CHAPTER 6
FLUID DYNAMICS

FLUID :-
A substance that can flow is called Fluid.
e.g :- Liquids , Gases.

FLUID STATICS :- The study of properties of fluids at rest is known as Fluid Statics.
It is based on Newton's first and third law.

FLUID DYNAMICS :- The study of properties of fluids in motion is known as Fluid Dynamics.
The study of fluids in motion is relatively complicated, but analysis can be simplified by the use of Newton's laws and conservation of mass and conservation of energy.

- The law of conservation of mass gives us the equation of continuity.
- The law of conservation of energy is the basis of Bernoulli's theorm.

6.1 VISCOUS DRAG AND STOKE'S LAW

1- VISCOSITY

(a). Definition :- The internal frictional effect between different layers of a flowing fluid is described interms of viscosity of the fluid.

(b). Explanation :-

(i). Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that do not flow easily, such as thick tar and honey etc; have large coefficient of viscosities. Substances which flow easily, like water have small coefficient of viscosities. Since liquids and gases have non zero viscosity, a force is required if an object is to be moved through them.

(ii). **Example:-** When we put our hand out of the window of a fast moving car. We feel that air exerts a considerable force opposite to our motion. This shows that there is a frictional force in fluids due to the viscosity of fluids.

(iii). **Symbol:-** The coefficient of viscosity is denoted by greek letter 'η'. $\left(\eta = \frac{F}{6\pi r v}\right)$

(iv). **Unit:-** Its S.I unit is $kg\, m^{-1} s^{-1}$ $\left(\text{i.e } \frac{N}{m\, m s^{-1}}\right)$

(V). **Dimensions:-** Its dimensions are $[ML^{-1}T^{-1}]$

2. DRAG FORCE:-

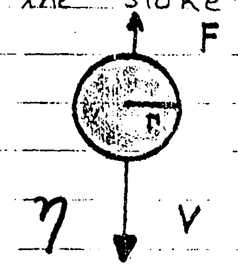
An object moving through a fluid experience a retarding force called a drag force.

- The drag force (fluid resistance) increases as the speed of the object increases.

3. STOKE'S LAW:-

The drag force 'F' on a sphere of radius 'r' moving slowly with speed 'v' through a fluid of viscosity 'η' is given by the Stoke's law as

$$\boxed{F = 6\pi \eta r v}$$



- At very high speeds the force is no longer simply proportional to speed.

6.2 TERMINAL VELOCITY

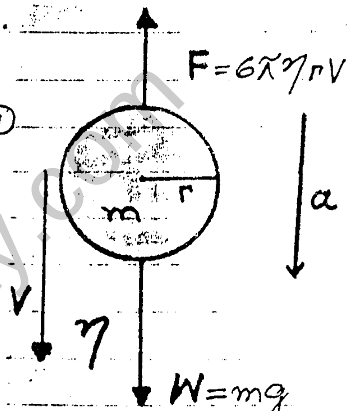
1- **Definition:-** When the magnitude of the drag force becomes equal to weight of the body, the net force acting on the body is zero. Then body will fall with constant velocity called Terminal Velocity.

2. Explanation:-

(a)- Consider a water droplet such as that of fog falling vertically downward under the influence of force of gravity and drag force. The air drag on the water droplet increases with speed. The droplet accelerates rapidly under the over powering force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases.

The net force on the droplet is

$$\boxed{\text{Net Force} = \text{Weight} - \text{Drag force}} \quad \text{--- (1)}$$



As the speed of the droplet continues to increase, the drag force also increases and eventually approaches the weight in the magnitude.

Finally, when, the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant velocity called Terminal velocity (V_t).

To find the terminal velocity 'V_t' in this case, we use Stokes Law for the drag. Equating it to the weight of the droplet.

$$\text{Weight} = \text{Drag Force}$$
$$mg = 6\pi\eta r V_t$$

$$\boxed{V_t = \frac{mg}{6\pi\eta r}} \quad \text{--- (2)}$$

Equation (2) shows that the terminal velocity of an object depends upon mass of the object if radius remains constant i.e $V_t \propto m$ if r = Constt.
i.e the massive objects of same radius fall faster through the same fluids.

(b)- **Dependence on Size**

Let 'ρ' be the density and 'V' be the volume of fog (water droplet), then

The mass of the droplet $m = \rho V$

where volume of spherical droplet $= V = \frac{4}{3}\pi r^3$

$\therefore \quad m = \frac{4}{3}\pi r^3 \rho$

Putting this value in equation (2)

$$V_t = \frac{\frac{4}{3}\pi r^3 \rho g}{6\pi \eta r}$$

$$\boxed{V_t = \frac{2g\rho r^2}{9\eta}}$$

or

$$V_t \propto r^2 \qquad \frac{2g\rho}{9\eta} = \text{Constt.}$$

Example 6.1:- A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal Velocity. Given that 'η' for air $= 19\times10^{-6}\ \text{Kg}\,m^{-1}s^{-1}$ and density of water $\rho = 10^3\ \text{Kg}\,m^{-3}$

Solution:-

$r = 0.010\ \text{cm} = 0.010\times10^{-2}\ m = 1.0\times10^{-4}\ m$

$\rho = 1000\ \text{Kg}\,m^{-3}$

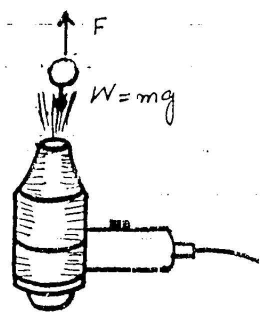
$\eta = 19\times10^{-6}\ \text{Kg}\,m^{-1}s^{-1}$

As we have

$$V_t = \frac{2g\rho r^2}{9\eta}$$

$$= \frac{2\times9.8\times1000\times(1.0\times10^{-4})^2}{9\times19\times10^{-6}}\ m s^{-1}$$

$$\boxed{V_t = 1.1\ m s^{-1}}$$



Can you do that? A table Tennis ball can be made suspended in the stream of air coming from the nozzle of a hair dryer when weight of the ball is balanced by the force produced by air of hair dryer

i-e $F = W$

6.3 FLUID FLOW

Introduction Moving fluids are of great importance. To study the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can be either

- Streamline / Laminar or
- Turbulent.

1- STREAMLINE OR LAMINAR FLOW

(a). Definition:- The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

(b). Explanation:-

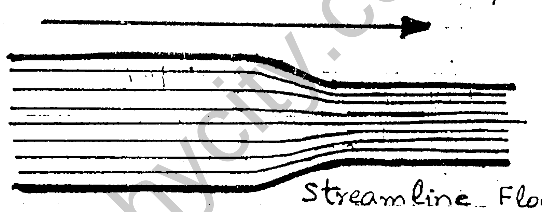


Fig. (a)

(i)- Line of Flow:-

The path followed by a particle of the fluid is called line of flow.

(ii)- Stream Line:-

If each particle of the fluid moves along a smooth path, then line of flow is called a stream line as shown in figure (a).

(iii). Steady Flow Condition:-

If the different streamlines cannot cross each other. This condition is called Steady Flow Condition as shown in fig. (a).

- The direction of streamlines is the same as the direction of the velocity of fluid at that point.

(c). Examples:-

(i). Dolphins have streamlined bodies to assist their movement in water.

(ii)- Formula One racing cars have a streamlined design.

2. TURBULENT FLOW:-

(a)- **Introduction:-** Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular.

(b)- **Definition:-** The irregular or unsteady flow of the fluid is called turbulent flow.

(c)- **Explanation:-**

Under this condition (unsteady or irregular flow) the velocity of the fluid changes abruptly as shown in fig (b). In this case the exact path of the particles of the fluid can not be predicted.

Fig.(b) Turbulent flow

- **Behaviour Of An Ideal Fluid**

A fluid is said to be an ideal fluid which satisfies the following conditions.

1- The fluid is non-viscous i-e there is no internal frictional force between adjacent layers of fluid.

2- The fluid is incompressible i-e its density is constant.

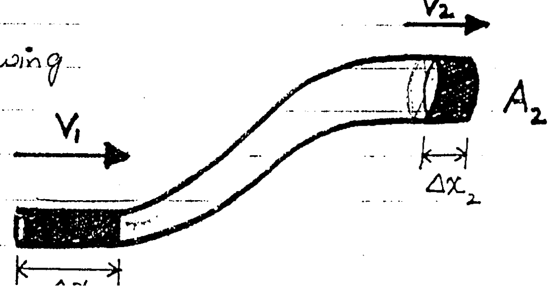
3- The fluid motion is steady.

6.4 EQUATION OF CONTINUITY

1- **Definition:-** For a non viscous and an incompressible fluid in steady flow, the net rate of flow of mass inward across any closed surface is equal to the net rate of flow of mass outward. This is the statement of equation of continuity.

2- **Expression:-**

Consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move



along the streamlines in a steady state flow as shown in figure.
In the small time 'Δt', the fluid at the lower end of the tube moves a distance Δx_1, with velocity v_1. If 'A_1' is the area of cross section of this end, then

Volume of fluid in lower shaded region $= A_1 \Delta x_1$

Let density of fluid be 'ρ_1'

Therefore the mass of the fluid contained in lower shaded region is:

$$\Delta m_1 = \rho_1 A_1 \Delta x_1 \qquad (\text{mass} = \text{density} \cdot \text{Volume})$$

As $\quad S = vt$

So $\quad \Delta x_1 = V_1 \Delta t$

$$\therefore \quad \Delta m_1 = \rho_1 A_1 V_1 \Delta t \longrightarrow (1)$$

Similarly the fluid that moves with velocity 'V_2' through the upper end of the pipe (area of cross section A_2) in the same time 'Δt' has mass

$$\Delta m_2 = \rho_2 A_2 V_2 \Delta t \longrightarrow (2)$$

If the fluid is incompressible and the flow is steady the mass of the fluid is conserved.
That is, the mass that flows into the bottom of the pipe through 'A_1' in a time 'Δt' must be equal to mass of the liquid that flows out through 'A_2' in the same time.

Therefore $\quad \Delta m_1 = \Delta m_2$

or $\quad \rho_1 A_1 V_1 \Delta t = \rho_2 A_2 V_2 \Delta t$

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

This equation is called the equation of continuity.
Since density is constant for the steady flow of incompressible fluid, the equation of continuity becomes:

$$\rho A_1 V_1 = \rho A_2 V_2 \qquad \because \rho_1 = \rho_2 = \rho \text{ (Let)}$$

$$\therefore \boxed{A_1 V_1 = A_2 V_2}$$

This equation is another form of equation of continuity. This equation shows that
"The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the **volume flow per second** of the fluid or simply **flow rate**."

i.e

$$AV = \text{Constant} = \text{Volume flow rate}$$

NOTE :-

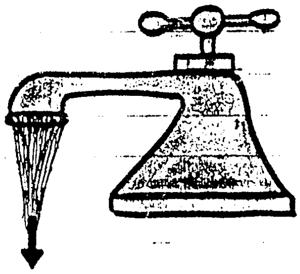
$$V = \frac{\text{Constt.}}{A}$$

$$\Rightarrow \quad V \propto 1/A$$

In steady incompressible flow the speed of flow varies inversely with the cross sectional area.

Tidbits :-

As water falls, its speed increases due to force of gravity and so its cross-sectional area decreases as mandated by the continuity equation.



Example :- 6.2 :- A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kg m^{-3} and its flow is steady.

Solution :- Mass flow per second $= \frac{\Delta m}{\Delta t} = \frac{30}{60} = 0.5 \text{ kg } s^{-1}$.

Cross sectional area $= A = \pi r^2$

As $\Delta m = \rho A V \Delta t$

then $\rho A V = \frac{\Delta m}{\Delta t}$

$$\therefore \quad V = \frac{\Delta m / \Delta t}{\rho A} \quad \text{---} (1)$$

Putting values in eq. (1)

$$V = \frac{0.5 \text{ kg } s^{-1}}{1000 \text{ kg } m^{-3} \times 3.14 \times (10 \times 10^{-3} m)^2}$$

$$V = 1.6 \, m s^{-1}$$

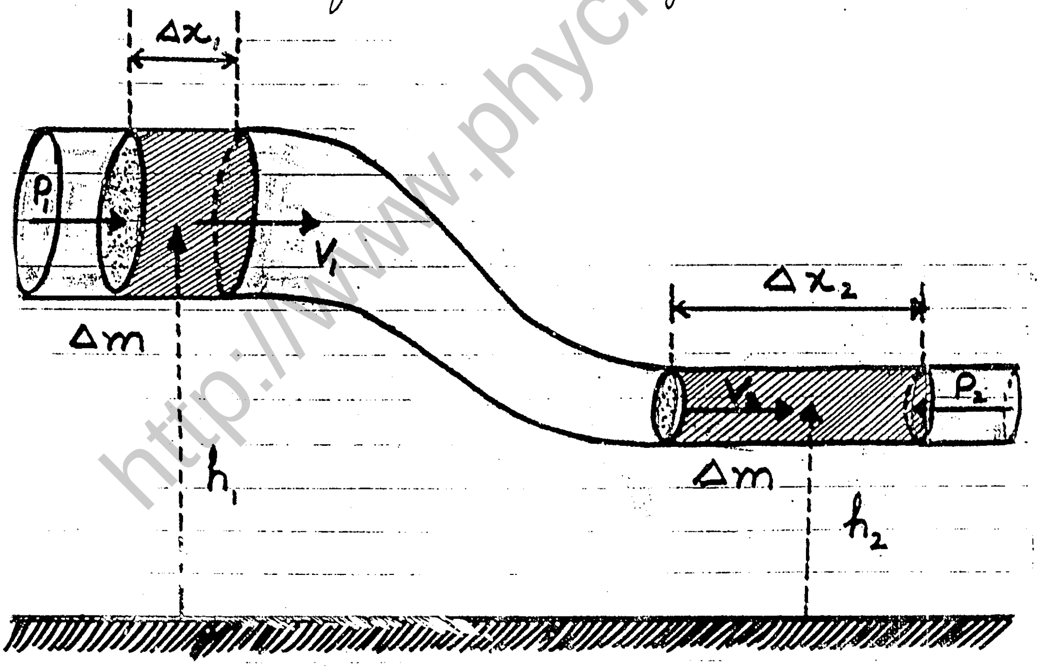
6.5 BERNOULLI'S EQUATION

1- Introduction:- Bernoulli's Equation named after a Swiss physicist Daniel Bernoulli who discovered it. Bernoulli's equation is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.

2. Statement:- It is defined as,

"For an incompressible non viscous fluid under going steady flow, the sum of pressure, kinetic energy per unit volume and potential energy per unit volume remains constant at all points on a streamline".

3. Explanation:- In deriving Bernoulli's equation, we assume that the fluid is incompressible, non viscous and flows in a steady state manner.



Let us consider the flow of the fluid through the pipe in time 't' as shown in figure.

As $P_1 = F/A_1$, the force on the upper end of the fluid is P_1A_1, where P_1 is the pressure and 'A_1' is the area of cross section at the upper end. The work done on the fluid, by the fluid behind it in moving it through a distance Δx_1, will be

$$\text{Work} = W_1 = \vec{F_1} \cdot \vec{d} = \vec{F_1} \cdot \vec{\Delta x_1} = F_1 \Delta x_1 \cos 0^\circ$$

$$\boxed{W_1 = P_1 A_1 \Delta x_1}$$

Similarly the work done on the fluid at the lower end is

$$\boxed{W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2}$$

Where 'P_2' is the pressure, 'A_2' is the area of cross section of the lower end and Δx_2 is the distance moved by the fluid in the same time interval 't'. The work W_2 is taken to be $-ve$ as this work is done against the fluid force, exerted by the liquid which is at right of (ahead of) the lower shaded water as shown in the figure.

The net work done

$$W = W_1 + W_2$$

or $$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad \text{———} \quad (1)$$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$\Delta x_1 = v_1 t \quad \text{and} \quad \Delta x_2 = v_2 t$$

$$\therefore \quad W = P_1 A_1 v_1 t - P_2 A_2 v_2 t \quad \text{———} \quad (2)$$

From equation of continuity

$$A_1 v_1 = A_2 v_2 = \text{Volume flow per second}$$
$$= \frac{V}{t}$$

Hence $A_1 v_1 t = A_2 v_2 t = V$ (volume)

So we have

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V \quad \text{———} \quad (3)$$

If 'm' is the mass and 'ρ' is the density of water, then

$$V = \frac{m}{\rho}$$

So equation (3) becomes

$$W = (P_1 - P_2) \frac{m}{\rho} \quad \text{———} \quad (4)$$

Part of this work is utilized by the fluid in changing its K.E and a part is used in changing its gravitational P.E.

Change in K.E $= \Delta(K.E) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Change in P.E $= \Delta(P.E) = mgh_2 - mgh_1$

where 'h_1' and 'h_2' are the heights of the upper and lower ends respectively.

Applying law of conservation of energy to this volume of the fluid, we get

$$\text{Work} = \Delta K.E + \Delta P.E$$

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2)\frac{1}{\rho} = \frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

Re-arranging the equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

This is Bernoulli's equation and can be expressed as

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constt.}}$$

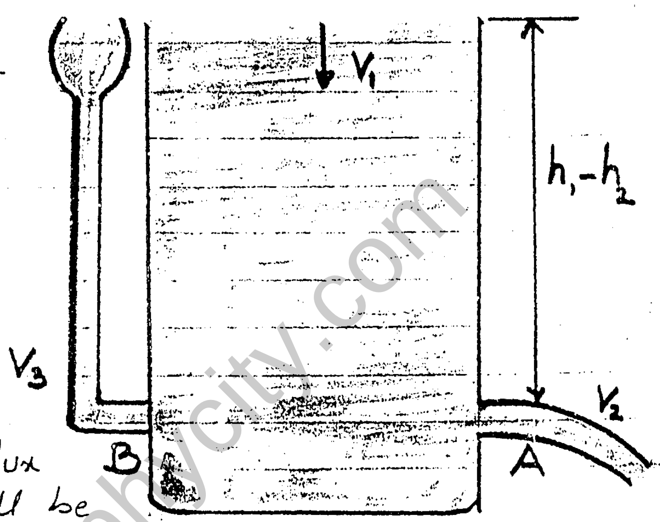
i.e $\quad P + \frac{1}{2}\,mv^2/V + mgh/V = \text{Constt.}$

APPLICATIONS OF BERNOULLI'S EQUATION

1- TORRICELLI'S THEORM:-

A simple application of Bernoulli's equation is shown in figure.

Suppose a large tank of fluid has two small orifices A and B on it as shown in figure. Let us find the speed with which the water flows from the orifice 'A'.



Since the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of water. i-e v_2 and $v_3 > v_1$

We can therefore take v_1 as approximately zero.

As Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As $v_1 = 0 \Rightarrow$

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

But $P_1 = P_2 =$ atmospheric pressure

So
$$\cancel{P_1} + \rho g h_1 = \cancel{P_1} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow \boxed{v_2 = \sqrt{2g(h_1 - h_2)}}$$

$\left(\begin{array}{l} \text{As } 2as = v_f^2 - v_i^2 \\ \sqrt{2as} = v_2 \end{array}\right)$

Statement:-

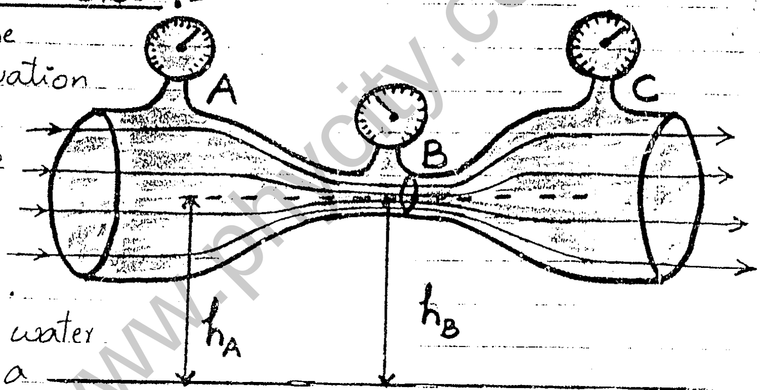
This is Torricelli's theorem which states

"The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity"

NOTE :- The speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The top level of the tank has moved down a little and the P.E. has been transferred into K.E of the efflux of fluid. If the orifice had been pointed upward as at 'B' shown in figure, then the K.E would allow the liquid to rise to the level of water tank. In actual practice, viscous energy losses would alter the result to some extent.

2 - Relation Between Speed And Pressure Of The Fluid :-

A result of the Burnoulli's equation is that the pressure will be low where the speed of the fluid is high.



Suppose that water flows through a pipe system as shown in figure. The water will flow faster at B than it does at 'A' or 'C'. Assuming the flow speed at 'A' to be $0.20\,ms^{-1}$ and at 'B' to be $2.0\,ms^{-1}$, we compare the pressure at 'B' with that at 'A'.

Applying Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

But the average P.E is the same at both places

i-e $$\rho g h_A = \rho g h_B \qquad (\because h_A = h_B)$$

$$\therefore \quad P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

Putting $V_A = 0.20\,m s^{-1}$, $V_B = 2.0\,m s^{-1}$

and $\rho = 1000\ Kg\,m^{-3}$.

we get $P_A - P_B = 1980\ N\,m^{-2}$

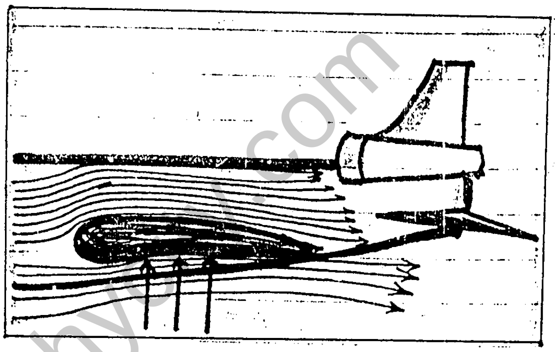
This shows that the pressure in the narrow pipe where streamlines are closer together is much smaller than in wider pipe.

Conclusion :-

Where the speed is high, the pressure will be low.

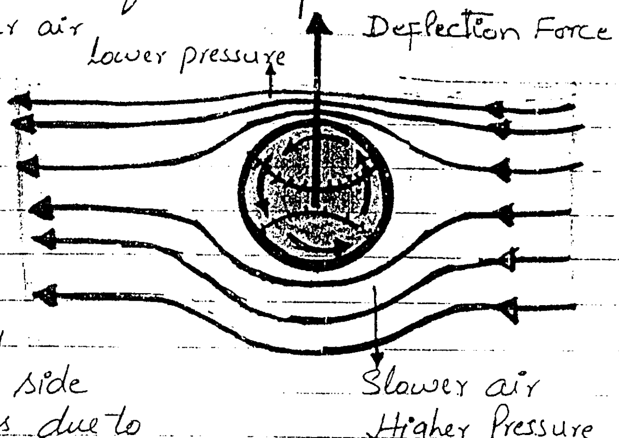
Examples :-

(a) - The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in figure. The wing is designed to deflect the air so that



streamlines are closer together above the wing than below it. It can be seen in figure that where the streamlines are forced closer together, the speed is faster. Thus air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.

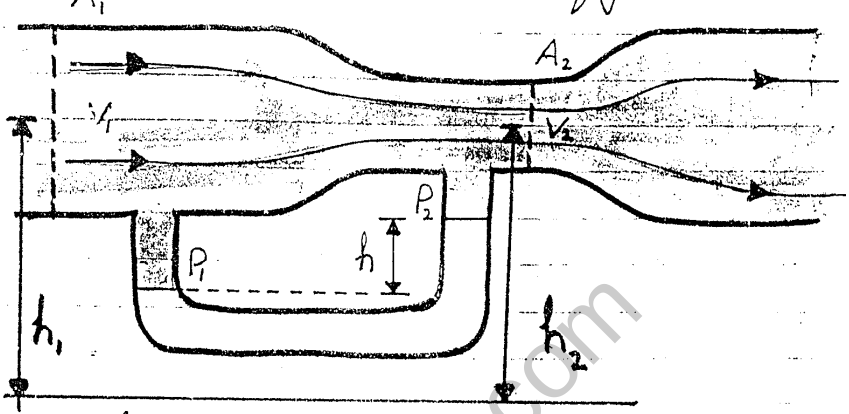
(b) - When a tennis ball is hit by a racket in such a way that it spins as well as moves forward, the velocity of the air on one side of the ball increases due to



spin and air speed in the same direction increases so pressure decreases. This gives an extra curvature to ball known as swing.

3. VENTURI RELATION :-

If one of the pipe has a much smaller diameter than the other as shown in figure.



Applying Bernoulli's equation.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As $\rho g h_1 = \rho g h_2 \quad (\because h_1 = h_2)$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho(v_2^2 - v_1^2).$$

As the cross-sectional area 'A_2' is small as compared to the area 'A_1', then from equation of continuity

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2$$

$v_1 < v_2$, thus for flow from a large pipe to a small pipe we can neglect 'v_1' on the right hand of the equation.

$\because v_1 < v_2 \quad$ So $\quad v_1 \simeq 0$

$$\therefore \boxed{P_1 - P_2 = \frac{1}{2}\rho v_2^2}$$

This relation is known as Venturi relation which is used in Venturimeter, a device used to measure speed of liquid flow.

(4) BLOOD FLOW

Blood is an incompressible fluid having a density ...

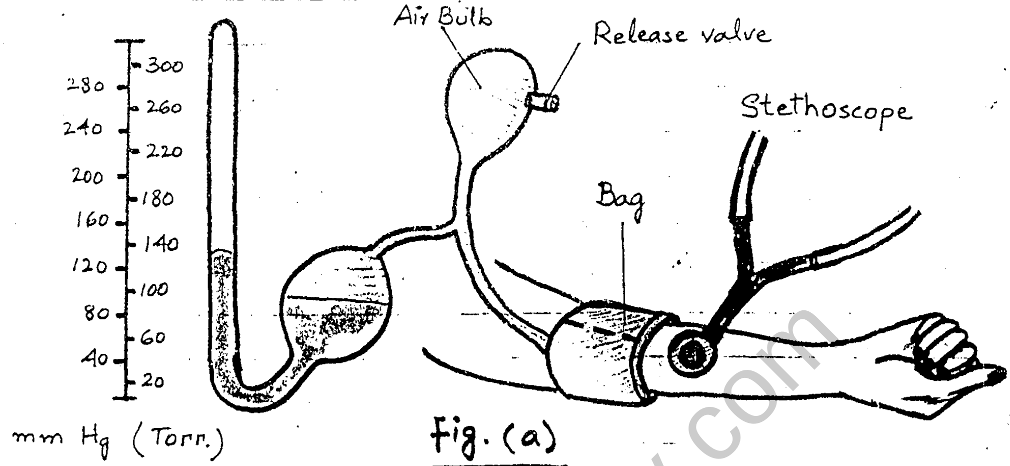


Fig. (a)

nearly equal to that of water. A high concentration (~ 50 %) of red blood cells increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose.

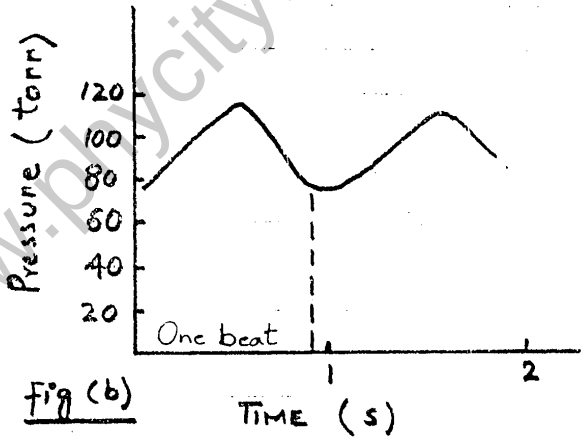


fig (b)

Under normal circumstances the volume of the blood is sufficient to keep the vessels inflated at all times, even in the relaxed states between heart beats. This means there is tension in the walls of the blood vessels and consequently the pressure of blood inside is greater than the external atmospheric pressure.

Figure (b) shows the variation in blood pressure as the heart beats.

For Healthy Person :- The pressure varies from a high (systolic pressure) of 120 torr to a low (diastolic pressure) of about 75 – 80 torr between beats in normal healthy person. The numbers tend to increase with age, corresponding to the decrease in the flexibility of the vessel walls.

Torr :- The unit torr or mm of Hg is opted instead of SI unit of pressure because of its extensive use in medical equipments.

$$1 \text{ torr} = 133.3 \text{ Nm}^{-2}$$

Sphygmomanometer :- It is an instrument which measures blood pressure dynamically.

Measurement Of Blood Pressure :-

An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the bag gradually decreases the external pressure.

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture produces a high flow speed. As a result the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow switches from turbulent to laminar, and the gurgle in the stethoscope disappears. This is the signal to record diastolic pressure.

EXAMPLE 6.3 Water flows down through a closed vertical funnel. The flow speed at the top is 12 cm s^{-1}. The flow speed at the bottom is twice the speed at the top. If the funnel is 40 cm long and the pressure at the top is 1.013×10^5 N m^{-2}, what is pressure at the bottom?

SOLUTION :-

Speed of water at top $= V_1 = 12\ \text{cm s}^{-1} = 0.12\ \text{m s}^{-1}$

Speed of water at bottom $= V_2 = 2V_1$

$V_2 = 24\ \text{cm s}^{-1} = .24\ \text{m s}^{-1}$

$P_1 = 1.013\times10^5\ \text{N m}^{-2}$

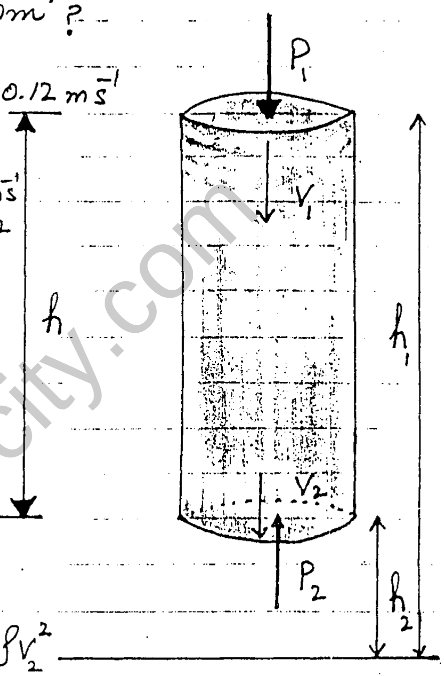
$P_2 = ?$

Length of funnel $= h = h_1 - h_2$

$= 40\ \text{cm}$

$= 0.4\ \text{m}$

Density of water $= \rho = 1000\ \text{Kg m}^{-3}$



Using Bernoulli's Equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \rho g h_1 - \rho g h_2 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

$$= P_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= P_1 + \rho g h + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= (1.013\times10^5\ \text{N m}^{-2}) + (1000\ \text{Kg m}^{-3} \times 9.8\ \text{m s}^{-2} \times 0.4\ \text{m})$$

$$+ \frac{1}{2}(1000\ \text{Kg m}^{-3})\{(0.12\ \text{m s}^{-1})^2 - (0.24\ \text{m s}^{-1})^2\}$$

$$\boxed{P_2 = 1.05\times10^5\ \text{N m}^{-2}}$$ Answer