S/A

CHAPTER # 8

WAVES

-: WAVE :-

Def:- A mechanism which transfers energy from one place to the other without transfering any matter is called a wave. e.g. Sound waves, light waves and waves on a stretched string.

Waves has the following three types.

(i) Mechanical waves
(ii) Electromagnetive waves
(iii) Matter waves

(i) MECHANICAL WAVES :-

Def:- The waves which propagate by the vibrations of material particles are called mechanical waves.

e.g. Sound waves, waves on the water surface etc.

(ii) ELECTROMAGNETIC WAVES

Def:- The waves which propagate out in space due to oscillating electric and magnetic field are called electromagnetic waves

e.g light waves, Satellite signal etc.

-: MATTER WAVE :-

Def:- The waves associated with the highly accelerating particles are called matter waves. e.g. The waves linked with the elementary particles such as electron moving at a speed of $10^7\ m\ sec^{-1}$ are called matter waves.

PROGRESSIVE WAVES:-

Def:- The waves which transfer energy in moving away from the source of disturbance are called progressive waves or travelling waves.

There are two types of progressive waves.

(i) Transverse waves

(ii) Longitudinal or Compressional waves.

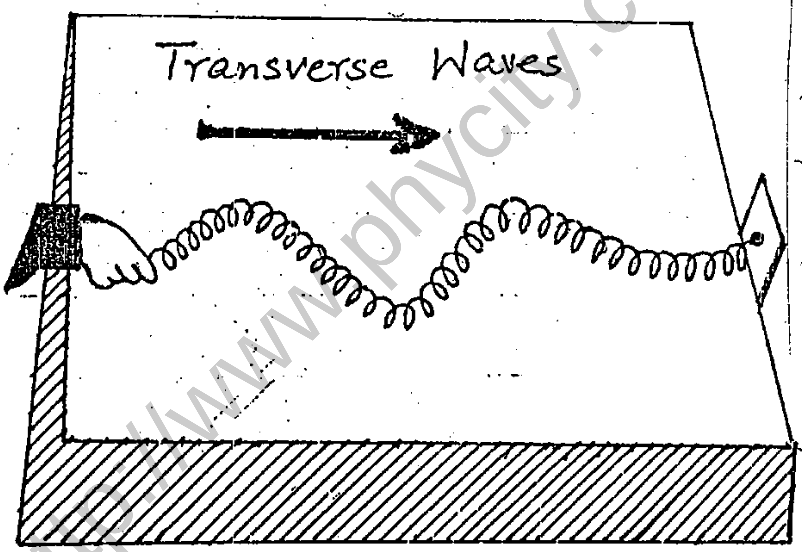
-: TRANSVERSE WAVES :-

Def:- The waves in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves are called transverse waves.

Transverse waves can be demonstrated as under.

-: EXAMPLE :-

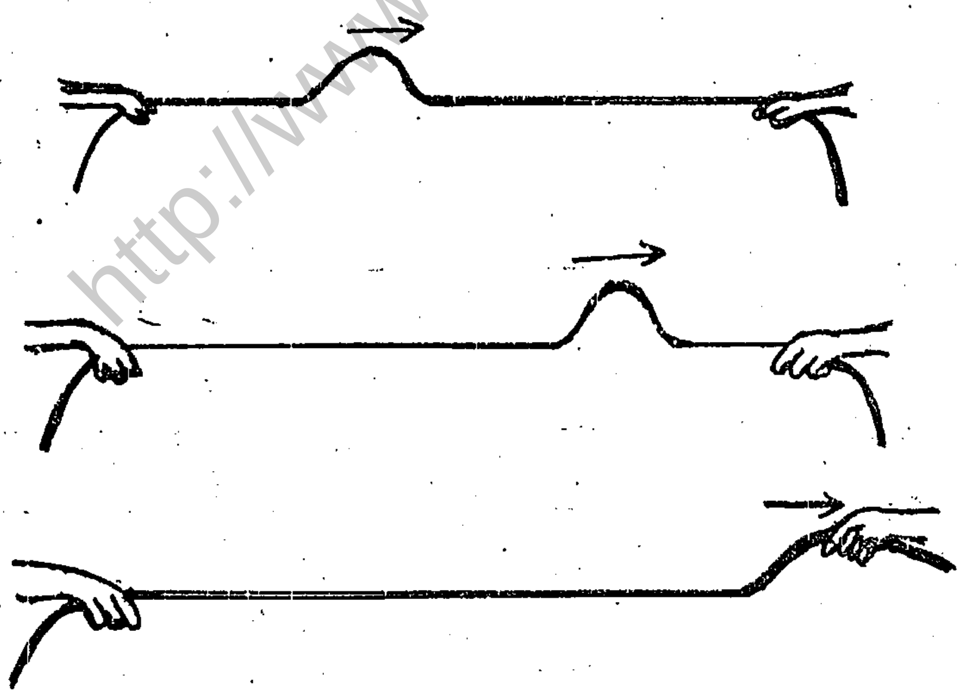
In order to clear the Concept about transverse waves, Consider a large and loose spring Coil on a smooth table with one end fixed. If the free end of the Spring is moved upside down, a wave pattern with an upward displacement is formed as shown in fig



If some points are marked on the spring then it is observed that the points move along vertical direction about the mean position (original position of the spring) while the wave propagate in the forward direction at right angle to the direction of motion of points marked on the spring. Such a wave is called transverse wave.

EXAMPLE #2

Consider two persons are holding a string at its ends. If one person gives a sudden up and down jerk, a hump (pulse) is generated and starts travelling towards the other person due to the disturbance of rope. This hump tends to move the hand of 2nd person by imparting its momentum and energy. The pattern of the hump moving forward as wave or pulse is an example of transverse wave. These waves travel in the form of crests and troughs.



Other examples of transverse waves are water waves, light waves and waves on a string.

LONGITUDINAL WAVES

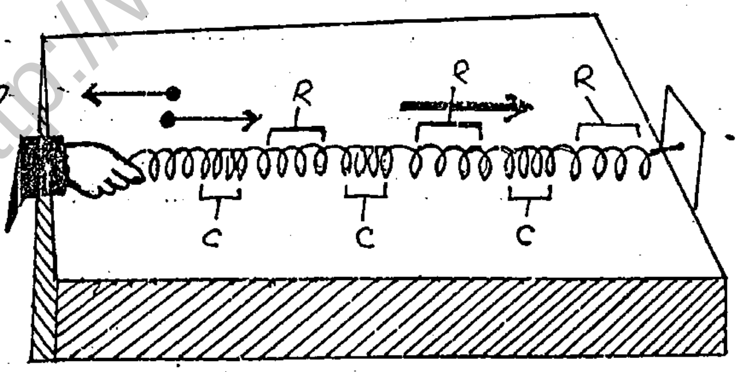
-: Def:- The waves in which particles of the medium vibrate parallel to the direction of propagation of waves are called longitudinal or Compressional waves.

-: EXPLANATION:-

Let us Consider a spring with one end fixed on a smooth horizontal table. If the free end of the spring is moved to and fro along the direction of the spring, a wave with a back and forth displacement is generated and moves forward in the form of compressions and rarefactions. Such waves are called longitudinal waves.

e.g. Sound waves.

C = Compression
R = Rarefaction



PERIODIC WAVES

-: Def:- The Continuous, regular and rhythmic disturbances in a medium produced due to repeated motion of its particles in successive intervals of time are called periodic waves e.g waves produced by mass spring system are periodic waves.

-: TRANSVERSE PERIODIC WAVES :-

Def:- The waves travelling along the length of a string attached to a simple harmonic oscillator vibrating about its equilibrium position in the form of crests and troughs are called transverse periodic waves.

Explanation

when a source fastened to a string executes simple harmonic motion along vertical direction about its equilibrium position with frequency f and amplitude A then it is observed that every point along the string vibrates back and forth about its mean position with same frequency f and amplitude A. These waves travel in the form of crests and troughs along a direction perpendiculer to the direction of motion of particles of the string. Here we mention some features concerning to these waves.

CREST :-

The portion of the string displaced above its mean position is called crest.

-: TROUGH :-

The section of the string displaced below its equilibrium position is called trough.

-: AMPLITUDE :-

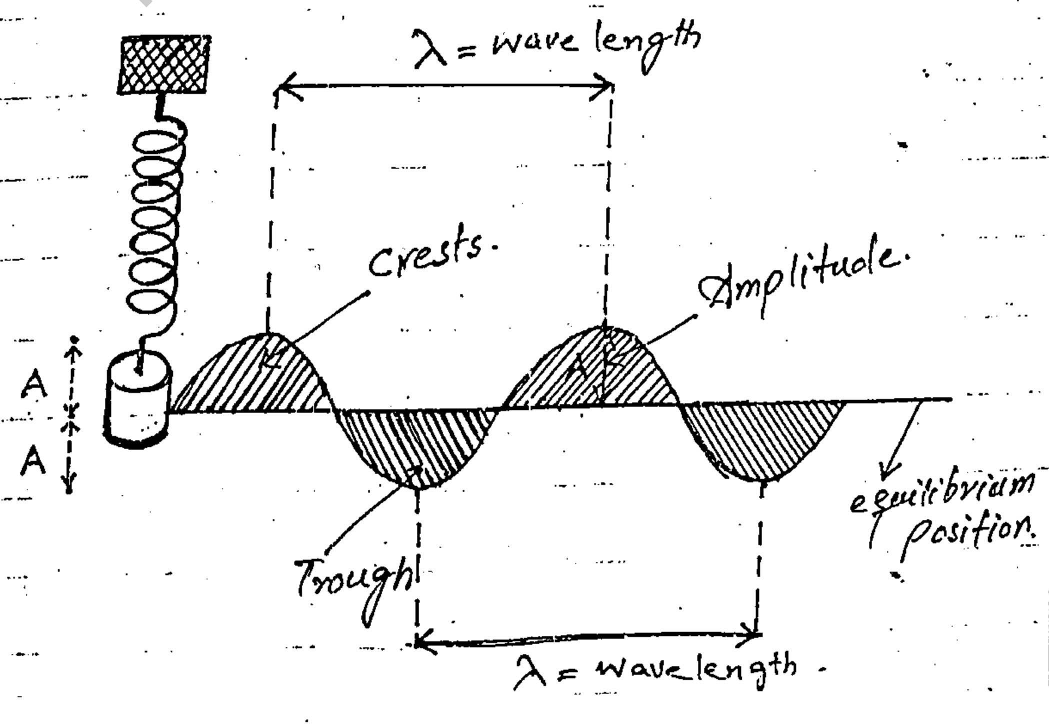
The maximum value of displacement in a crest or trough is called amplitude of the wave and is denoted by A

-: FREQUENCY :-

The number of waves passing through a certain point of a medium in one second is called frequency of the wave and is denoted by f.

-: WAVE LENGTH :-

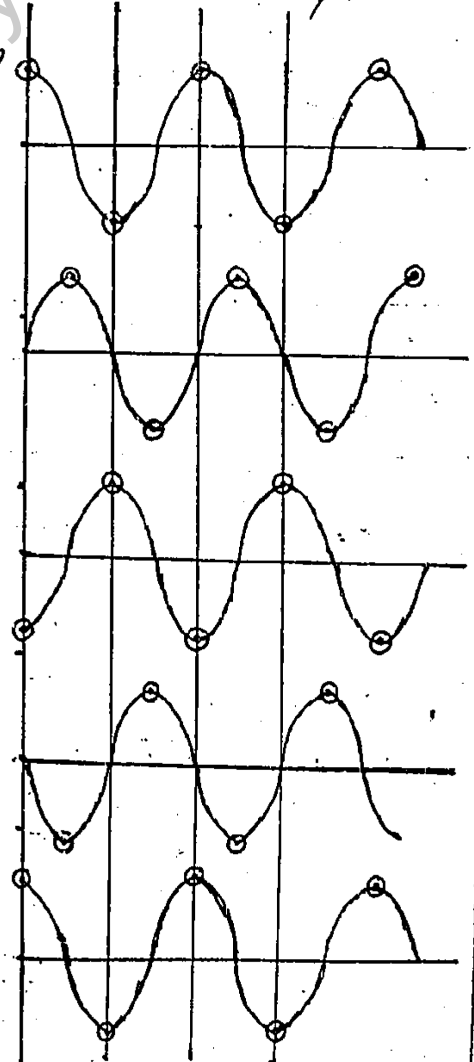
The distance between any two consecutive crests or troughs is called wave length and is denoted by a Greek letter lamda (λ)



EQUATION OF WAVE SPEED
($V = f\lambda$)

Although it is not Convinient to determine the speed of a wave by timing the motion of a wave crest over a measured distance. However speed of the wave Can be found indirectly from its frequency and wavelength as under.

Let us consider a periodic wave is travelling across a certain medium having a frequency f and time period T equal to that of a source producing it. The snap shots of periodic wave after every $\frac{1}{4}$ period are shown in the fig. Initially at $t=0$ Progress of a Crest is Started out on extreme left. The time that this crest to move through a distance equal to one wavelength λ is equal to the time required by the point in the medium to Complete its one vibration. It means that the wave Crest travels a distance equal to λ in a time period T. The Speed of the wave is given as



$$V = \frac{S}{t} \quad (\because S = vt)$$

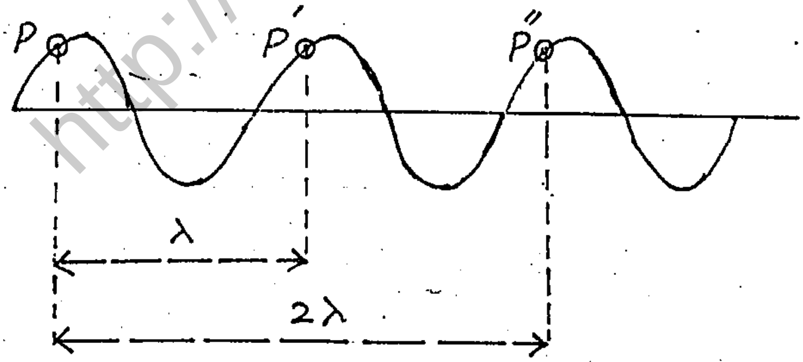
Here $S = \lambda$ when $t = T$

$$\therefore \quad V = \frac{\lambda}{T} = \lambda \times \frac{1}{T}$$

$$V = f\lambda \quad (\because f = \frac{1}{T})$$

Hence the above equation proves that the speed of the wave is equal to the products of its frequency and wavelength.

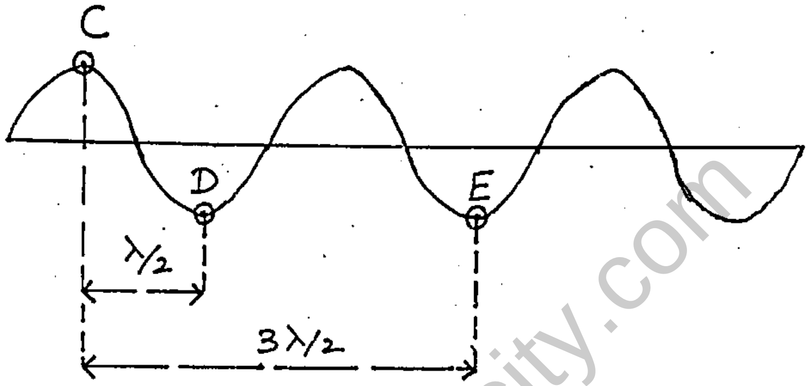
★★ There are certain points on the wave form of Simple Harmonic Motion which are in their same state of vibration i.e. having same displacement and velocity Such points are said to be **in phase**
e.g. $P, P' \,\&\, P''$ etc are all in phase path difference b/w these points is λ, 2λ, 3λ ..... or integral multiple of λ



On the other hand there are some certain points C & D on the wave such that when point C reaches on the top with its maximum upward displacement, at that instant point D approaches the bottom of the wave with maximum downward displacement. Such points are said to be **Out of Phase**

Any two points separated from one another by $\lambda/2$, $3\lambda/2$, $5\lambda/2$ ..... or odd integral multiple of half wavelength

i.e. Path difference $= (n+\frac{1}{2})\lambda$ or $(2n+1)\lambda/2$



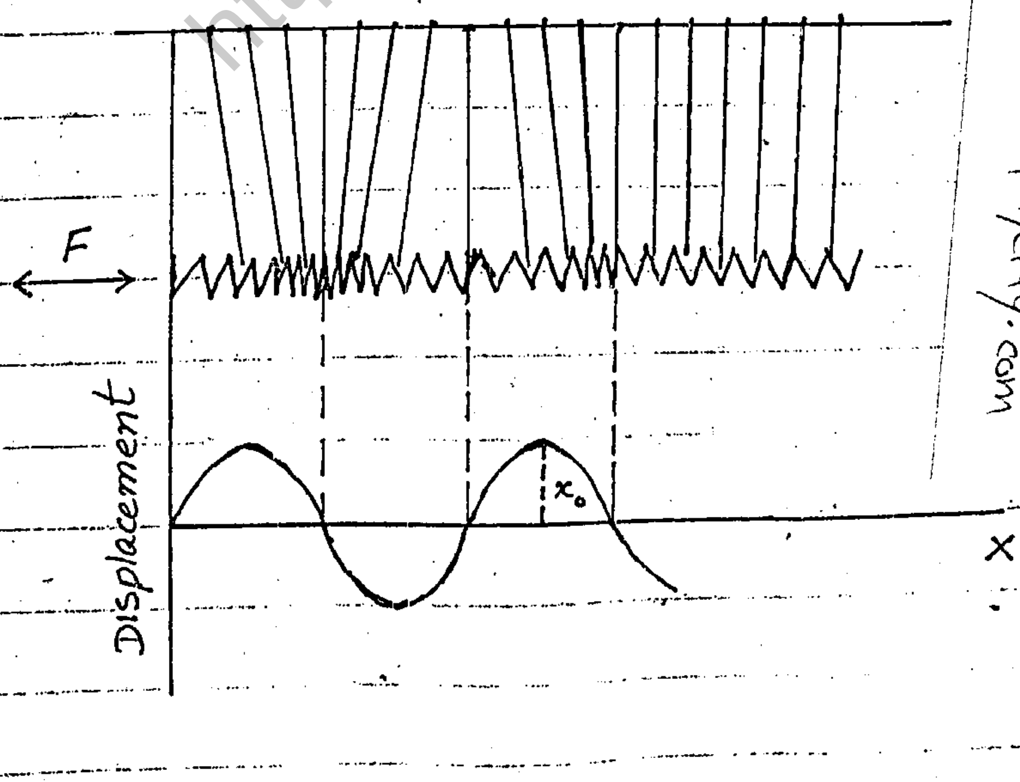
Here points D and E are out of phase with respect to point C

LONGITUDINAL COMPRESSIONAL WAVES

Def:- The waves in which particles in the path of the waves move back and forth along the line of propagation of the wave are called longitudinal waves e.g. Sound waves are longitudinal waves.

-: EXPLANATION :- Consider a coil of spring that is suspended by vertical threads so that it can vibrate horizontally. when the left end of the spring is moved to and fro by constant force F. then it compresses and stretches the spring as a result of that alternative compressions and rarefactions start travelling along the spring. Such waves are called compressional waves.

In the absence of applied force the threads will be exactly vertical. The disturbances passing through the spring causes displacements in the elements of the spring as well as in the vertical threads as shown in the fig below. This is the direct measure of displacement of spring elements from mean position as shown by the graph.



SPEED OF SOUND IN AIR

Sound waves are longitudinal in nature. Speed of sound depends upon the Compressibility (E) and inertia (density ρ) of the medium. Newton was the first one who suggested that the velocity of sound waves V having elasticity E and density ρ is given as

$$V = \sqrt{\frac{E}{\rho}} \quad \text{—— (1)}$$

The equation indicates that speed of sound will be greater for a medium which is more elastic and less compressible. As solids are more elastic than gases therefore sound travels faster in solids than in gases because they respond more quickly to the sound disturbances due to having packed molecules.

As the compressibility of gases is more than liquids and solids hence they have less elastic modulus. For the calculation of elastic modulus E for air, Newton assumed that when a sound wave travels through air, the temperature of air remains constant. Let on the passage of compression, the pressure of air changes from P to $(P+\Delta P)$ and its volume changes from V to $(V-\Delta V)$. Then according to Boyle's law

$$PV = (P+\Delta P)(V-\Delta V)$$

$$PV = PV - P\Delta V + \Delta P V - \Delta P \Delta V$$

As the product $\Delta P \Delta V$ is very small hence it can be neglected.

$$\therefore \quad \cancel{PV} = \cancel{PV} - P\Delta V + \Delta P V$$

$$P\,\Delta V = \Delta P\, V$$

$$P = \frac{\Delta P\, V}{\Delta V} = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\text{stress}}{\text{Vol. Strain}}$$

$$P = E$$

where E is the elastic modulus at constant temperature.

Using the value of E in equation ①

$$V = \sqrt{\frac{P}{\rho}}$$

where P is the atmospheric pressure and is given as $P = \rho' g h$

$$\therefore \quad V = \sqrt{\frac{\rho' g h}{\rho}} \qquad \text{———} \quad ②$$

Here

ρ' = density of mercury = 13.6 g/cm^3

g = 981 cm/s^2

h = height of mercury Column = 76 cm

ρ = density of air = .001293 g/cm^3

On substituting these values in equation ②

$$V = \sqrt{\frac{13.6 \times 981 \times 76}{.001293}}$$

$$= 28003\, cm\, s^{-1}$$

$$\simeq 280\, m\, s^{-1}$$

This velocity is 16% less than the experimental value equal to 332m/s which was Corrected by a French Scientist Laplace.

-: LAPLACE'S CORRECTION :-

According to Laplace, Sound waves travel through air under adiabatic Condition i.e. Compression and rarefaction occur so rapidly that the heat produced during Compression is Confined in to its own region and have no time to flow to its neighbouring cooler regions which is expanded during the passage of adjacent rarefaction. Hence the temperature of the medium doesn't remain Constant and the amount of heat confined within the medium remains Conserved. Hence the relationship b/w pressure and volume under adiabatic Condition is given as.

$$PV^{\gamma} = \text{Constant}$$

where γ is a Constant and is equal to the ratio of specific heat at Constant pressure to the specific heat at Constant volume i.e

$$\gamma = \frac{C_p}{C_v} = \frac{\text{sp. heat at Const. Pressure}}{\text{sp. heat at Const. volume}}$$

If now the pressure of gas changes from P to $(P+\Delta P)$ and its volume changes from V to $(V-\Delta V)$ then under adiabatic condition

$$P V^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

$$P \cancel{V^{\gamma}} = (P+\Delta P)\cancel{V^{\gamma}}\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

$$P = (P+\Delta P)\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

By applying the Binomial expansion

$$(1+x)^n = 1 + nx + \ldots\ldots \text{neglecting higher term}$$

the term $\left(1-\frac{\Delta V}{V}\right)^{\gamma}$ is expanded as

$$\left(1-\frac{\Delta V}{V}\right)^{\gamma} = 1 - \gamma\frac{\Delta V}{V} + \ldots\ldots \text{neglecting higher terms}$$

$$\therefore\ P = (P+\Delta P)\left(1-\gamma\frac{\Delta V}{V}\right)$$

$$\cancel{P} = \cancel{P} - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma\frac{\Delta P\Delta V}{V}$$

$$\gamma P\frac{\Delta V}{V} = \Delta P - \gamma\frac{\Delta P\Delta V}{V}$$

The quantity $\gamma\frac{\Delta P\Delta V}{V}$ is negligible and hence is neglected.

$$\gamma P\frac{\Delta V}{V} = \Delta P$$

$$\gamma P = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\text{Stress}}{\text{Vol. strain}} = E$$

$$\Rightarrow\ E = \gamma P \quad \text{——— } (3)$$

Using the value of E from eq 3 in eq (1)

$$V = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma}\sqrt{\frac{P}{\rho}}$$

For air at S.T.P $\gamma = 1.41$

Hence $V = \sqrt{1.41} \times 28003$

$= 333\ m\,s^{-1}$

which is very close to the experimental value ($332\,ms^{-1}$). Hence Laplace Correction is valid.

EFFECT OF PRESSURE ON VELOCITY OF SOUND

Any increase in pressure Causes an increase in the density of a gas. But as the density is proportional to the elasticity of the gas. So the ratio of E/ρ remains unchanged Causing no effect on the speed of sound.

-: EFFECT OF DENSITY :-

At S.T.P the value of γ is same for all gases. From the equation $V = \sqrt{\frac{\gamma P}{\rho}}$ it is clear that the velocity v of sound is inversely proportional to the square root of density. Due to this effect, the velocity of sound in hydrogen is four times its speed in oxygen because oxygen is 16 times denser than hydrogen.

EFFECT OF TEMPERATURE

when a gas is heated at Constant pressure, its density is decreased due to increase in volume as a result the velocity of sound is increased. Since we know that the equation of speed of sound is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Let V_t and V_0 be the speeds of sound at t°_c and 0°_c and ρ_t and ρ_0 be its respective densities at these temperatures then

$$V_t = \sqrt{\frac{\gamma P}{\rho_t}}$$

&

$$V_0 = \sqrt{\frac{\gamma P}{\rho_0}}$$

$$\frac{V_t}{V_0} = \sqrt{\frac{\gamma P}{\rho_t}} \div \sqrt{\frac{\gamma P}{\rho_0}}$$

$$= \sqrt{\frac{\gamma P}{\rho_t}} \times \sqrt{\frac{\rho_0}{\gamma P}}$$

$$= \sqrt{\frac{\rho_0}{\rho_t}} \quad \text{———} \quad (A)$$

Let V_0 be the initial volume of a gas. when this gas is heated through a certain temperature t then its final volume is given as

$$V_t = V_0(1+\beta t)$$

where β is called the co-efficient of volume expansion and its value for all gases is equal to $\frac{1}{273}$

$$\therefore \quad V_t = V_0\left(1+\frac{t}{273}\right) \quad \text{———} \quad (B)$$

As we know that the equation of density ρ is $\quad \rho = \frac{m}{V}$

$$\Rightarrow V = \frac{m}{\rho}$$

$$V_t = \frac{m}{\rho_t} \quad \& \quad V_0 = \frac{m}{\rho_0}$$

Using these values in equation (B)

$$\frac{m}{\rho_t} = \frac{m}{\rho_0}\left(1 + \frac{t}{273}\right)$$

$$\rho_0/\rho_t = \left(1 + \frac{t}{273}\right) \text{ ——— (C)}$$

putting the value of equation (C) in eq. (A)

$$\frac{V_t}{V_0} = \sqrt{1 + \frac{t}{273}} \text{ ——— (D)}$$

$$= \sqrt{\frac{273 + t}{273}}$$

$$\frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}}$$

where T and T_0 be the absolute temperatures at t°_c and 0°_c. It is evident from the above equation that the velocity of sound in a medium is proportional to the square root of its absolute temperature.

EQUATION OF VELOCITY OF SOUND AT A CERTAIN TEMPERATURE t

From equation (D)

$$\frac{V_t}{V_0} = \sqrt{1 + \frac{t}{273}}$$

$$\frac{V_t}{V_0} = \left(1 + \frac{t}{273}\right)^{1/2}$$

Applying Binomial theorem on R.H.S

$$\frac{V_t}{V_0} = \left(1 + \frac{1}{2} \times \frac{t}{273}\right) \qquad \because (1+x)^n = 1 + nx$$

$$\frac{V_t}{V_0} = 1 + \frac{t}{546}$$

$$V_t = V_0 \left(1 + \frac{t}{546}\right)$$

$$= V_0 + \frac{V_0 t}{546}$$

$\because$ $V_0 = 332\ m s^{-1}$ for Sound at $0°c$

$$V_t = V_0 + \frac{332}{546} t$$

$$V_t = V_0 + 0.61 t$$

This equation indicates that the velocity of sound is increased by a factor $0.61 m s^{-1}$ for every $1°c$ rise in temperature.

EXAMPLE 8.1

Find the temperature at which the velocity of sound in air is two times its velocity at $10°c$.

SOLUTION :-

$T_1 = 10°c + 273 = 283 K$

$T_t = ?$

$V_1 = V$

$V_t = 2V$

As we know that

$$\frac{V_t}{V_1} = \sqrt{\frac{T_t}{T_1}}$$

Squaring both sides.

$$\frac{V_t^2}{V_1^2} = \frac{T_t}{T_1}$$

$$\Rightarrow T_t = \frac{V_t^2 \times T_1}{V_1^2}$$

$$= \frac{(2V)^2 \times 283}{(V)^2} = \frac{4\cancel{V^2} \times 283}{\cancel{V^2}}$$

$$T_t = 4 \times 283 = 1132 K$$

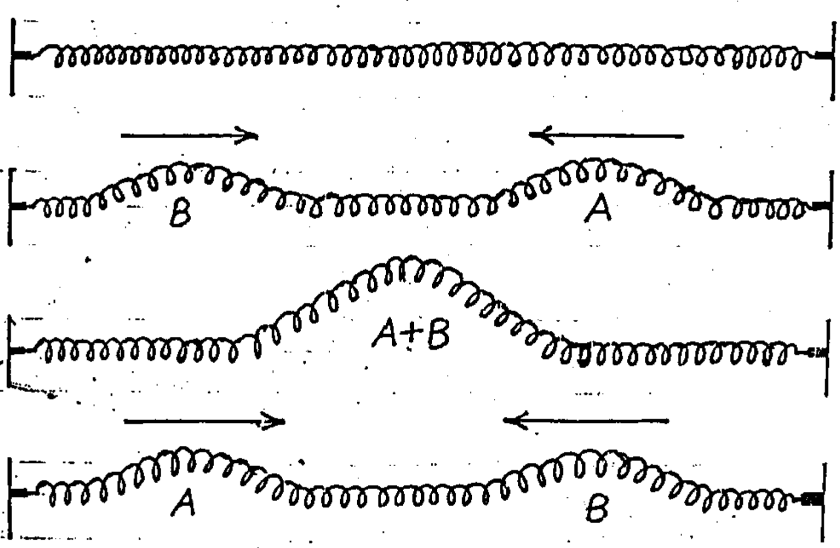
or $859°c$

PRINCIPLE OF SUPERPOSITION

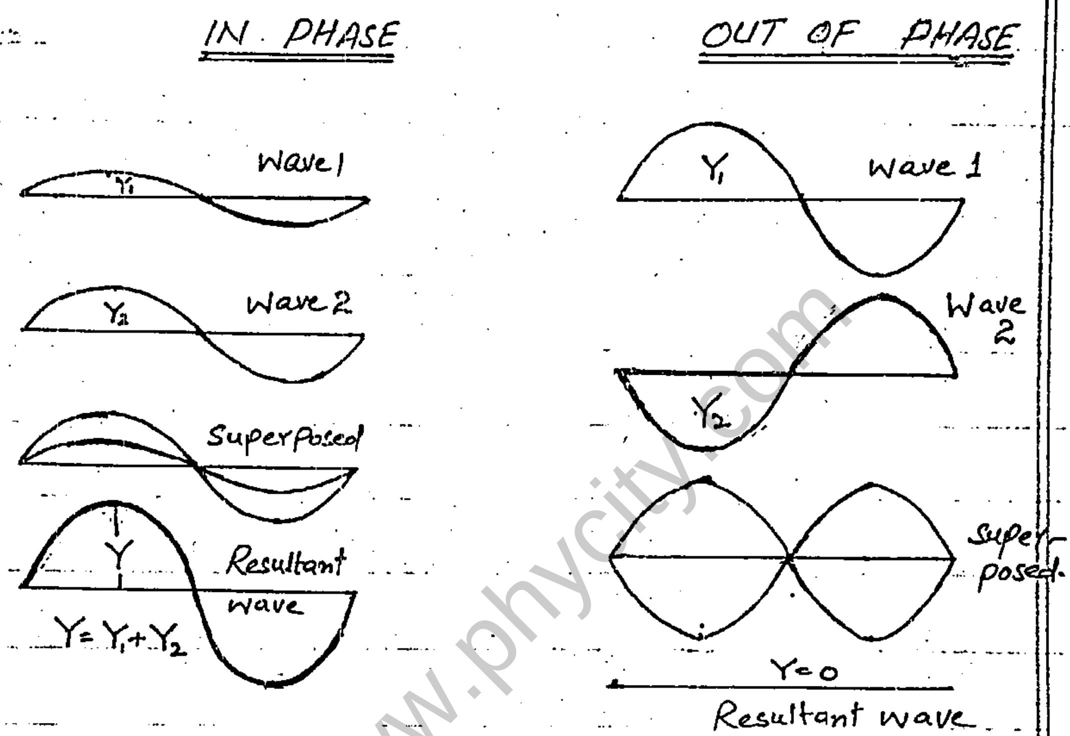
Def:- If a particle of a medium is acted upon by two waves simultaneously, then the resultant displacement of the particle is the algebric sum of their individual displacements. This is called the principle of superposition.

EXPLANATION:-

Suppose two waves are approaching each other on a coil of spring from opposite direction. They overlap and pass through each other without any change in their shape. But it is observed that when they overlap at a certain point during their motion. The resultant pulse will have a displacement equal to the sum of the individual displacements of the waves approaching each other. This fact is valid for all kinds of waves and can be illustrated as shown in the fig below.



The resultant wave pattern for two waves having same frequency when superpose with each other inphase or out of phase are shown in the figure



If now we suppose that a particle of a medium is simultaneously acted upon by n waves such that the displacements due to each of the individual waves be $Y_1, Y_2, \ldots Y_n$ then according to the principle of superposition the resultant displacement Y of the particle under the simultaneous action of these waves is given by

$$Y = Y_1 + Y_2 + \cdots\cdots + Y_n$$

INTERFERENCE

Def:- The superposition of two waves having same frequency and amplitude travelling in the same direction results in a phenomenon called interference.

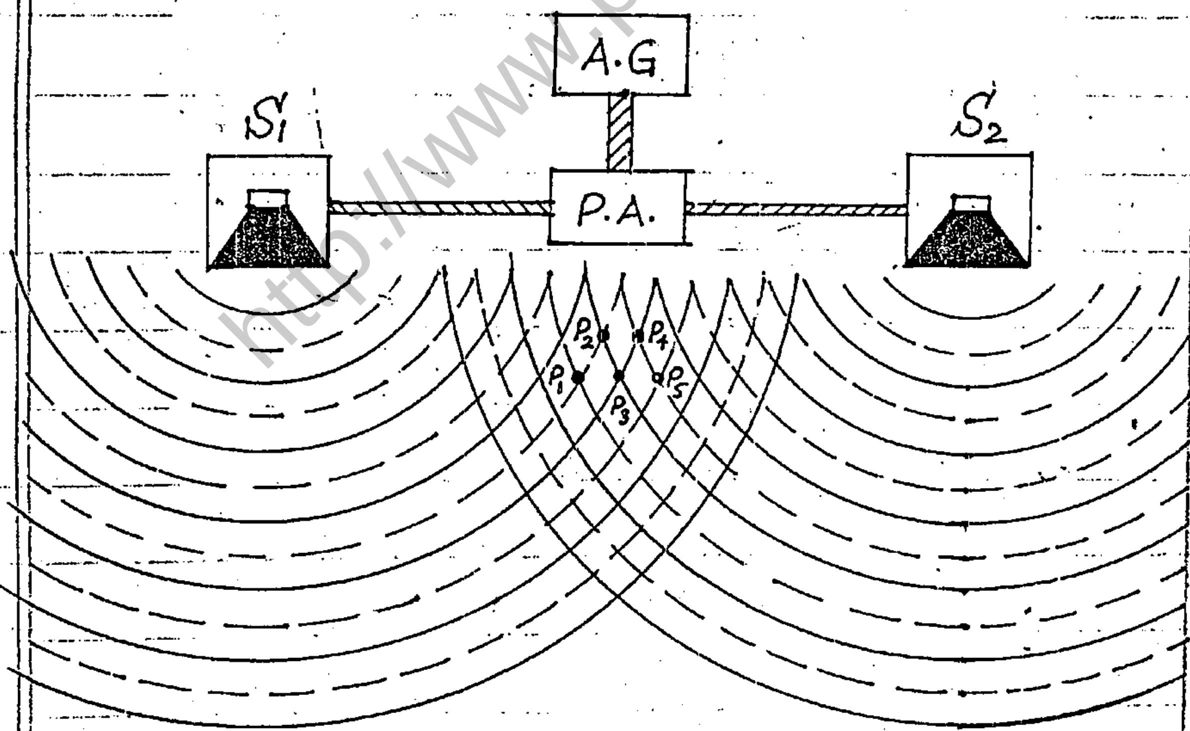
OR

When two identical waves moving along a medium in same direction in such a manner that they reinforce each other at some points and cancel the effect of each other at some other points, such a phenomenon is called interference.

EXPLANATION:-

In order to explain the phenomenon of interference, let us consider two loud speakers S_1 and S_2 attached to a sound source transmitting a signal of constant frequency in air. A microphone connected with a CRO (Cathode Ray Oscilloscope) is used as a detector for displaying the resultant intensity of sound waves at different points in front of speakers. Continuous lines indicates the compression while dotted line represents the rarefaction of sound. As points P_1, P_3 and P_5 are in phase it means that the compression of sound wave from one source reinforce the compression emitted from second source at point P_3. Similarly rarefactions from both sources S_1 & S_2 supports each other at points P_1 and P_5. As a result of which a loud sound is produced and a resultant wave with large amplitude is seen on the screen of CRO.

Similarly the waves approaching at points P_2 and P_4 are out of phase. Compression from one source cancels the effact of rarefaction from other soure and a wave with zero displacement is displayed on the screen.



CONDITION FOR CONSTUCTIVE INTERFERENCE

The path difference between the two waves arriving at point P_1 from two different sources is given as

$$\Delta S = S_2P_1 - S_1P_1$$
$$= 4\tfrac{1}{2}\lambda - 3\tfrac{1}{2}\lambda$$
$$= \lambda$$

Similarly the path difference b/w the two waves will be 0 and $-\lambda$ at points P_3 and P_5 respectively. So it is clear that whenever the path difference between the two waves is $0, \pm\lambda, \pm 2\lambda \ldots\ldots$ or integral multiple of wavelength i.e $n\lambda$, a maximum sound is heard.

CONSTRUCTIVE INTERFERENCE:-

When the path difference between the two waves is an integral multiple of wavelength i.e. $n\lambda$ where $n = 0, \pm 1, \pm 2, \ldots\ldots$, The two wave reinforce each other to give a resultant wave of larger displacement. This effect is called Constructive Interference.

CONDITION FOR DESTRUCTIVE INTERFERENCE

The path differce between the two waves approaching at point P_2 out of phase is given as

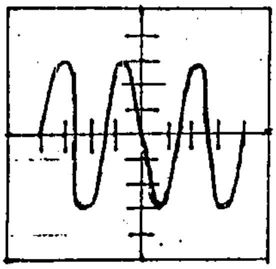
$$\Delta S = S_2P_2 - S_1P_2$$
$$= 4\lambda - 3\tfrac{1}{2}\lambda = \tfrac{1}{2}\lambda$$

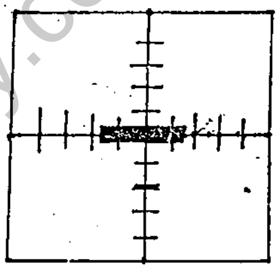
Similarly the path difference at point P_4 is $-\frac{1}{2}\lambda$

DESTRUCTIVE INTERFERENCE

When the path difference between the two waves is $\pm\lambda/2, \pm 3\lambda/2 + \cdots\cdots$ or odd integral multiple of half the wavelength i.e $(2n+1)\lambda/2$ the displacement of the two waves cancel each other's effect. This effect is called destructive interference.

The wave form for constructive and destructive interference as displayed on CRO are shown below.


Constructive Interference


Destructive Interference

BEATS

Def:- periodic variation of sound between maximum and minimum loudness are called beats.

OR

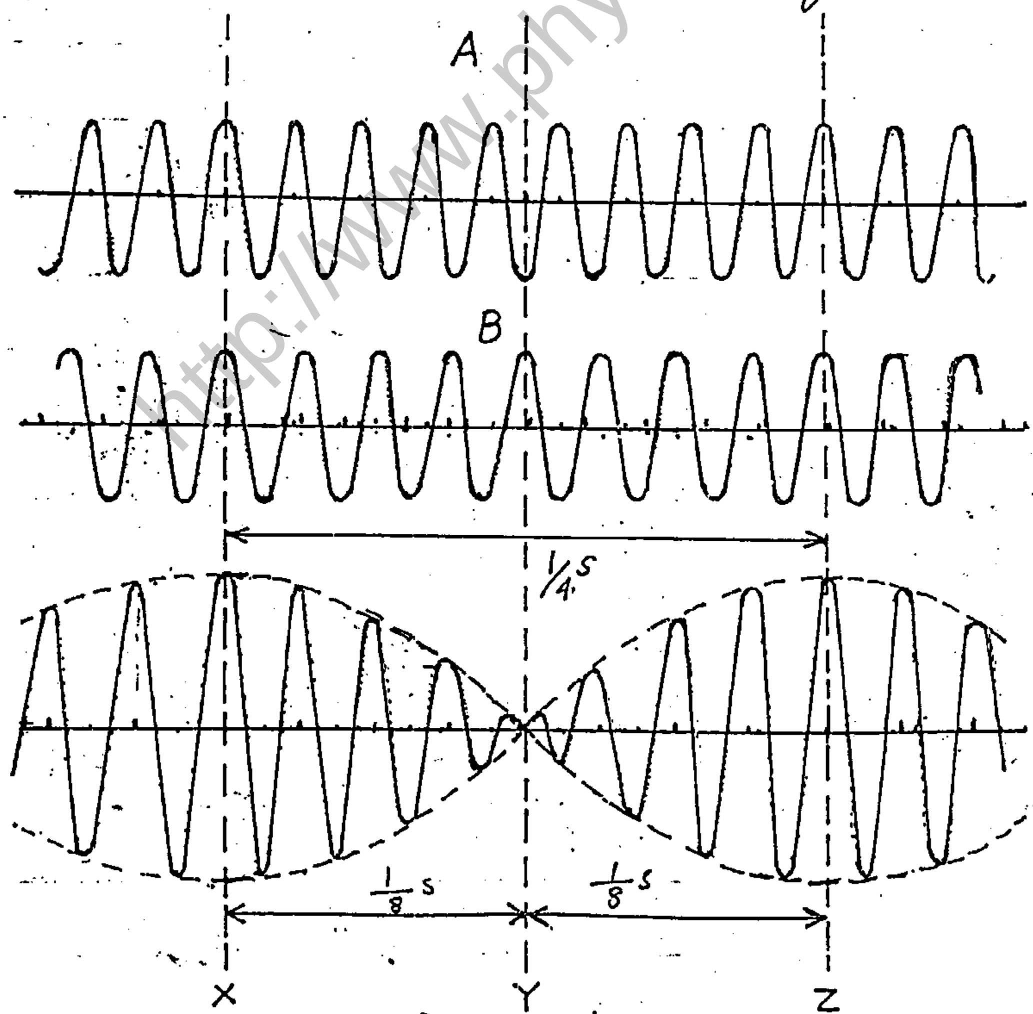
When two tuning forks of slightly different frequencies are sounded together then the periodic alteration in the intensity of sound is called beats.

EXPLANATION

When two tuning forks A & B

of same frequency say 256 Hz are sounded together, then it is very difficult to differentiate between their notes. But if tuning fork B is loaded with some wax or plasticine then its frequency will be lowered slightly, say it becomes 252 Hz

If now these tuning forks are sounded together, a note of slightly varying intensity is heard. which is called beats The wave form of the two tuning forks emitted in the form of compression and rarefaction along with the resultant displacement is shown in the fig below.



Resultant

At some particular instant X the displacement of the two waves is in the same direction. Both waves are in phase and thus reinforce each other to produce constructive interference as a result of which a loud sound is heard. At this instant the amplitude of the resultant waves becomes maximum at X.

After a time of $\frac{1}{8}$ s, the displacement of a wave emitted by one tuning fork is opposite to the displacement of the wave due to 2nd tuning fork. Both these waves will be out of phase and hence cancel the effect of each other due to destructive interference and a faint sound or no sound is heard at Y.

After $\frac{1}{4}$ s $\left(\frac{1}{8}s + \frac{1}{8}s\right)$ the displacements are again in phase and a loud sound is heard at Z

It means that in one second the intensity of sound increases or decreases four times. "One loud sound and one faint sound form one beat." So 4 beats will be heard by the listner. Also the frequency difference between the two tuning forks is 4.

#RESULT#

∴ From above discussion it is concluded that "number of beats per second is equal to the difference between the frequencies of the tuning forks."

Let f_A and f_B be the frequencies of two tuning forks and n be the number of beats per second then mathematically

$$f_A - f_B = \pm n$$

∴ Beats cannot be heard clearly if the frequency difference is more than about 10 Hz.

:APPLICATION:-

Beats are used for tuning the musical instruments like piano or violin. A desired frequency is achieved by beating a note against a note of known frequency. The length of the string is adjusted by loosening or tightening it until no variation in the intensity of sound is observed. At this instant no beat is heard and the instrument is said to be tuned at desired frequency.

REFLECTION OF WAVES

Def:- When a wave travelling along one medium comes across the boundary of second medium. Some part of it is reflected back. This bouncing back of the wave is called reflection of a wave.

-: EXPLANATION :-

When the wave is reflected from the boundary of second medium, the change in its phase depends upon the nature of the boundary as discussed below.

-: REFLECTION FROM THE BOUNDARY OF A DENSER MEDIUM :-

Let we take a slinky spring (A spring which has small initial length but a relatively large extended length) is tied to a rigid support on a smooth horizontal table. When a sharp jerk is given at its free end A, an upward pulse or crest will travel from free end to the boundary. It will exert a force on the fixed end. As the fixed end is rigid and acts as a denser medium Hence as a reaction a force is exerted on the spring and a downward pulse or trough starts travelling backwards towards end A.

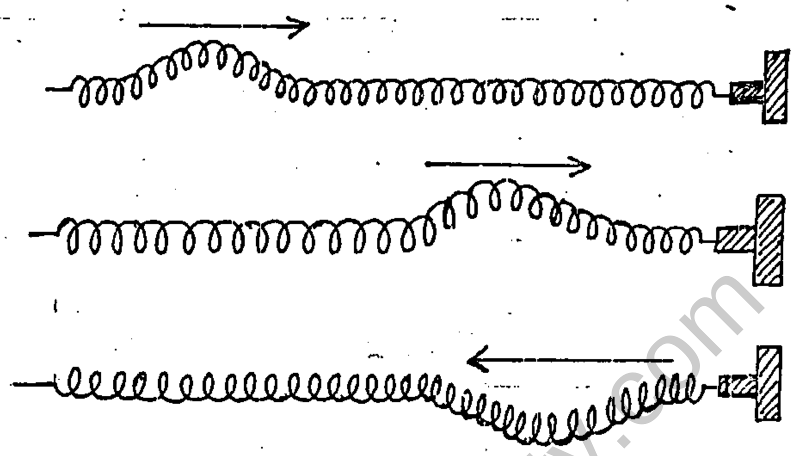
So it can be concluded that

-: **RULE** :- whenever a transverse wave travelling in a rarer medium encounters a denser medium, it bounces back such that the direction of displacement is reversed. i.e.

An incident crest on reflection becomes a trough and vice versa.

-: EFFECT ON PHASE :-

In this type of reflection a wave undergoes a phase change of 180°



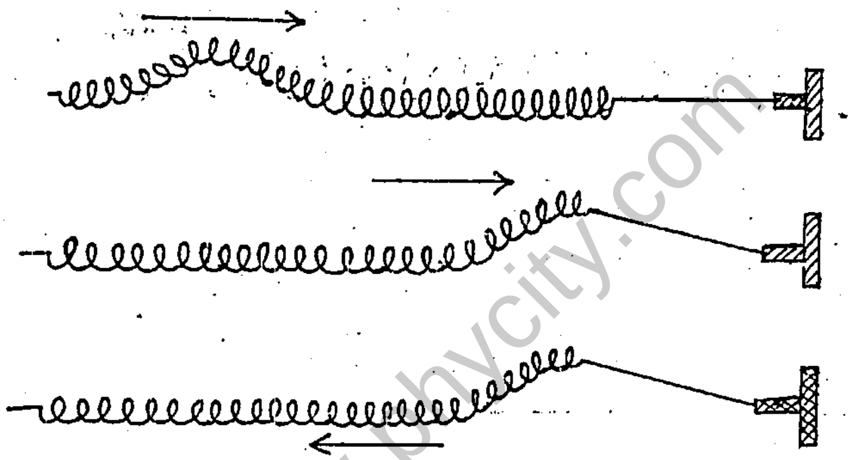
-: REFLECTION FROM THE BOUNDRY OF RARER MEDIUM :-

Attach one end of the slinky spring to a light string (rarer medium). Now if the spring is given a small jerk at end A, a crest travel along the spring as shown in the fig. When this crest reaches the spring string boundary, it will pluck the string upward. As a reaction an upward pulse or crest is generated and travels backward towards end A. So from above discussion it is concluded that

-: RULE :- When a transverse wave travelling in a denser medium is reflected from the boundary of a rarer medium, the direction of the displacement remains the same i.e. crest is reflected as crest and trough is reflected as trough

-: EFFECT ON PHASE :-

If a transverse wave travelling in a denser medium is incident on a rarer medium, it is reflected without any change in phase.



STATIONARY WAVES

Def:- The waves produced by the interference of two identical waves travelling inside a medium in opposite direction are called stationary or standing waves.

EXPLANATION:- Generally stationary waves are generated by the superposition of a wave travelling down a string with its reflection travelling in opposite direction.

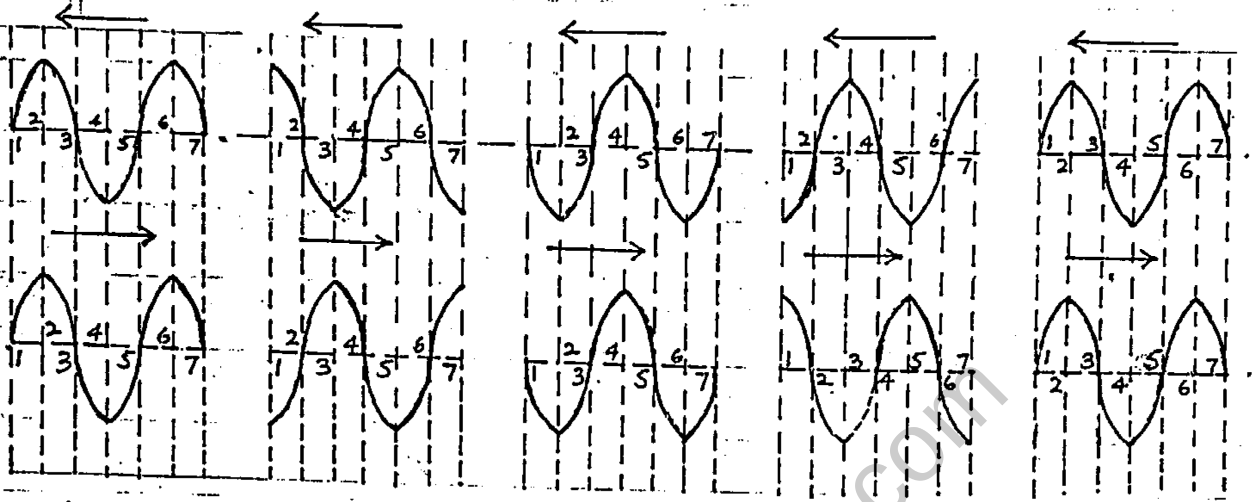
PRODUCTION OF STATIONARY WAVES

Let us consider the superposition of two waves moving along a string in opposite direction. Fig (a, b) shows the snapshot of the two waves at instants $t = 0, T/4, T/2, 3T/4$ and T where T is the time period of the waves.

Fig

(a)

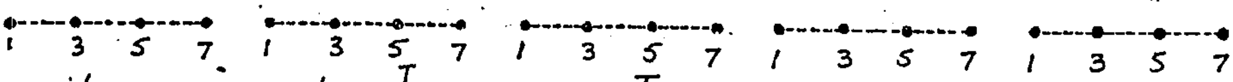
(b)



$t=0$ $\quad t=\frac{T}{4}$ $\quad t=\frac{T}{2}$ $\quad t=\frac{3T}{4}$ $\quad t=T$

We are interested to determine the displacement of points 1, 2, 3, 4, 5, 6 and 7 at these instants when the wave superpose. The distance between any two points is $\lambda/4$ where λ is the wave-length of the wave. By applying the principle of superposition, it is evident from fig (c) that the resultant displacement of points 1, 3, 5, 7 is equal to zero. Such points are called nodes.

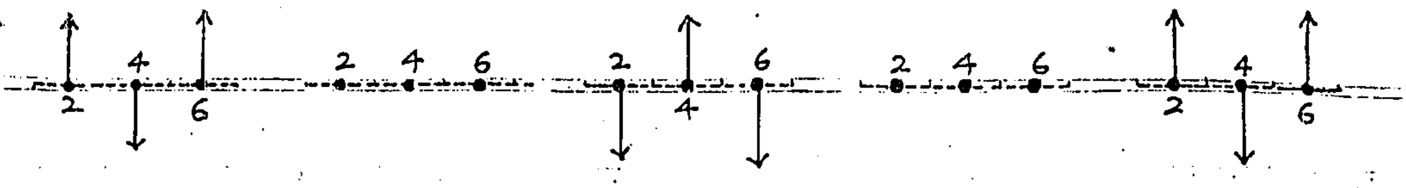
Fig (c)



$t=0$ $\quad t=\frac{T}{4}$ $\quad t=\frac{T}{2}$ $\quad t=\frac{3T}{4}$ $\quad t=T$

Fig (d) shows that the points 2, 4 and 6 are oscillating with an amplitude which is equal to the amplitudes of the component waves. These points are called antinodes

Fig (d)



$t=0$ $t=T/4$ $t=T/2$ $t=3T/4$ $t=T$

Such a pattern of nodes and antinodes is called stationary waves.

As for as the energy of the wave is concerned, it is oscillating between potential energy and kinetic energy. As nodes are at rest, so energy can not flow past these points and is standing in the medium between the nodes. When antinodes are at their extreme position, the energy stored in these points is wholy potential and is changed in to kinetic energy when they pass through their equilibrium position simultaneously

:- CHARACTRISTICS OF STATIONARY WAVES :-

Some main features of these waves are

(1) In such waves, the string is oscillating in the form of loops instead of waves travelling along the string.

(2) The distance between the two consecutive nodes or antinodes is equal to $\lambda/2$ where λ is the wavelength of transverse stationary waves.

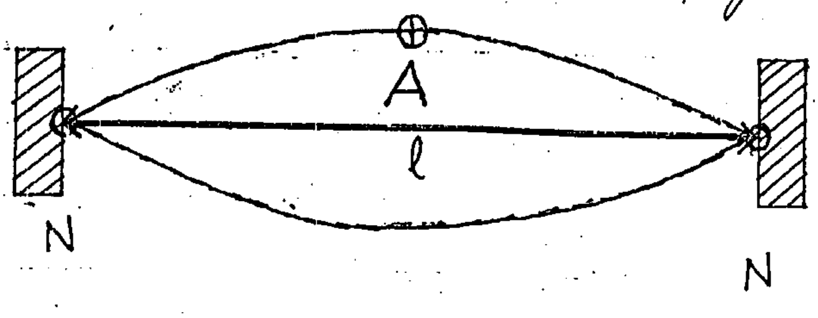
(3) The distance between a node and a neighbouring antinode is equal to $\lambda/4$

STATIONARY WAVES IN A STRETCHED STRING

Consider a string of length l which is kept stretched by clamping its end so that the tension in the string is F. The behaviour of the string when it is oscillating in the form of different loops is explained as under.

-: FIRST MODE OF VIBRATION :-

In this mode of vibration, the string is plucked at its middle point. Two transverse waves will originate from this point. One of them will move towards the left end of the string while the other moves towards the right end. When these waves reach the two clamped ends (denser medium), they are reflected back and thus giving rise to the stationary waves. The two clamped ends of the string are at rest and act as nodes denoted by N while the central point oscillating with maximum amplitude behaves as an antinode denoted by A. In this mode the string seems to be oscillating in one loop as shown in the fig.



As the distance between the two ends is equal to $\lambda/2$ which is also equal to the length of the string therefore

$$l = \lambda_1/2$$

$$\Rightarrow \lambda_1 = 2l \quad \text{———} \quad A'$$

Let f_1 be the frequency of the transverse wave for this mode of vibration then the equation of velocity of transverse wave is

$$V = f_1 \lambda_1$$

$$\Rightarrow f_1 = \frac{V}{\lambda_1}$$

$$f_1 = \frac{V}{2l} \quad \text{———} \quad A$$

Let F be the tension and m be the mass per unit length of string then the equation of speed V of the waves along the string is

$$V = \sqrt{\frac{F}{m}} \quad \text{———} \quad B$$

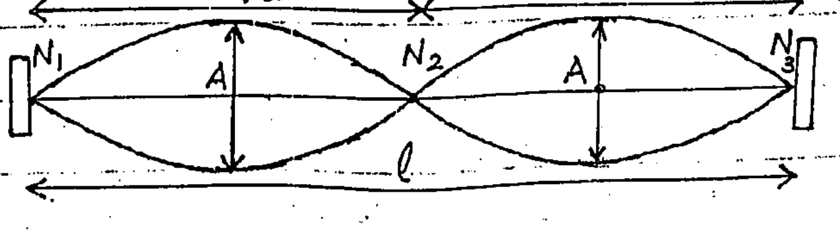
Hence by using the value of V in eq A we get

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

f_1 is also called fundamental frequency.

2ND MODE OF VIBRATION

If the same string is plucked from one quarter of its length, again stationary waves will be set up with nodes and antinodes as shown in the fig.



In this mode the string is vibrating in two loops. Let λ_2 be the wavelength of the wave along the string then the length l of the string is

$$l = N_1N_2 + N_2N_3$$

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

$$\Rightarrow \lambda_2 = \frac{2l}{2} \quad \text{------ } B'$$

Hence the wave length of the wave becomes half to that in the first Case. If f_2 be the frequency of vibration then the velocity of the wave is given as

$$V = f_2 \lambda_2$$

$$\Rightarrow f_2 = \frac{V}{\lambda_2} = \frac{V}{2l/2}$$

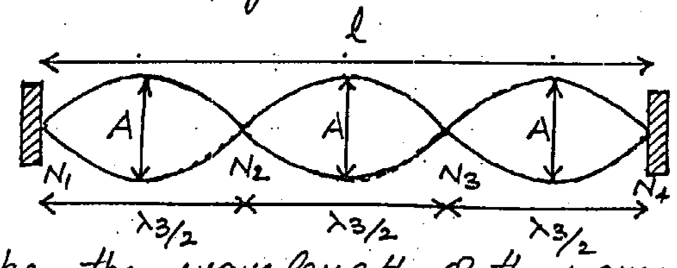
$$= 2\frac{V}{2l}$$

$$f_2 = 2f_1 \quad \text{------ } C$$

The above equation indicates that when the string oscillates in two loops, its frequency becomes twice that of fundamental frequency f_1.

-: 3RD MODE OF VIBRATION :-

In this mode the string is plucked at $\frac{1}{6}$th of its length and it starts vibrating in three loops as shown in the fig.



If λ_3 be the wavelength of the wave for this mode Then the length l of the string is given as.

$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = \frac{3\lambda_3}{2}$$

$$\Rightarrow \lambda_3 = \frac{2l}{3} \quad \text{---} \quad c'$$

from equation of velocity

$$V = f_3 \lambda_3$$

$$\Rightarrow f_3 = \frac{V}{\lambda_3} = \frac{V}{2l/3} = 3\frac{V}{2l}$$

$$f_3 = 3f_1$$

Hence frequency becomes thrice that of f_1 (Ist mode)

Now if the string is made to vibrate in n loops then the equations for its frequency and wavelength are

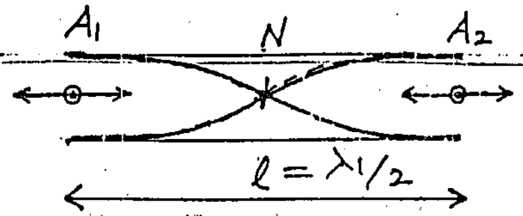
$$f_n = n f_1$$

$$\lambda_n = \frac{2l}{n}$$

From above equations it is concluded that with the increase in number of loops, the frequency goes on increasing and wave length starts decreasing in such a way that their product remains constant and is always equal to the speed V of the wave.

The discrete set of frequencies like $f_1, f_2 = 2f_1, 3f_1 \ldots nf_1$ at which stationary waves can exist is called harmonic series. Here f_1 and f_2 are called first harmonic and second harmonic respectively.

N.B. # The musical instrument like sitar or violin can be tunned by changing either the tension or length of string by tightening the pegs on the neck of the instrument. After being tuned the musician vary the frequency by moving their



Let λ_1 and f_1 be the wavelength and frequency for this mode of vibration then length of air column between two consecutive antinodes is given as

$$l = A_1A_2 = \lambda_1/2$$

$$\Rightarrow \lambda_1 = 2l$$

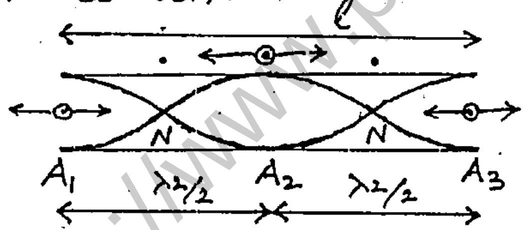
According to the equation of velocity

$$V = f_1\lambda_1$$

$$\Rightarrow f_1 = \frac{V}{\lambda_1} = \frac{V}{2l} \quad \text{——— (1)}$$

-: 2ND MODE OF VIBRATION:-

The wave pattern for this mode of vibration is shown as under.



If λ_2 be the wavelength for this mode the length of air column l is given as.

$$l = A_1A_2 + A_2A_3$$

$$= \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

$$\Rightarrow \lambda_2 = \frac{2l}{2} = l$$

From equation of velocity

$$V = f_2\lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2} = \frac{V}{l}$$

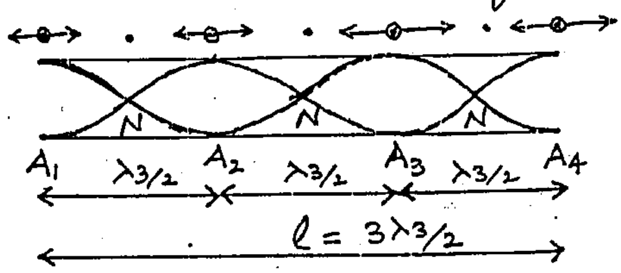
$$f_2 = 2\frac{V}{2l} = 2f_1$$

$$f_2 = 2f_1 \quad \text{——— (2)}$$

So frequency in this case is twice than that of first case.

3RD MODE OF VIBRATION

Wave pattern of vibrating air column for this mode is shown in the fig



Let λ_3 be the wavelength in this mode the length l of vibrating air column is given as

$$l = A_1A_2 + A_2A_3 + A_3A_4$$

$$= \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \lambda_3/2 = \frac{3\lambda_3}{2}$$

$$\Rightarrow \lambda_3 = \frac{2l}{3}$$

$$\because \quad V = f_3\lambda_3$$

$$\Rightarrow f_3 = \frac{V}{\lambda_3} = \frac{V}{2l/3} = 3\times\frac{V}{2l}$$

$$f_3 = 3f_1 \quad \text{———— (3)}$$

where f_3 is the frequency for third mode.

If the air column is vibrating for nth mode then its frequency is given as

$$f_n = nf_1 \quad \text{or} \quad n\frac{V}{2l} \quad \text{———— (4)}$$

where $n = 1, 2, 3, 4 \ldots\ldots$

Here v is the speed of longitudinal wave along the spring and l is the length of open organ pipe.

CLOSED ORGAN PIPE :-

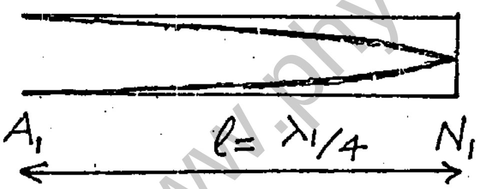
Def:- A pipe having one end open and reflected end closed is called closed organ pipe.

STATIONARY WAVES IN CLOSED ORGAN PIPE

In closed organ pipe the closed reflected end behaves as a node because the movement of the molecules is restricted at reflected end. Now we discuss the different modes of vibration in closed organ pipe.

-: IST MODE OF VIBRATION :-

The wave pattern produced for this fundamental mode of vibration is shown in the figure.



The end where the air molecules have complete freedom of motion behaves as antinode while the other end where the motion of air molecules is restricted acts as a node. Let f_1 be the frequency and λ_1 be the wavelength for this mode then the length of the pipe which is equal to the distance between the node and antinode, is given as

$$l = A_1N_1 = \frac{\lambda_1}{4}$$

$$\Rightarrow \lambda_1 = 4l$$

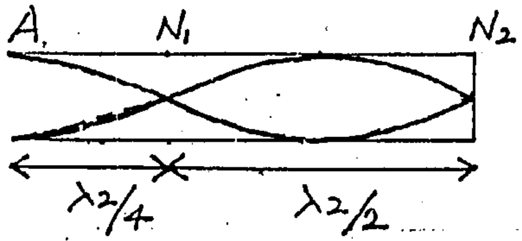
Since the equation of velocity of the wave

$$\Rightarrow f_1 = \frac{V}{\lambda_1}$$

$$f_1 = \frac{V}{4l} \quad \text{———} \quad A'$$

2ND MODE OF VIBRATION

The wave pattern for this mode is given as



Let f_2 be the frequency and λ_2 be the wavelength for this second mode (second harmonic) the length of air column according to the above figure is

$$l = AN_1 + N_1N_2$$

$$l = \lambda_2/4 + \frac{\lambda_2}{2} = \frac{\lambda_2 + 2\lambda_2}{4} = \frac{3\lambda_2}{4}$$

$$\Rightarrow \lambda_2 = \frac{4l}{3}$$

$$\because V = f_2\lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2}$$

$$f_2 = V/\tfrac{4l}{3}$$

$$= 3 \times \frac{V}{4l}$$

$$f_2 = 3f_1 \quad \text{———} \quad B'$$

Hence the frequency for this mode is equal to three times that of fundamental mode.

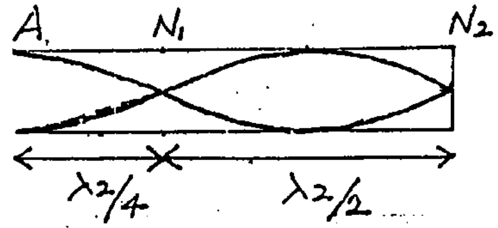
Similarly for 3rd mode of vibration it can be proved that $f_3 = 5f_1$ and so on. So from above discussion it can be concluded that in closed organ pipe only odd harmonics can be produced. For nth mode the is $V = f_1 \lambda_1$

$$\Rightarrow f_1 = \frac{V}{\lambda_1}$$

$$f_1 = \frac{V}{4l} \text{ ——— } A'$$

2ND MODE OF VIBRATION

The wave pattern for this mode is given as



Let f_2 be the frequency and λ_2 be the wavelength for this second mode (second harmonic) the length of air column according to the above figure is

$$l = AN_1 + N_1N_2$$

$$l = \lambda_2/4 + \frac{\lambda_2}{2} = \frac{\lambda_2 + 2\lambda_2}{4} = \frac{3\lambda_2}{4}$$

$$\Rightarrow \lambda_2 = \frac{4l}{3}$$

$$\because V = f_2\lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2}$$

$$f_2 = V/\tfrac{4l}{3}$$

$$= 3 \times \frac{V}{4l}$$

$$f_2 = 3f_1 \text{ ——— } B'$$

Hence the frequency for this mode is equal to three times that of fundamental mode.

Similarly for 3rd mode of vibration it can be proved that $f_3 = 5f_1$ and so on. So from above discussion it can be concluded that in closed organ pipe only odd harmonics can be produced. For nth mode the equation of the frequency is

$$f_n = n f_1$$
$$= n \frac{v}{4l} \quad \text{---------- } \acute{c}$$

where $n = 1, 3, 5, 7, \ldots$

From equation 4 and equation ć it is clear that open organ pipe is richer in harmonics as compared to closed organ pipe.

DOPPLER'S EFFECT

Christian Johann Doppler observed that there is a slight change in the frequency of light emitted from a distant star as compared to a similar source on the earth. On the basis of his conclusions, he stated that

:STATEMENT:-

The apparent change in the frequency of the light waves due to relative motion between the source of waves and the observer is called Doppler's Effect.

This effect is also applicable for sound waves

:EXPLANATION:-

When an observer is standing on a railway plateform, the pitch of the whistle of an approaching locomotive is heard to be higher But when the same locomotive moves away from the observer

This change in frequency can be determined easily by taking the relative motion between the source and observer along a straight line. The effect can be illustrated under the following four cases.

CASE I :– WHEN THE OBSERVER IS MOVING TOWARDS A STATIONARY SOURCE :–

Let a source is emitting sound waves of frequency f and wavelength λ inside a medium having velocity V. Initially if the source and the listener are at rest then the number of waves emitted by the source per second is given by.

$$f = \frac{V}{\lambda} \qquad (\because V = f\lambda)$$

If the observer A is moving towards the source with a velocity u_0. The relative velocity of the source w.r.t. the observer is $V-(-u_0)$

$$= V + u_0$$

The apparent frequency experienced by the observer in this condition is given as

$$f_A = \frac{V+u_0}{\lambda}$$

$$\because \lambda = \frac{V}{f} \qquad \therefore f_A = \frac{V+u_0}{V/f}$$

$$f_A = \left(\frac{V+u_0}{V}\right) f$$

As the term $\frac{V+u_0}{V} > 1$

This change in frequency can be determined easily by taking the relative motion between the source and observer along a straight line. The effect can be illustrated under the following four cases.

CASE I :- WHEN THE OBSERVER IS MOVING TOWARDS A STATIONARY SOURCE :-

Let a source is emitting sound waves of frequency f and wavelength λ inside a medium having velocity V. Initially if the source and the listener are at rest then the number of waves emitted by the source per second is given by

$$f = \frac{V}{\lambda} \qquad (\because V = f\lambda)$$

If the observer A is moving towards the source with a velocity U_0. The relative velocity of the soure w.r.t. the observer is $V-(-U_0)$

$$= V + U_0$$

The apparent frequency experienced by the observer in this condition is given as

$$f_A = \frac{V+U_0}{\lambda}$$

$$\because \lambda = \frac{V}{f}$$

$$\therefore f_A = \frac{V+U_0}{V/f}$$

$$f_A = \left(\frac{V+U_0}{V}\right) f$$

As the term $\frac{V+U_0}{V} > 1$

So $f_A > f$

Hence the apparent frequency heard by the listner is increased

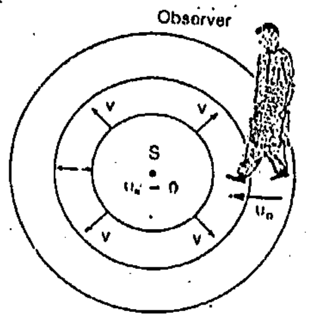


Fig 8.18
An observer moving with velocity u_o towards a stationary source hears a frequency f_A that is greater than the source frequency.

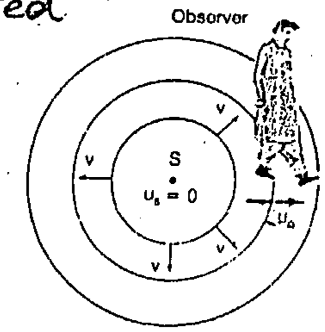


Fig 8.19
An observer moving with velocity u_o away from stationary source hears a frequency f_B that is smaller than the source frequency.

:CASE 2 :- WHEN OBSERVER IS RECEDING FROM A STATIONARY SOURCE

Since we know that when source and the observer are at rest then the number of waves received by the observer per second are

$$f = \frac{V}{\lambda}$$

$$\Rightarrow \lambda = \frac{V}{f}$$

If now the observer B is moving away from a stationary source with velocity U_o. The relative velocity of waves and the observer will become $(V - U_o)$. Thus the observer receives waves at a reduced rate. Hence the no. of waves received in one second are

$$f_B = \frac{V - U_o}{\lambda}$$

By using the value of λ

$$f_B = \frac{V - U_o}{V/f}$$

$$f_B = \left(\frac{V - U_o}{V}\right) f$$

As $\frac{V-U_o}{V} < 1$

$$\Rightarrow f_B < f$$

Hence the apparent frequency will be reduced.

CASE 3:- WHEN SOURCE APPROACHES A STATIONARY OBSERVER

Let a source is emitting f number of waves in one second with a velocity V having a wavelength λ. when the source and the observer are at rest then the wavelength λ of the waves received by the listner (observer) will be

$$\lambda = \frac{V}{f} = \frac{\text{distance occupied by the waves}}{\text{no. of waves}}$$

Now if the source is moving with velocity U_s towards the stationary observer then in one second the same no. of waves are compressed in comparatively shorter space so the shift in wave length $\Delta\lambda$ is given as

$$\Delta\lambda = \frac{U_s}{f}$$

Hence the new modified wavelength for the observer is

$$\lambda_c = \lambda - \Delta\lambda$$

$$= \frac{V}{f} - \frac{U_s}{f} = \frac{V-U_s}{f}$$

therefore the apparent (modified) frequency for the observer is given as

$$f_c = \frac{V}{\lambda_c}$$

$$f_c = \frac{V}{V-U_s/f}$$

$$f_c = \left(\frac{V}{V-U_s}\right) f$$

$$\text{As} \quad \frac{V}{V-U_s} > 1$$

Hence the apparent frequency will be increased.

#CASE 4# SOURCE IS RECEDING FROM THE LISTENER

If now the source is moving away from the listener with a velocity U_s then the space occupied by the waves received by the observer is increased. The change or shift in wavelength is given as

$$\Delta\lambda = \frac{U_s}{f}$$

The modified (apparent) wavelength will appear to increase and is given as

$$\lambda_D = \lambda + \Delta\lambda$$

$$= \frac{V}{f} + \frac{U_s}{f}$$

$$= \frac{V+U_s}{f}$$

So the apparent frequency is

$$f_D = \frac{V}{\lambda_D} = \frac{V}{V+U_s/f}$$

$$f_D = \left(\frac{V}{V+U_s}\right) f$$

$$\text{As} \quad \frac{V}{V+U_s} < 1$$

$$\Rightarrow f_D < f$$

Hence the frequency of sound received by the observer is decreased when source moves away from the listener.

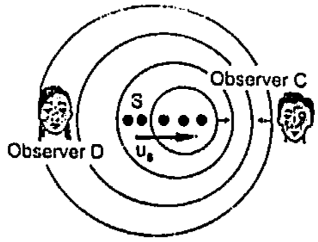


Fig. 8.20

A source moving with velocity u_s towards a stationary observer C and away from stationary observer D. Observer C hears an increased and observer D hears a decreased frequency.

APPLICATIONS OF DOPPLER EFFECT

The main applications of Doppler's effect are discussed as under.

(1) Radar is a device for transmitting and receiving radio waves (electromagnetic waves) and is used to determine the speed and the elevation of an aeroplane. when a plane is approaching a radar, the wavelength of reflecting waves becomes shorter. If the plane is receding, its wavelength become larger. By observing the Doppler shift, the speed of the satellite is also determined.

(2) Sonar is derived from sound navigation and ranging. It is a technique which is used to detect and locate the objects like Submarine, anti-submarine weapons and mines under water. By observing the Doppler shift in the sound signal, the speed and location of submarines and other military weapons can be estimated.

(3) Astronomers use this effect to determine the speed of stars and galaxies by comparing the spectrum of light from stars with light from a laboratory source. when a star is approaching the earth, its frequency of light is increased which is evident from the shifting of spectrum towards the blue end of shorter wavelength. when a star is moving away from earth, the spectrum of its light is shifted towards the red end of longer wavelength which shows a decrease in frequency due to the receding motion of star.

(4) In radar speed trap, microwaves (electromagnetic waves) emitted from a transmitter in short bursts are reflected from a moving car. The reflected beam from any moving car is received by the transmitter. With the measurement of Doppler shift in the reflected beam, the speed of a moving car is calculated by computer programme.

(5) A bat determine the nature and location of objects by sending ultrasonics They are likely to possess an ability to find the speed of surrounding objects by sensing Doppler shift

EXAMPLE 8.2.

A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

SOLUTION:-

Data

$$f_A = 320\ Hz$$

Before loading

Beat frequency $= n = 4$

After loading

Beat frequency $= n' = 6$

$$f_B = ?$$

Before loading

$$\because f_B - f_A = \pm n$$

$$f_B = f_A \pm n$$

$$= 320 \pm 4$$

$$= 324\ Hz,\ 316\ Hz$$

Let 324 Hz be the original frequency of tuning fork B. when it is loaded, its frequency is decreased to 323, 322 ..... due to which the no. of beats or beat frequency is decreased. Hence it will not satisfy the new beat frequency $n' = 6$. So this frequency is not Correct. Hence the actual frequency is 316 Hz. when this tuning fork is loaded, its frequency is decreased. Let the frequency is decreased by 2 and it becomes 314 Hz, then the no. of beats producing b/w the two tuning forks will become equal to 6 as the frequency difference b/w the two tuning forks is also six. Hence the frequency of tuning fork B after loading is 314 Hz.

EXAMPLE 8.3

A steel wire hangs vertically from a fixed point, supporting a weight of 80N at its lower end. The diameter of the wire is .50mm and its length from the fixed point to the weight is 1.5m. Calculate the fundamental frequency emitted by the wire when it is plucked?

Density of steel wire $= 7.8 \times 10^3 \, Kg\,m^{-3}$

-: SOLUTION :- **Data**

Density of steel wire $= d = 7.8 \times 10^3 \, Kg/m^3$

Diameter of wire $= D = .50 \, mm$

$= .5 \times 10^{-3} \, m$

$= 5 \times 10^{-4} \, m$

radius of the wire $= \frac{D}{2} = 2.5 \times 10^{-4} \, m$

weight = load $= F = 80 \, N$

length of wire $= l = 1.5m$

fundamental frequency $= f = ?$

SOLUTION: ~~From the equation of~~ density of wire.

$$d = \frac{m}{V}$$

$$\Rightarrow m = d \times V$$

$\because$ volume = Area × length

$$V = A \times l$$

$$\therefore m = d \times A \times l$$

$$m' = \frac{m}{l} = d \times A = d \times \pi r^2$$

$$= 7.8 \times 10^3 \times 3.14 \times (2.5 \times 10^{-4})^2$$

$$m' = 1.53 \times 10^{-3}\ Kg\,m^{-1}$$

As the equation of fundamental frequency is

$$f = \frac{1}{2l}\sqrt{\frac{F}{m'}}$$

$$= \frac{1}{2 \times 1.5}\sqrt{\frac{80}{1.53 \times 10^{-3}}}$$

$$= 76.22\ Hz \simeq 76\ Hz$$

EXAMPLE # 8.4

A pipe has a length of 1m. Determine the frequency of fundamental and first two harmonics

(a) if the pipe is open at both ends

(b) if the pipe is closed at one end

SOLUTION (a) velocity of sound in air = $340 ms^{-1}$

$f_1 = ?$ $f_2 = ?$, $f_3 = ?$

when the pipe is open at both end the general equation for frequency is given as.

$$f_n = n f_1$$
$$= n \frac{v}{2l}$$
$$\Rightarrow f_1 = 1 \times \frac{v}{2l} = \frac{1 \times 340}{2 \times 1}$$
$$f_1 = 170 \text{ Hz}$$

Also $f_2 = 2 f_1 = 2 \times 170 = 340 \text{ Hz}$

$f_3 = 3 f_1 = 3 \times 170 = 510 \text{ Hz}$

(b)

when one end of the pipe is closed then $f_n = n f_1 \qquad n = 1, 3, 5, \ldots$

$$= n \frac{v}{4l}$$
$$f_1 = 1 \frac{v}{4l} = \frac{340}{4 \times 1} = 85 \text{ Hz}$$
$$f' = 3 f_1 = 3 \times 85 = 255 \text{ Hz}$$
$$f'' = 5 f_1 = 5 \times 85 = 425 \text{ Hz}$$

f_1, f', f'' are odd harmonics inside a closed organ pipe.

EXAMPLE 8.5

A train is approaching a station at 90 km h^{-1} sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the plateform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed

(Speed of Sound $= 340 \text{ m s}^{-1}$

SOLUTION:- DATA:-

Frequency of Source $= f_o = 1000\,Hz$

velocity of Sound $= V = 340\,ms^{-1}$

Speed of train (Source) $= U_s = 90\,km\,h^{-1}$

$$= \frac{90 \times 1000}{3600} = 25\,ms^{-1}$$

(a) $f' = ?$

(b) $f'' = ?$

(a) when a train is approaching towards a stationary listener then the apparent frequency f' is given as

$$f' = \left(\frac{V}{V - U_s}\right) f$$

$$= \left(\frac{340}{340 - 25}\right) \times 1000$$

$$= \frac{340}{315} \times 1000 = \frac{340000}{315} = 1079.36\,Hz$$

$$\simeq 1079.4\,Hz$$

(b) when the train is moving away from the stationary listener then using the relation

$$f'' = \left(\frac{V}{V + U_s}\right) f$$

$$= \frac{340}{340 + 25} \times 1000 = \frac{340000}{365}$$

$$= 931.50\,Hz$$