E = - \frac{\Delta V}{\Delta r}$$

⑥

Potential Gradient :- The quantity  $\frac{\Delta V}{\Delta r}$  gives the maximum value of the rate of change of potential with respect to displacement because the charge has been moved along a field line along which the distance between the plates is minimum. It is known as potential gradient. Thus  $E = -$  gradient of potential.

or  $E = -\text{grad. } V$

The negative sign indicates that the direction of  $E$  is along the decreasing potential.

(i.e. potential increases in a direction opposite to field)

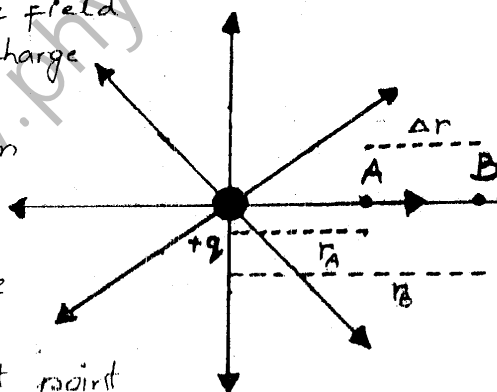
#### 4. ELECTRIC POTENTIAL AT A POINT DUE TO A POINT CHARGE

Consider an electric field  $E$  due to a point charge  $+q$ . In order to derive an expression for the potential at a certain point in the field a unit positive charge is moved from infinity to that point by keeping it in equilibrium.

Since electric intensity varies inversely as the square of distance from the charge (i.e.  $E \propto \frac{1}{r^2}$ ), it does not remain constant. So take two points A and B infinitesimally close to each other, so that  $E$  remains almost constant between them. The distance of points A and B from  $q$  are  $r_A$  and  $r_B$  respectively. The distance of midpoint of space interval between A and B is  $r'$  from  $q$ .

As from figure

$$\Delta r = r_B - r_A \quad \text{--- (1)}$$



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The mid point between A and B is given as

$$r = \frac{r_A + r_B}{2}$$

The magnitude of electric intensity at this point is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

As the points A and B are very close then

$$r_A = r_B = r$$

$$\therefore r^2 = r \cdot r = r_A r_B$$

Using this value in above equation.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A r_B}$$

As we know that

$$\Delta V = -E \Delta r$$

If a unit positive charge is moved from B to A, the work done is equal to the potential difference between A and B

$$i.e. \quad V_A - V_B = -E (r_A - r_B)$$

$$V_A - V_B = E (r_B - r_A)$$

Putting value of E from eq 0 to above eq.

$$V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[ \frac{r_B - r_A}{r_A r_B} \right]$$

$$V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

To calculate absolute potential or potential at 'A' point B is assumed to be infinity point so that  $V_B = 0$  and hence

$$\frac{1}{r_B} = \frac{1}{r_\infty} = \frac{1}{\infty} = 0$$

This gives

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A}$$

The general expression for electric potential  $V_r$  at a distance 'r' from q is

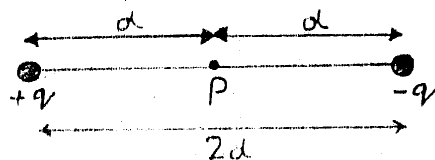
$$V_r = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

**Example: 12.3** Two opposite point charges each of magnitude  $q$  are separated by a distance  $2d$ . What is the electric potential at a point  $P$  mid-way between them?

**Solution :-**

The general expression for electric potential at a point is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$



The electric potential at pt.  $P$  due to  $+q$  is

$$V^+ = \frac{1}{4\pi\epsilon_0} \frac{q}{d}$$

The electric potential at pt.  $P$  due to  $-q$  is

$$V^- = \frac{1}{4\pi\epsilon_0} \frac{(-q)}{d} = -\frac{1}{4\pi\epsilon_0} \frac{q}{d}$$

The electric potential at  $P$  due to opposite charges is

$$\begin{aligned} V &= V^+ + V^- \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{d} - \frac{1}{4\pi\epsilon_0} \frac{q}{d} \end{aligned}$$

$$V = \text{Zero}$$

## 12.10 ELECTRON VOLT

As we know that when a particle of charge  $q$  moves from point 'A' with potential  $V_A$  to a point B with potential  $V_B$ , keeping electrostatic equilibrium, the change in potential energy of the particle is

$$\Delta U = q(V_B - V_A) = q\Delta V$$

When this particle is moved freely in the electric field, then no external force acts on the charge to maintain equilibrium.

then it will move from a point of higher potential  $V_B$  to a point at a lower potential  $V_A$ , which gradually increases its velocity. So in this case its change in P.E appears in the form of change in K.E. Therefore according to law of conservation of energy

$$\Delta U = \Delta \text{K.E} = q \Delta V$$

If the particle under consideration is electron then

$$\Delta \text{K.E} = e \Delta V = (1.6 \times 10^{-19} \text{C}) \Delta V$$

Let  $\Delta V = 1 \text{ volt}$ , hence

$$\Delta \text{K.E} = (1.6 \times 10^{-19} \text{C}) \times (1 \text{ volt})$$

$$= 1.6 \times 10^{-19} (\text{C} \times \text{V})$$

$$\Delta \text{K.E} = 1.6 \times 10^{-19} \text{ Joule}$$

The amount of energy equal to  $1.6 \times 10^{-19} \text{ J}$  is called one electron-volt and is denoted by  $1 \text{ eV}$ .

**Definition :-** "The amount of energy acquired or lost by an electron as it transverses a potential difference of one volt."

Thus

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

**Example 12.4:** A particle carrying a charge of  $2e$  falls through a potential difference of  $3.0 \text{ V}$ . Calculate the energy acquired by it.

**Solution :-**

$$q = 2e, \quad \Delta V = 3.0 \text{ V}$$

The energy acquired by the particle is

$$\Delta \text{K.E} = q \Delta V = (2e)(3.0 \text{ V}) = 6.0 \text{ eV}$$

$$\text{As we have } 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \Delta \text{K.E} = 6.0 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{Energy acquired by the particle} = 9.6 \times 10^{-19} \text{ J}$$

Answer

## 12.11 A COMPARISON BETWEEN ELECTRIC AND GRAVITATIONAL FORCES

### ELECTRIC FORCE

1- The electric force between two charges is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

This shows that

$$F \propto q_1 q_2$$

and  $F \propto \frac{1}{r^2}$

2. The electric constant has a value

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

3. The electrostatic force could be attractive or repulsive.

4. Electric force is medium dependant and can be shielded

5. The electrostatic force is a conservative force

### GRAVITATIONAL FORCE

1- The gravitational force b/w the two point masses is

$$F = G \frac{m_1 m_2}{r^2}$$

This shows that

$$F \propto m_1 m_2$$

and  $F \propto \frac{1}{r^2}$

2. The gravitational constt. has a value

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

3. The gravitational force is only attractive.

4. The gravitational force does not depend on the medium.

5. The gravitational force is also a conservative force.

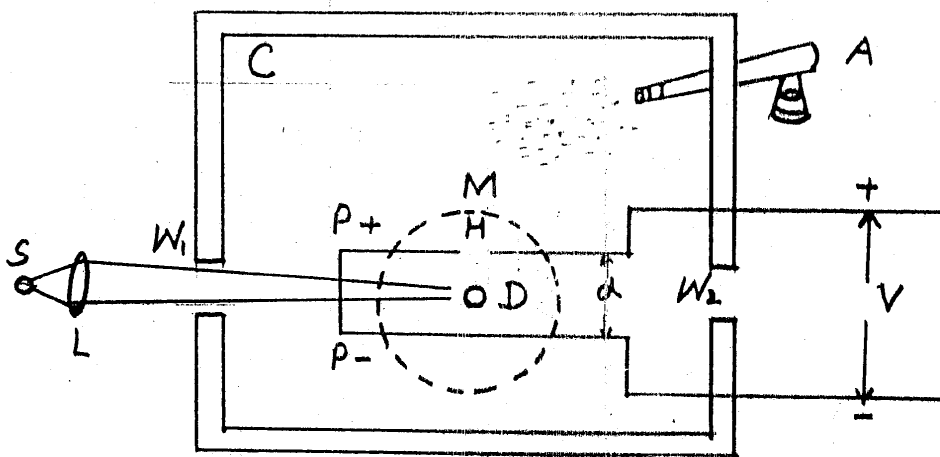
6- This force is weak as compared to the electric force.

## 12.12 CHARGE ON AN ELECTRON MILLIKAN'S METHOD

1- Introduction :- In 1909 R.A Millikan devised a technique that resulted in precise measurement of the charge on an electron.

2- Diagram :- A schematic diagram of the Millikan oil drop experiment is shown in fig.





### 3. Construction and Working

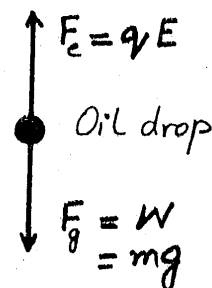
Two parallel plates  $PP'$  are placed inside a container  $C$  to avoid disturbance due to air currents. The separation between the plates is ' $d$ '. The upper plate ' $P$ ' has a small hole  $H$  as shown in figure. A voltage is applied to the plates due to which the electric field between the plates is setup. The magnitude of its value is

$$E = \frac{V}{d}$$

An atomizer  $A$  is used for spraying oil drops into the container through a nozzle. The oil drop gets charge because of friction between walls of atomizer and oil drops. These oil drops are very small, and are actually in the form of mist. Some of these drops happen to pass through the hole in the upper plate. The path of motion of these drops can be carefully observed by means of illumination produced by source of light ' $S$ ' focussed by a lens  $L$  and a microscope  $M$ .

### 4. Theory

A given droplet between the two plates could be suspended in air if the gravitation force  $F_g = mg$  acting on the droplet is equal to the electric force  $F_e = qE$  as shown in figure (b). The  $F_e$  can be adjusted equal to  $F_g$  by



3.

by adjusting the voltage (i.e.  $E = \frac{V}{d}$ )

In this case  $F_e = F_g$

$$\text{or } qE = mg$$

If 'V' is the value of p.d. between the plates for this setting, then above equation can be written as

$$q \frac{V}{d} = mg$$

or

$$q = \frac{mgd}{V} \quad \text{--- ①}$$

### Determination of the mass :-

In order to determine the mass  $m$  of the droplet the electric field between the plates is switched off. The droplet falls under the action of gravity through air. It attains terminal speed  $v_t$  almost at the instant the electric field is switched off. Its terminal speed  $v_t$  is determined by timing the fall of the droplet over a measured distance.

Since the drag force  $F$  due to air acting upon the droplet, then it is falling with constant terminal speed is equal to its weight.

By using Stoke's law

$$F = 6\pi\eta r v_t = mg \quad \text{--- ②}$$

where  $r$  is the radius of the droplet and  $\eta$  is the coefficient of viscosity for air.

If  $\rho$  is the density of the droplet, then

$$\text{mass} = \text{volume} \times \text{density}$$

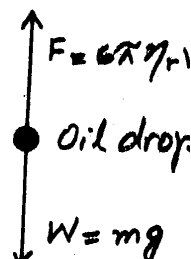
$$m = \frac{4}{3}\pi r^3 \rho$$

Using this value of mass in equation ②

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_t$$

$$\text{or } r^2 = \frac{9\eta v_t}{2\rho g}$$

By knowing the value of  $r$  from above equation, the mass ' $m$ ' can be calculated by using eq ②.



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This value of  $m$  is substituted in eq (1), we get the value of charge  $q$  on the droplet.

**5. Conclusion :-** Millikan measured the charge on many drops and found that each charge was an integral multiple of minimum value of charge equal to  $1.6 \times 10^{-19} \text{ C}$ . He therefore, concluded this minimum value of the charge is the charge on an electron.

**Example :- 12.5** In Millikan oil drop experiment an oil drop mass  $4.9 \times 10^{-15} \text{ kg}$  is balanced and held stationary by the electric field between two parallel plates. If the potential difference between the plates is  $750 \text{ V}$  and the spacing between them is  $5.0 \text{ mm}$ , calculate the charge on the droplet. Assume  $g = 9.8 \text{ m s}^{-2}$ .

**Solution :-**

$$\text{Mass of the drop} = m = 4.9 \times 10^{-15} \text{ kg}$$

$$\text{Potential difference} = V = 750 \text{ V}$$

$$\text{Spacing between plates} = d = 5.0 \text{ mm} = 5.0 \times 10^{-3} \text{ m}$$

Using the equation

$$q \frac{V}{d} = mg$$

$$\therefore q = \frac{mgd}{V}$$

$$= \frac{4.9 \times 10^{-15} \text{ kg} \times 9.8 \text{ m s}^{-2} \times 5.0 \times 10^{-3} \text{ m}}{750 \text{ V}}$$

$$\boxed{q = 3.2 \times 10^{-19} \text{ C}}$$

**NOTE**

$$\frac{\text{Volt}}{\text{metre}} = \frac{\text{newton}}{\text{Coulomb}}$$

**Proof**

$$\text{L.H.S} = \frac{\text{volt}}{\text{metre}}$$

$$\text{As } \Delta V = \frac{\text{Work}}{q} \quad \text{i.e. } \text{volt} = \frac{\text{joule}}{\text{Coulomb}}$$

$$\therefore \text{L.H.S} = \frac{\text{joule}}{\text{Coulomb} \cdot \text{metre}} = \frac{\text{newton} \times \text{metre}}{\text{Coulomb} \times \text{metre}} = \text{R.H.S}$$

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## 12.13 CAPACITOR

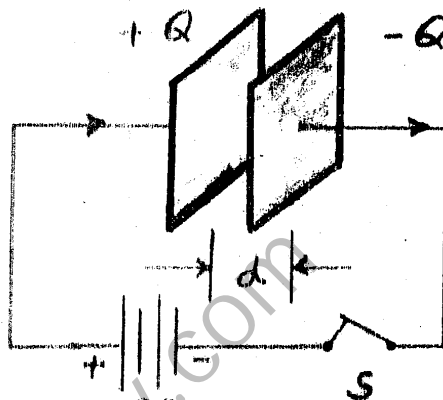
1- Definition:- "A device used for storing electric charge is called a capacitor or condenser."

2- Construction:-

A simple parallel plate capacitor consists of two conducting plates separated by vacuum, air or any other insulator called dielectric.

When the plates of such a capacitor are connected to a battery of voltage  $V$  volts, it

produces a potential difference of  $V$  volts between the two plates. The battery places a charge  $+Q$  on the plate which is connected with its positive terminal and a charge  $-Q$  on the other plate, connected to its negative terminal.



3- Formula  
It is experimentally observed that a charge  $Q$  stored by a capacitor is directly proportional to the potential difference between the plates.

i.e

$$Q \propto V \quad \text{or} \quad Q = CV$$

$$\text{or} \quad C = \frac{Q}{V}$$

where  $C$  is the constant of proportionality called capacitance of the capacitor.

### 4- Capacitance

It is a measure of the ability of capacitor to store charge.

(a)- Definition:- The amount of charge on one plate necessary to raise the potential of that plate by one volt with respect to the other is called capacitance of capacitor.

(b). Unit :- The SI unit of capacitance is Farad (F) after the English scientist Faraday.

Farad :- The capacitance of a capacitor is one farad if a charge of one coulomb, given to one of the plates of a parallel plate capacitor produces a potential difference of one volt between them.

As  $C = \frac{Q}{V}$   
 So 1 Farad  $= \frac{\text{Coulomb}}{\text{Volt}}$  or  $1F = \frac{C}{V}$   
 Its sub-multiple units are given below

1 micro-farad  $= 1\mu F = 10^{-6} F$

1 pico-farad  $= 1pF = 10^{-12} F$

(C). Factors :- Capacitance depends upon

- (i). geometry of the plates
- (ii). the medium between the plates.
- (iii). separation between plates.

### 12.14 CAPACITANCE OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor consisting of two plane metal plates, each of area  $A$ , separated by a distance ' $d$ ' as shown in figure. The distance ' $d$ ' is small so that the electric field  $E$  between the plates is uniform and confined almost entirely in the region between the plates.

Case (a). When air or vacuum lies b/w plates

Let initially the medium between the plates be air or vacuum. If  $Q$  is the charge on the capacitor and  $V$  is the potential difference between the parallel plates then

$$C_{vac} = \frac{Q}{V} \quad \text{--- (1)}$$

The magnitude of electric intensity ' $E$ ' is related with the distance ' $d$ ' as

$$E = \frac{V}{d} \quad \text{--- (2)}$$

As  $Q$  is the charge on either of the plates of area  $A$

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the surface charge density of charge on the plate

is 
$$\sigma = \frac{Q}{A} \quad \text{--- (3)}$$

As we know that electric intensity between two oppositely charged plates is given as

$$E = \frac{\sigma}{\epsilon_0} \quad \text{--- (4)}$$

Putting the value of  $\sigma$  from eq (3) to (4)

$$E = \frac{Q}{A\epsilon_0}$$

Putting this value in eq (2)

$$\frac{Q}{A\epsilon_0} = \frac{V}{d}$$

$$V = \frac{Qd}{A\epsilon_0}$$

Using this value in eq (1)

$$C_{vac.} = \frac{QAE\epsilon_0}{Ad}$$

or

$$C_{vac.} = \frac{A\epsilon_0}{d} \quad \text{--- (5)}$$

Case (b). When insulating material lies b/w plates.

When an insulating medium of relative permittivity  $\epsilon_r$  is placed between the plates then potential difference between the plates is reduced due to decrease in electric intensity.

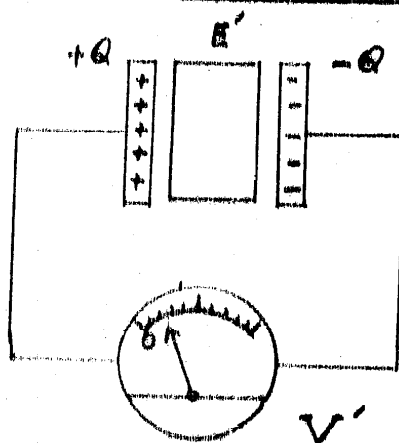
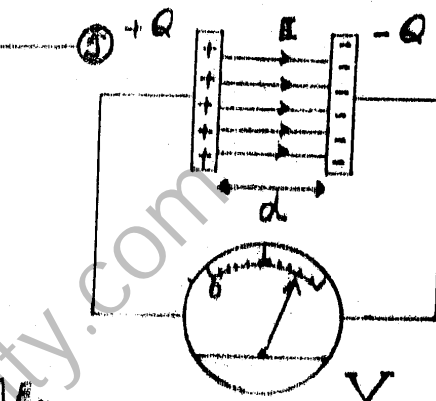
In this case

$$C_{med.} = \frac{Q}{V'}$$

The electric intensity  $E'$  between the plates for the same amount of charge is

$$E' = \frac{E}{\epsilon_r} = \frac{Q}{A\epsilon_0\epsilon_r} \quad \because E = \frac{Q}{A\epsilon_0}$$

The new potential difference becomes



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$$V' = E'd = \frac{Qd}{AE_0\epsilon_r}$$

So capacitance becomes

$$C_{med.} = \frac{Q}{V'} = \frac{QAE_0\epsilon_r}{Qd}$$

$$C_{med.} = \frac{AE_0\epsilon_r}{d} \quad \text{--- } \textcircled{Y}$$

or  $C_{med.} = \epsilon_r C_{vac.}$

This shows that capacitance of capacitor is enhanced (increased) by the factor  $\epsilon_r$ .

$$\epsilon_r = \frac{C_{med.}}{C_{vac.}}$$

Def. of Dielectric Constt. or Coefficient ( $\epsilon_r$ )

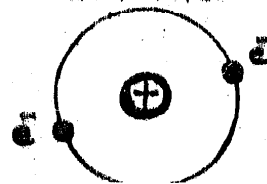
"The ratio of the capacitance of a parallel plate capacitor with an insulating substance as medium between the plates to its capacitance with vacuum (or air) as medium between them."

### 12.15: ELECTRIC POLARIZATION OF DIELECTRICS

If an insulating medium (dielectrics) such as glass, mica, wood or plastic etc is placed between the plates of a capacitor, the capacitance of capacitor increases due to electric polarization of dielectrics.

Dielectrics :- The dielectric consists of atoms and molecules which are electrically neutral on the average i.e they contain equal amounts of negative and positive charges. The distribution of these charges in atoms and molecules is such that the centre of the positive charge coincides with the centre of negative charge as shown in figure (a).

fig. (a)



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Effect of electric field :- When dielectric medium is placed between the plates of a capacitor, then electric field between the plates effects on the molecules of the dielectrics as, the negative charges (electrons) are attracted towards the positively charged plate of capacitor and the positive charges (nuclei) towards the negatively charged plate. The electrons in the dielectric are not free to move but it is possible that the electrons and nuclei can undergo slight displacement when subjected to an electric field. As a result of this displacement the centre of positive and negative charges now no longer coincide with each other and one end of molecules shows a negative charge and the other end, an equal amount of positive charge but the molecule as whole is still neutral as shown in fig. (b).

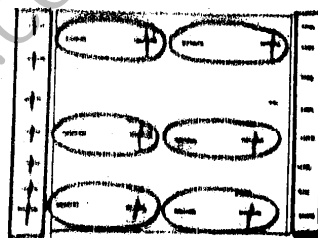
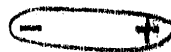


Fig (b)

Dipole :- Two equal and opposite charges separated by a small distance are said to constitute a dipole.



### ELECTRIC POLARIZATION

A process in which the molecules of the dielectric under the action of external electric field become dipoles is called Electric Polarization and the dielectric is said to be polarized. The charges on the dielectric faces are called induced charges or polarization charges.

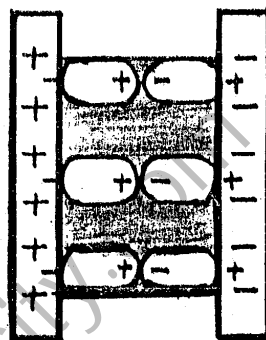
The effect of the polarization of dielectric is shown in figure (b). The positively charged plate attracts the negative end of the molecular dipoles and the negatively charged plate attracts the positive end.



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If the surface of the dielectric is in contact with the charged plates, then the surface which is in contact with the positively charged plate places a layer of negative charges on the plate. Similarly the surface of the dielectric in contact with the negatively charged plate places

a layer of positive charges as shown in figure below. These layers of the charges decrease the surface charge density  $\sigma = Q/A$  on the plate (due to decrease in net charge on the plates).



As electric intensity  $E$  between the plates is  $\sigma/\epsilon_0$ . So  $E$  decreases due to polarization of the dielectric. As  $V = Ed$  so electric intensity decreases the potential difference between plates. Finally, capacitance of capacitor increases due to presence of dielectric.

### 12.16 ENERGY STORED IN A CAPACITOR

Capacitor is a device to store electric charge, so it can also be used to store electrical energy. The charges on the plate possess electrical potential energy which arises due to work done by the battery on the charges to move them towards plate of the capacitor.

Initially when capacitor is uncharged, the potential difference between plates is zero and let finally it becomes  $V$  when ' $q$ ' charge is deposited on each plate.

$$\begin{aligned} \text{The average potential difference} &= V_{av.} = \frac{0+V}{2} = \frac{V}{2} \\ \text{Electrical potential energy} &= \text{Charge} \times \text{Average p.d} \\ \text{Energy} &= q V_{av.} \end{aligned}$$

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Energy of a charged capacitor =  $q \cdot \frac{V}{2}$

As  $q = CV$

$$\therefore \text{Energy} = \frac{1}{2} CV^2 \quad \text{--- (1)}$$

This equation shows the energy stored in a charged capacitor.

### Energy Stored in an Electric Field

If the medium between the plates is dielectric of dielectric constt.  $\epsilon_r$

then  $C = \frac{A\epsilon_0\epsilon_r}{d}$

Using it and  $V = Ed$  in equation (1)

$$\begin{aligned} \text{Energy} &= \frac{A\epsilon_0\epsilon_r E^2 d^2}{2d} \\ &= \frac{1}{2} \epsilon_0\epsilon_r E^2 (Ad) \end{aligned}$$

where

$Ad = \text{Volume between plates.}$

Energy Density:-

Energy density is defined as The energy stored in the electric field between the plates (or in the dielectric medium) per unit volume of dielectric is called energy density.

$$\therefore \text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$\text{Energy Density} = \frac{1}{2} \epsilon_r \epsilon_0 E^2$$

This equation is valid for any electric field strength. This equation indicates that energy stored in a unit volume of the dielectric is proportional to the square of the electric intensity

i.e. Energy density  $\propto E^2$ .

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## 12.17 CHARGING AND DISCHARGING OF A CAPACITOR

RC - Circuit :- An electric circuit which consists of both capacitor and resistor connected in series is called RC circuit.

### 1. Charging of a Capacitor :-

A RC-circuit is shown in the figure (a). When the switch  $S$  is set at terminal  $A$ , the RC combination is connected to a battery of voltage  $V_0$  which starts charging the capacitor through the resistor  $R$ .

The capacitor is not charged immediately, rather charges deposit on the plates gradually. The charges take some time to attain their equilibrium value (i.e. maximum value) on the capacitor.

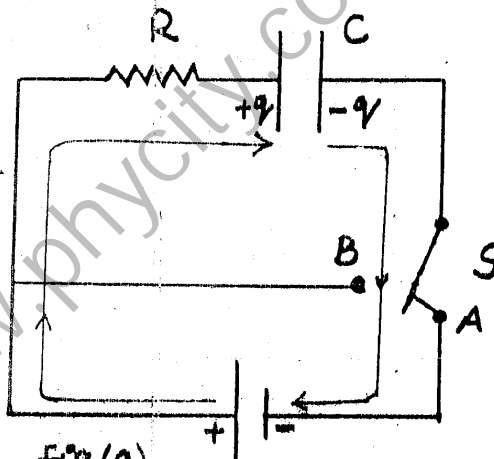


Fig. (a)

$$q_0 = CV_0$$

When amount of charge stored on the plates is plotted against time for different resistances then the graph obtained is shown in figure (b).

According to this

graph  
at  $t=0$ ,  $q=0$   
and charge increases gradually with time till it reaches its equilibrium value  $q_0 = CV_0$ .  
The voltage across capacitor at any instant is  $V = \frac{q}{C}$

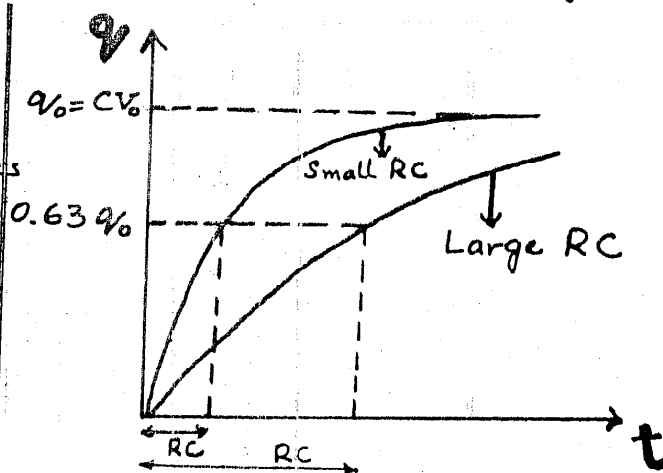


Fig. (b)

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Time Constant:

The time for the charging and discharging of the capacitor depends upon the resistance and capacitance used in the circuit.

The product of resistance and capacitance is termed as time constant. i.e.  $t = RC$ .

Def. It is defined as, "The time required by the capacitor to deposit 0.63 times the equilibrium charge  $q_0$  is called Time Constant."

It can be seen from the graph that the charge reaches its equilibrium value (max. value) sooner when the time constt. is small.

2. Discharging of the Capacitor

In this case the switch  $S$  is connected at point  $B$ .

The charge  $+q$  on the left plate can flow anti-clockwise through the resistance and neutralize the charge  $-q$  on the right plate.

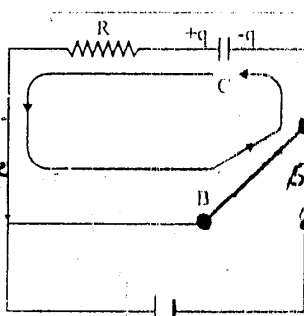


Fig. (a)

The amount of charge on the capacitor at any instant is

$$q = q_0 (1 - e^{-t/RC}) \quad \because t = RC$$

$$q = q_0 (1 - e^{-Rq_0/C})$$

$$q = q_0 (1 - e^{-1}) = q_0 (1 - \frac{1}{e})$$

$$q = q_0 (1 - \frac{1}{2.718})$$

$$q = 0.63 q_0 \quad \because e = 2.718$$

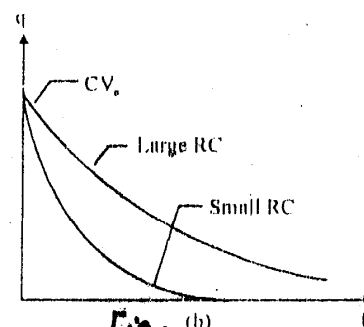


Fig. (b)

The graph shows that discharging begins at  $t = 0$  when  $q_0 = CV_0$  and decreases gradually to zero. Smaller values of time constt.  $RC$  lead to a more rapid discharge.

Example 12.6: The time constt. of a series RC circuit is  $t = RC$ . Verify that an ohm times farad is equivalent to second.

Solution:- Ohm's law in terms of potential difference  $V$ , current  $I$  and resistance  $R$  can be written as,

$$V = IR$$

Putting  $I = \frac{q}{t}$   $\therefore V = \frac{q}{t} R$

As  $R = \frac{Vt}{q}$  \_\_\_\_\_ ①

or  $q = CV$   
 $C = \frac{q}{V}$  \_\_\_\_\_ ②

Multiplying eq. ① with ②

$$RC = \frac{Vt}{q} \times \frac{q}{V}$$

$$\boxed{RC = t}$$

Hence 1 ohm  $\times$  1 farad = 1 second