CHAPTER : 5

CIRCULAR MOTION

"When a particle moves in a circle, its motion is called circular motion".

For example: (i) Motion of the wheel of a car,
(ii) Motion of planets around the sun,
(iii) Motion of satellites around the earth,
(IV) Motion of electrons around the nucleus, etc.

When a body is moving along a circular path with uniform speed, it has acceleration due to continuously changing direction. In this type of motion velocity '$\vec{v}$' and acc. $\vec{a}$ are always perpendicular to each other.

ANGULAR DISPLACEMENT :-

Def :- "It is the angle subtended at the centre of the circle, by an arc along which a particle moves from one position to the other, in a given time".

Explanation :- consider the motion of a single particle 'p' of mass 'm' in a circular path of radius 'r'. suppose this motion is taking place by attaching the particle 'p' at the end of a massless rigid rod of length 'r' whose other end is pivoted at the centre 'O' of the circular

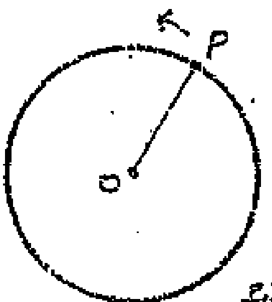


fig.-1(a)

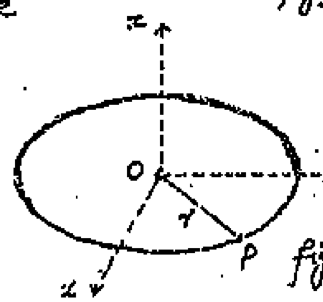


fig.-1(b)

path, as shown in fig-1(a). As the particle is moving on the circular path, the rod 'OP' rotates in the plane of the circle. The axis of rotation passes through the pivot 'O' and is normal to the plane of rotation.

Now consider a system of axes as shown in fig-1(b). The z-axis is taken along the axis of rotation with pivot 'O' as origin of coordinates. Axes x- and y- are taken in the plane of rotation.

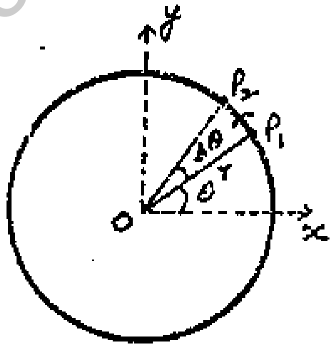


fig-1(c)

While 'OP' is rotating, let at any instant 't', its position is 'OP_1' making angle 'θ' with x-axis. At later time '$t+\Delta t$', let its position be 'OP_2' making angle '$\theta+\Delta\theta$' with x-axis as shown in fig-1(c). "The angle '$\Delta\theta$' between two positions of the particle defines the angular displacement of 'OP' during the time interval 'Δt'." The sign convention for '$\Delta\theta$' is as follows: "The angular displacement '$\Delta\theta$' is taken as '+ve' when the rotation of 'OP' (or particle) is counterclockwise and is taken as '-ve' when the rotation is clockwise."

The angular displacement '$\Delta\theta$' is a vector quantity and its direction is along the axis of rotation and is given by right hand rule, as follows :-

Right Hand Rule :-

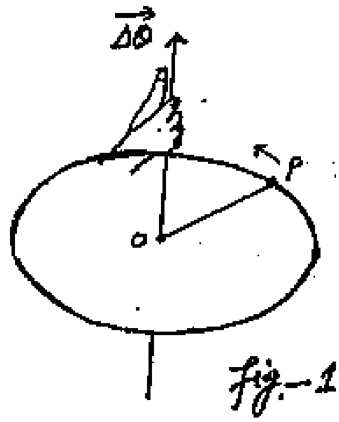


fig.-1(d).

"Grasp the axis of rotation in right hand with fingers curling in the direction of rotation. The erected thumb points in the direction of angular displacement, as shown in fig.-1(d)."

Units :- Three units are generally used to express angular displacement, namely:

(i) Degree '°' (ii) Revolution (rev.) (iii) Radian (rad.) (SI-unit).

(i) **Degree** :- "It is the angle subtended at the centre of the circle by an arc of length equal to $\frac{1}{360}$th of its circumference".

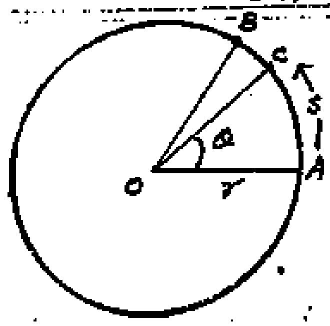
(ii) **Revolution** :- "one complete round trip of the body about a circular path is called one revolution".

$$1 \text{ rev.} = 360°.$$

(iii) **Radian** :- "It is the angle subtended at the centre of the circle by an arc of length equal to the radius of the circle".

Relation Between Linear and Angular Displacement :-

consider a particle moving in a circle of radius 'r' as shown in fig-2. Let the length



of the arc 'AC' = s ,
Angle subtended by arc 'AC' (in radians) = θ ,
Now take an arc 'AB' equal in length to the radius of the circle i.e. $\widehat{AB} = r$. By definition, the angle subtended by arc 'AB' is 1 radian. Now, the arcs are proportional to angles which they make at the centre of circle 'O',

fig.-2.

∴ Ratio of angles = Ratio of arcs

i.e. $\dfrac{\angle AOC}{\angle AOB} = \dfrac{\text{Arc } AC}{\text{Arc } AB}$

or $\dfrac{\theta}{1\,\text{rad}} = \dfrac{s}{r}$ or $\theta = \dfrac{s}{r} \times 1\,\text{rad}$.

∴ $\boxed{s = r\theta}$, where θ is in radians

i.e. Length of arc = Radius of circle × Angle in rad. made by the arc at the centre.

Relation between Degree, Rad. and Rev. :-

The circumference of circle '$2\pi r$' makes an angle of 360° at the centre of the circle.

∵ $s = r\theta$

or $\theta = \dfrac{s}{r} = \dfrac{2\pi \cancel{r}}{\cancel{r}}$

$\theta = 2\pi$ rad.

∴ 2π rad = 360°

1 rad $= \dfrac{360}{2\pi} = \dfrac{180^\circ}{\pi}$

or $\underline{1 \text{ rad} = 57.3^\circ}$.

Conversely :

$360^\circ = 2\pi$ rad

$1^\circ = \dfrac{2\pi}{360} = \dfrac{\pi}{180}$ rad.

or $\underline{1^\circ = 0.0174 \text{ rad}}$

Also:-

$\underline{1 \text{ rev.} = 360^\circ}$

$\underline{1 \text{ rev.} = 2\pi \text{ rad.}}$

ANGULAR VELOCITY :-

Defi. :- "The rate of change of angular displacement is called angular velocity". It is denoted by $\vec{\omega}$ (omega).

Explanation :- Consider a body is rotating around a circular path. If it traverses an angular displacement '$\vec{\Delta\theta}$' during the time interval 'Δt', then the average angular velocity '$\vec{\omega}_{av}$' during this time interval is given by:-

$$\vec{\omega}_{av} = \frac{\vec{\Delta\theta}}{\Delta t}$$

The instantaneous angular velocity i.e. the angular velocity at a particular instant of time is '$\vec{\omega}_{inst}$' and is taken in the limit of the ratio '$\frac{\Delta\theta}{\Delta t}$' as '$\Delta t$' approaches to zero and is written as:

$$\vec{\omega}_{inst} = \lim_{\Delta t \to 0} \frac{\vec{\Delta\theta}}{\Delta t}.$$

Direction :- The angular velocity is a vector quantity and its direction is taken parrallel to the angular displacement i.e. along the axis of rotation and is given by right hand rule.

* → The direction of angular velocity '$\vec{\omega}$' changes with the change of direction of rotation of the body.

Units:- The SI-unit of angular velocity is rad/s, but sometimes it is also measured in rev/s or rev/min (rpm).

ANGULAR ACCELERATION :-

Defi. :- " The rate of change of angular velocity is called angular acceleration".
It is denoted by $\vec{\alpha}$ (alpha).

Explanation :- Consider a rotating body (e.g. electric fan) whose angular velocity goes on increasing, when it is just switched on. We say it has an angular acceleration.

If 'ω_i' and 'ω_f' are the values of inst. angular velocities of a rotating body at instants 't_i' and 't_f' respectively, then the avg. angular acceleration during the interval '$t_f - t_i$' is given by :

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i} = \frac{\Delta\vec{\omega}}{\Delta t}$$

or

$$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t}$$

The instantaneous angular acceleration "$\vec{\alpha}_{inst}$" is the limit of the ratio '$\frac{\Delta\omega}{\Delta t}$' as '$\Delta t$' approaches to zero, and is given by :

$$\vec{\alpha}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}$$

Direction :- " The angular acceleration is also a vector quantity, whose direction is taken parallel or antiparallel to the direction of angular velocity i.e. it is along the axis of rotation, given by RHR.

*→ If a body is speeding up, then: "$\vec{\alpha}$ is parallel to $\vec{\omega}$" and $\vec{\alpha}$ is taken as positive. If the

body is slowing down, then: $\vec{\alpha}$ is antiparallel to '$\vec{\omega}$' and '$\vec{\alpha}$' is taken as '$-ve$'.

Units:- The SI-unit of angular acc. is rad/s^2 but sometimes it is measured in rev/s^2 or rev/min^2.

RELATION BETWEEN ANGULAR AND LINEAR VELOCITIES :-

For finding the relation between angular and linear velocities we must consider a rigid body rotating about z-axis with an angular velocity 'ω' as shown in fig-3(a).

Imagine a pt. 'p' in the rigid body at a perpendicular distance 'r' from the axis of rotation. 'OP' represents the reference line of the rigid body. "A reference line is a replacement of rotation of a rigid body."

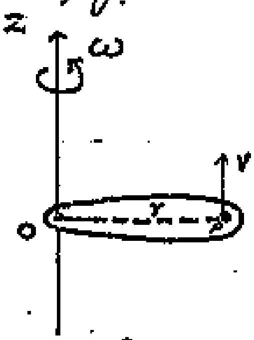


fig.-3(a)

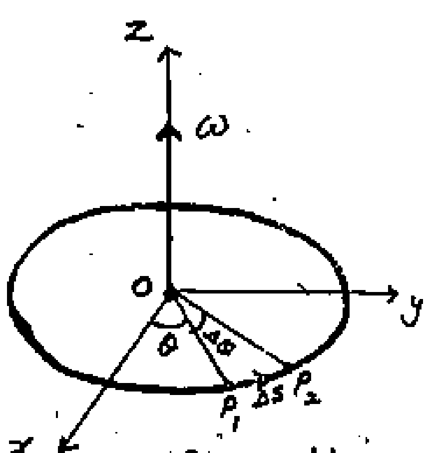


fig.-3(b).

Now, as the body rotates, the pt. 'p' moves along a circle of radius 'r' with a linear velocity '$\vec{v}$' whereas the line 'OP' rotates with angular velocity 'ω' as shown in fig-3(b).

Let during the course of its motion, the pt. 'p' moves through a distance '$P_1P_2 = \Delta S$' in a time interval 'Δt' while the reference line 'OP' has an angular displacement '$\Delta\theta$' (radians) during this interval.

As we know that : $S = r\theta$.

$$\text{or} \qquad \Delta S = r\,\Delta\theta$$

Dividing both sides by 'Δt', we have,

$$\frac{\Delta S}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t} \longrightarrow (1)$$

Taking limit both sides:

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = (r) \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} \longrightarrow (2)$$

The L.H.S. in eq.(2) is defi. of inst. linear velocity 'v', the magnitude of the velocity with which pt. 'P' is moving on the circumference of the circle. Similarly on R.H.S of eq.(2) '$\lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$' represents the inst. angular velocity 'ω' of the reference line 'OP'. So eq.(2) becomes: $V = r\omega \longrightarrow (3)$

It can be seen from fig.-3(b) that the pt. 'P' is moving along arc 'P_1P_2'. In the limit when $\Delta t \to 0$, the length of arc 'P_1P_2' becomes very small and its direction represents the direction of tangent to the circle at pt. P_1. Thus the velocity of pt. 'P' along the circumference of the circle has a magnitude 'v' and its direction is always along the tangent to the circle.

That is why the linear velocity of the pt. 'P' is also known as "tangential velocity".

Relation between linear and angular accelerations :-

The equation : "$v = r\omega$" shows that a change in angular velocity 'ω' will produce change in linear velocity 'v'. These changes in both velocities will result in angular and linear accelerations. So the relation between two acc. is derived as follows:

We know that : $v = r\omega$

$\therefore \quad \Delta v = r\,\Delta\omega \qquad (r = \text{const})$

Dividing both sides by 'Δt' we get:

$$\frac{\Delta v}{\Delta t} = r\,\frac{\Delta\omega}{\Delta t}$$

Taking limit both sides, we have:

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}.$$

By using definitions of inst. angular and linear acc. in above eq., we get:

$$a = r\alpha .$$

This eq. gives the relationship between the magnitude of linear and angular accelerations. The linear acc. 'a' is also called tangential acc. 'a_t'. Therefore : "$a_t = r\alpha$".

Equations of angular Motion:-

The equations of angular motion are exactly the same to those in linear motion when we replace 'θ', 'ω' and 'α' by 's', 'v' and 'a' respectively. A comparison of the two is given here:

	Linear Motion		Angular Motion
(i)	$v_f = v_i + at$	(i)	$\omega_f = \omega_i + \alpha t$
(ii)	$S = v_i t + \frac{1}{2}at^2$	(ii)	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
(iii)	$2aS = v_f^2 - v_i^2$	(iii)	$2\alpha\theta = \omega_f^2 - \omega_i^2$.

The angular equations of motion hold true only when the axis of rotation is fixed.

Example 5.1:- An electric fan rotating at 3 rev/s is switched off. It comes to rest in '18' s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Sol.:- Given that: $\omega_i = 3$ rev/s, $\omega_f = 0$

$t = 18$ s, $\alpha = ?$, $\theta = ?$

For 'α' using eq.:- $\omega_f = \omega_i + \alpha t$

$0 = 3 + \alpha(18)$, $18\alpha = -3$

$\therefore \alpha = \frac{-3}{18} = -0.167$ rev/s^2

For 'θ' using eq.: $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Putting values: $\theta = 3(18) + \frac{1}{\cancel{2}}\left(-\frac{3}{\cancel{18}}\right)(\cancel{18})^{\cancel{2}}$ (9)

$\theta = 54 - 27 = 27$ rev

$\therefore \theta = 27$ rev.

CENTRIPETAL FORCE :-

Defi. :- "The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force".

Examples of Centripetal Force :-

(1) When a ball tied at the end of a string is whirled in a horizontal circle, the tension in the string provides the centripetal force.

(2) The planets moving in nearly circular orbits around the sun has a centripetal force due to gravitational attraction between Sun and planet.

(3) Motion of nuclear particles in accelerators, motion of flywheels, etc., is due to centripetal force.

Equation of centripetal force :-

Consider a particle which moves from pt. 'A' to pt. 'B' with uniform speed 'v' as shown in fig-4(a). The velocity of the particle changes its direction but not its magnitude. The change in velocity is

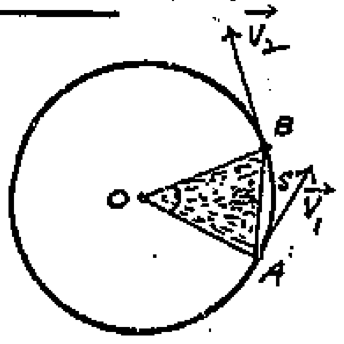


fig-4(a)

shown in fig-4(b). Hence the acc. of the particle is :

$$a = \frac{\Delta V}{\Delta t} \longrightarrow (1)$$

where 'ΔV' is the change in velocity due to change in direction during time interval 'Δt', in moving the particle from pt. 'A' to pt. 'B'. Let the velocities at pts. 'A' and 'B' are $\vec{V_1}$ and $\vec{V_2}$ respectively as shown in fig-4(a).

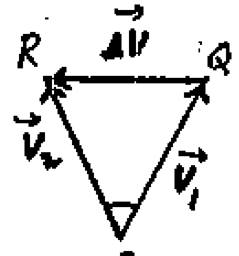


fig-4(b)

Since the speed of particle is 'V', so the time taken to travel a distance 's' as shown in fig-4(a) is given by:

$$\Delta t = \frac{S}{V} \longrightarrow (2)$$

putting this value in eq. (1), we have:

$$a = \frac{\Delta V}{S/V} = V\,\frac{\Delta V}{S}$$

or $$a = V\left(\frac{\Delta V}{S}\right) \longrightarrow (3)$$

Consider now 'triangle PQR' such that 'PQ' is parallel and equal to 'V_1' and 'PR' is parallel and equal to V_2 as shown in fig-4(b). We know that the radius of the circle is always perpendicular to the tangent, so 'OA' is perpendicular to 'V_1' and 'OB' is perpendicular to V_2 as in fig-4(a).

Therefore : $\angle AOB = \angle QPR$ between V_1 and V_2.

Further as : $|\vec{V_1}| = |\vec{V_2}| = V$ (or $V_1 = V_2 = V$) and $OA = OB$,

so both triangles are isosceles. From geometry, "two isosceles triangles are similar, if the angles between their equal arms are equal". Hence, the 'ΔOAB' of fig-4(a) is similar to the 'ΔPQR' of fig.-4(b).

So we can write : $\frac{\Delta v}{v} = \frac{AB}{r} \longrightarrow (4)$

If the pt. 'B' is close to the pt. 'A' on the circle i.e. when $\Delta t \longrightarrow 0$, the arc AB is nearly equal to the length of the line 'AB'.

or $\widehat{AB} = \overline{AB}$, Thus: $\overline{AB} = s$

Therefore eq. (4) becomes:

$$\frac{\Delta v}{v} = \frac{s}{r} \Rightarrow \Delta v = v\left(\frac{s}{r}\right) \longrightarrow (5)$$

putting this value in eq. (3) we get:

$$a = v\left(\frac{\Delta v}{s}\right) = v\left(\frac{v\left(\frac{s}{r}\right)}{s}\right)$$

$$= \frac{v^2}{r} \times \frac{\cancel{s}}{\cancel{s}} = \frac{v^2}{r}$$

$$\therefore a = \frac{v^2}{r} \longrightarrow (6)$$

The eq. (6) gives inst. acceleration. As this acc. is caused by the centripetal force, so it is called the centripetal acc. denoted by 'a_c'.

$$\therefore a_c = \frac{v^2}{r} \longrightarrow (7)$$

This acc. is directed along the radius towards the centre of the circle.

From fig.-4(a) and 4(b); since 'PQ' is $\perp$lar to 'OA' and 'PR' is $\perp$lar to OB, so 'QR' is $\perp$lar to 'AB'. As the acc. of the object moving in a circle is parallel to 'Δv' when $AB \longrightarrow 0$, so centripetal acc. is directed along the radius towards the centre of the circle. Thus it is concluded that :

"The inst. acc. of an object travelling with uniform speed in a circle is directed towards the centre of the circle and is called Centripetal acceleration".

The centripetal force has the same direction as the centripetal acc. and its value is given by :

$$F_c = ma_c = \frac{mv^2}{r}$$

$$\because V = r\omega \quad , \quad so, F_c = m\left(\frac{r^2\omega^2}{r}\right) = mr\omega^2$$

$$\therefore \quad F_c = mr\omega^2 .$$

<u>In vector form</u> : $\vec{a}_c = \frac{v^2}{r}(-\hat{r}) = -\frac{v^2}{r}\hat{r}$

<u>or</u> $\vec{a}_c = -\frac{v^2}{r}\left(\frac{\vec{r}}{r}\right) = -\frac{v^2}{r^2}\vec{r}$ $\quad (\because \hat{r} = \frac{\vec{r}}{r})$.

$\therefore \vec{a}_c = -\frac{v^2}{r^2}\vec{r}$ $\quad$ ($\because$ Direction $-\hat{r}$ is towards the centre of circle).

and $\vec{F}_c = -m\frac{v^2}{r^2}\vec{r}$

Also : $\vec{F}_c = -m\omega^2\vec{r}$.

<u>Example 5.2</u> :- A 1000 kg car is turning round a corner at 10 m/s as it travels along an arc of a circle. If the radius of the circle is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car on circular path?

Sol. :- The required force is centripetal force F_c.

As : $F_c = \frac{mv^2}{r} = \frac{1000 \times (10)^2}{10} = 10000 \text{ N}$, $\therefore F_c = 10^4 \text{ N}$.

Example 5.3 :- A ball tied to the end of a string, is swung in a verticle circle of radius 'r' under the action of gravity as shown in fig-5. What will be the tension in the string when the ball is at the pt. 'A' of the path and its speed is 'v' at this point ?

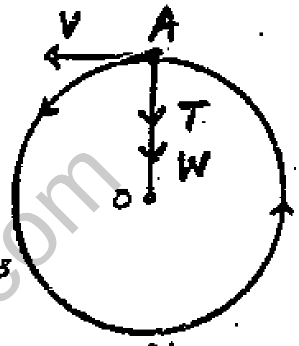


fig.-5.

Sol. :- As shown in fig-5, both forces i.e. tension 'T' and weight 'W' are acting downward along the radius towards the centre at highest pt. 'A' in the verticle circle. So their vector sum must provide the required centripetal force.

$$\therefore \quad F_c = T + W$$

$$F_c = \frac{mv^2}{r} \quad \Rightarrow \quad T + W = \frac{mv^2}{r}$$

$$\text{or} \quad T = \frac{mv^2}{r} - W = \frac{mv^2}{r} - mg$$

$$\therefore \quad T = m\left(\frac{v^2}{r} - g\right) \longrightarrow *$$

If specially, $\frac{v^2}{r} = g$, then $T = m(g-g) = 0$ and $F_c = W$ i.e. Centripetal force is just equal to the weight of the body.

MOMENT OF INERTIA :-

Defi. :- "The quantity 'mr^2' is Known as the moment of inertia and is denoted by 'I'."

→ (1) The moment of inertia plays the same role in angular motion as the 'mass' in linear motion.

→ (2) The moment of inertia depends upon mass 'm' and 'r^2'.

Explanation :-

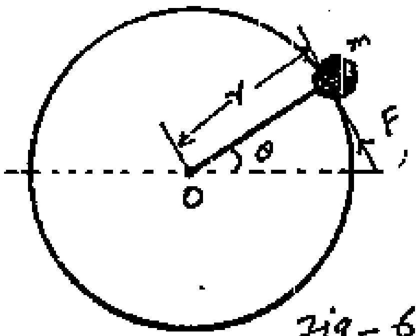


Fig.-6.

Consider a mass 'm' attached to a massless rod at 'O' as shown in Fig.-6. Let the moving system at pt. 'O' (Pivot), is frictionless.
Let this system lie in a horizontal plane.
A force 'F' is acting on the mass perpendicular to the rod. According to 2nd law of motion:

$$F = ma \longrightarrow (1)$$

In doing so the force will cause the mass to rotate about 'O'.

$$\because a_T = r\alpha \longrightarrow (2) \quad (a_T = a)$$

$$\therefore F = mr\alpha \longrightarrow (3)$$

As turning effect is produced by torque 'τ', so for writing eq. (3) in terms of torque, we multiply it by 'r' on both sides. Thus.

$$rF = mr\alpha(r) = mr^2\alpha$$

i.e. $\tau = rF$

<u>or</u> $\tau = mr^2\alpha$

<u>or</u> $\tau = I\alpha \qquad (\because mr^2 = I)$

Moment of Inertia of a rigid body :-

Most rigid bodies have different mass concentrations at different distances from the axis.

of rotation, which means the mass distribution is not uniform. The rigid body is made up of 'n' small pieces of masses m_1, m_2, ... at distances r_1, r_2, ... from the axis of rotation 'O' as shown in fig.-7(a), (b).

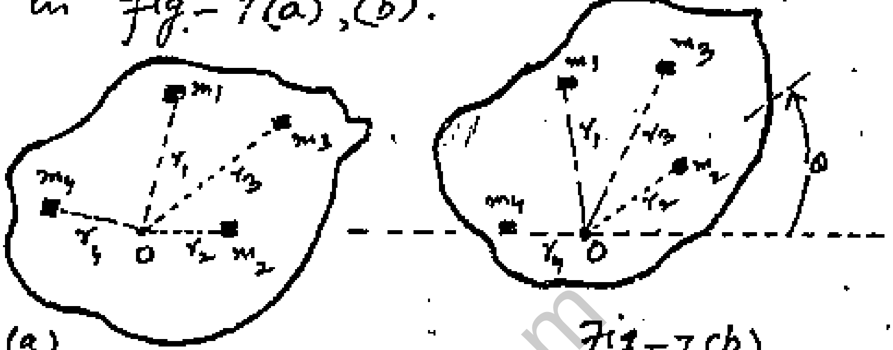


Fig.-7(a) Fig.-7(b)

*["Each small piece of mass within a large, rigid body undergoes the same angular acc. about the pivot point].

Let the body be rotating with the angular acc. 'α', so the magnitude of torque acting on m_1 is : $\tau_1 = m_1 r_1^2 \alpha_1 \longrightarrow (1)$

Similarly, the torque on m_2 is : $\tau_2 = m_2 r_2^2 \alpha_2 \longrightarrow (2)$

and so on.

Since the body is rigid, so all the masses are rotating with the same angular acc.

i.e. $\alpha_1 = \alpha_2 = \ldots\ldots = \alpha$.

$\therefore$ Total torque is given by:

$$\tau_{Total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)\alpha$$
$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

or $$\tau = I\alpha \longrightarrow (3)$$

where '$I = \sum_{i=1}^{n} m_i r_i^2$' is the moment of inertia of the rigid body.

*— The moments of inertia of various bodies about the axis of rotation that passes through their Central Points (Book Page→109.) are given below:

(1) Thin rod : $I = \frac{1}{12} mL^2$, ($\because L$ = length).

(2) Thin ring (or hoop) : $I = \frac{1}{2} mr^2$, (r = radius)

(3) Solid Disc (or cylinder) : $I = \frac{1}{2} mr^2$, (").

(4) Sphere : $I = \frac{2}{5} mr^2$. (").

Unit :- The SI-unit of moment of Inertia is $Kg\,m^2$.

ANGULAR MOMENTUM :-

Defi. :- "A particle is said to posses an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis."

OR "The cross product of position vector '$\vec{r}$' and linear momentum '$\vec{p}$' is Called angular momentum denoted by $\vec{L}$."

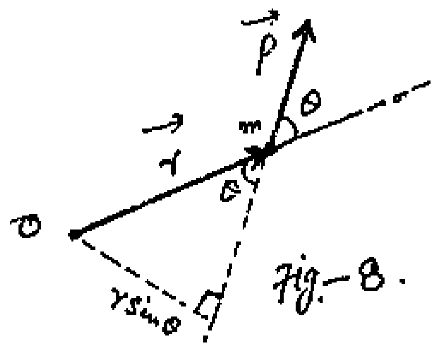


Fig.-8.

Explanation :-

Consider a particle of mass 'm' moving with velocity '$\vec{v}$'

having linear momentum '$\vec{p}$' as shown in fig.-8. Its angular momentum $\vec{L}$ relative to the origin 'O' is written as:

$$\vec{L} = \vec{r} \times \vec{p} \longrightarrow (1)$$

where '$\vec{r}$' is the position vector of the particle at that instant relative to the origin 'O'. Angular momentum is a vector quantity and its <u>direction</u> is perpendicular to the plane formed by '$\vec{r}$' and '$\vec{p}$', which can be found by right hand rule of vector product. The <u>magnitude</u> of angular momentum is given by:

$$L = rp\sin\theta = rmv\sin\theta$$

<u>or</u> $$L = mvr\sin\theta \longrightarrow (2)$$

where 'θ' is the angle between $\vec{r}$ and $\vec{p}$.

<u>**Unit**</u> :- The SI-unit of angular momentum is $Kg\,m^2 s^{-1}$ or $J\,s$.

<u>**Example**</u> :- <u>If a particle is moving in a circle</u> of radius 'r' with uniform angular velocity 'ω', then angle between 'r' and 'v' is $90°$. Hence:

$$L = mvr\sin 90 = mvr$$

But : $$v = r\omega$$

so, $$L = m(r\omega)r = mr^2\omega$$

$$\therefore \quad L = mr^2\omega \longrightarrow (3)$$

Also : $$L = I\omega \longrightarrow (4) \quad (\because I = mr^2)$$

Angular Momentum of a rigid body:-

consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in fig-9. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity 'ω'.

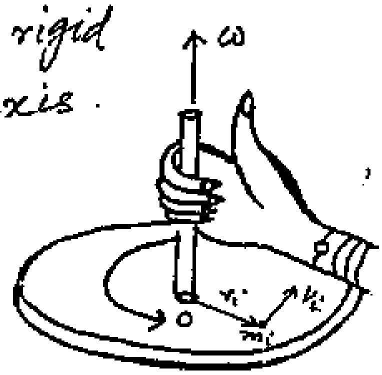


fig- 9.

As the rigid body is made up of n-small pieces of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis of rotation 'O'. Hence:

The magnitude of angular momentum due to $m_1 = L_1 = m_1 r_1 v_1$

similarly " " " " " $m_2 = L_2 = m_2 r_2 v_2$

and so on, and:

" " " " " " $m_i = L_i = m_i r_i v_i$

Since the direction of $\vec{L_i}$ is same as that of $\vec{\omega}$.

$\therefore$ Total angular momentum of the rigid body is:

$$L_{Total} = L_1 + L_2 + \cdots + L_i$$

$$= m_1 v_1 r_1 + m_2 v_2 r_2 + \cdots + m_i v_i r_i$$

$$\because \quad v = r\omega$$

So: $$L_{Total} = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \cdots + m_i r_i^2 \omega$$

$$= (m_1 r_1^2 + m_2 r_2^2 + \cdots + m_i r_i^2)\omega$$

or $$L_{Total} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

$$= I\omega \qquad \because \sum_{i=1}^{n} m_i r_i^2 = I$$

$\therefore$ $L_{Total} = I\omega$, where I = moment of inertia of rigid body.

Types of angular momentum :-

Angular momentum is of two types:

(i) spin angular momentum :-

The spin angular momentum is the angular momentum of a spinning body. It is denoted by 'L_s'. For example: an acrobat spins around a fixed axis AA', have spin ang. momentum as shown in fig-10(a).

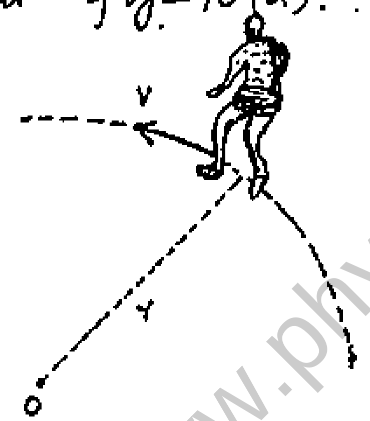


Fig.—10 (b)

fig-10(a)

(ii) orbital angular momentum :-

The orbital angular momentum is associated with the motion of a body along a circular path. It is denoted by 'L_o'. For example: A sprinter racing around a circular track have orbital angular momentum. usually in orbital angular momentum cases orbital radius is large as compared to the size of the body.

Example 5.4 :- The mass of earth is 6×10^{24} kg. The distance 'r' from earth to the sun is 1.5×10^{11} m. As seen from the direction of North Star, the earth in one year, so its orbital speed v_o is:

$$v_o = \frac{2\pi r}{t} \qquad (\because s = vt)$$

So, orbital angular momentum of earth is $= L_o = m v_o r$

$$\Rightarrow L_o = m\left(\frac{2\pi r}{t}\right) r = \frac{2\pi r^2 m}{t}$$

$$\therefore L_o = \frac{2\times 3.14\times(1.5\times10^{11})^2\times(6\times10^{24})}{3.16\times10^{7}}$$

<u>or</u> $L_o = 2.68\times10^{40}\ \mathrm{Kg\,m^2 s^{-1}}$.

The sign of 'L_o' is +ve because the revolution is counterclockwise.

<u>Law of Conservation of Angular Momentum</u> :-

<u>Statement</u> :- "This law states that if no external torque acts on a system, the total angular momentum of the system remains constant."

Mathematically:

$$L_{Total} = L_1 + L_2 + \cdots\cdots = \text{constant}.$$

<u>Explanation</u> :- The law of conservation of angular momentum is one of the fundamental principles of physics. It has been verified for macro and microlevel i.e. for very large as well as for very small objects. The effect of the law of conservation of angular momentum is readily apparent if a single isolated spinning body alters its moment of inertia. For example:

<u>Motion of a diver</u> :-

The 'diver' pushes off the board with a small angular velocity about a horizontal axis through his centre of gravity 'G'. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia 'I_1' about this axis. The moment of inertia is considerably reduced to a new value 'I_2' when the legs and arms are drawn into the closed "tuck position". As the angular momentum is conserreed, so:

$$L_1 = I_1 \omega_1 \qquad (\because L = I\omega)$$

$$L_2 = I_2 \omega_2$$, where ω is angular velocity

$$\therefore \; L_1 = L_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2$$

Hence, the diver must spin faster when moment of inertia becomes smaller to conserree angular momentum. This enables the diver to take extra somersaults. In this example, we discussed the conservation of magnitude of angular momentum.

Direction of Angular Momentum:-

The angular momentum is a vector quantity with direction along the axis of rotation, which remains fixed. This is clear from the following fact:

"The axis of rotation of an object will not change its orientation unless an external torque causes it to do so."

This fact is of great importance for the earth as it moves round the sun. No other sizeable torque is experienced by the earth, because the major force acting on it is the pull of the sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

Applications of this law:

The law of Conservation of angular momentum has many applications. For example:

(i) In creation of stars (ii) In motion of ice skaters (iii) In ballet dance (IV) In Gymnastics.

ROTATIONAL KINETIC ENERGY:-

"If a body is spinning about an axis with constant angular velocity 'ω', each point of the body is moving in a circular path and, therefore, has some K.E." This K.E. is called rotational K.E.

Expression for Rotational K.E.:-

To derive an expression for the total rotational K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass $m_1, m_2, \ldots$ If a piece of mass 'm_i' is at a distance 'r_i' from the axis of rotation as shown in Fig.-11, it is moving in a circle with speed:

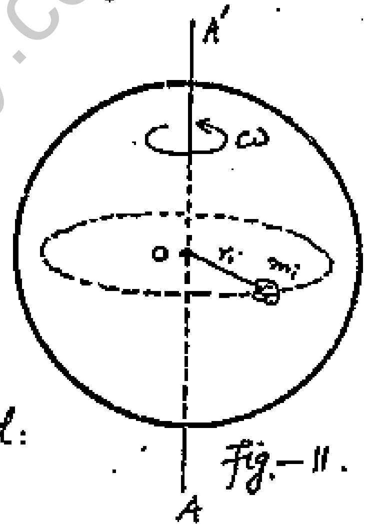


Fig.-11.

$$v_i = r_i \omega$$

Thus the K.E. of this piece is:

$$K.E_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i (r_i \omega)^2$$

$$\therefore \quad K.E_i = \frac{1}{2} m_i r_i^2 \omega^2 \longrightarrow (1)$$

The rotational K.E. of the whole body is the sum of the kinetic energies of all parts. So we have:

$$K.E_{rot} = \frac{1}{2}(m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \ldots\ldots)$$

$$= \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots)\omega^2$$

$$K.E_{rot} = \frac{1}{2} I \omega^2 \longrightarrow (2)$$

where $I = m_1 r_1^2 + m_2 r_2^2 + \cdots$

$\quad\quad$ = moment of inertia of the body.

If rolling or spinning bodies are present in a system, their rotational K.E. must be included as part of the total K.E.

Use of rotational K.E. :-

Rotational K.E. is put to practical use by fly wheels, which are essential parts of many engines. A fly wheel stores energy between the power stokes of the pistons.

Rotational K.E. of a Disc and a Hoop:-

(1) The rotational K.E. of a disc :-

we know that : $K.E_{rot} = \frac{1}{2} I\omega^2 \longrightarrow (1)$

For a disc : $I = \frac{1}{2} mr^2$

so : $K.E_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{1}{4}mr^2\omega^2$

$\because \; v = r\omega \Rightarrow v^2 = r^2\omega^2$

$\therefore \; K.E_{rot} = \frac{1}{4}mv^2 \longrightarrow (2)$

(2) The rotational K.E. of a hoop :-

Hoop :- A 'hoop' can be thought of as a very thin-walled hollow cylinder.

For a hoop, since : $I = mr^2$

then : $K.E_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(mr^2)\omega^2$

so : $= \frac{1}{2}mr^2\omega^2$

$\because \; v = r\omega \Rightarrow v^2 = r^2\omega^2$

$\therefore \; K.E_{rot} = \frac{1}{2}mv^2 \longrightarrow (3)$

When both (Disc and hoop) starts moving down an inclined plane of height 'h', their motion consists of both rotational and translational motions as shown in fig-12.

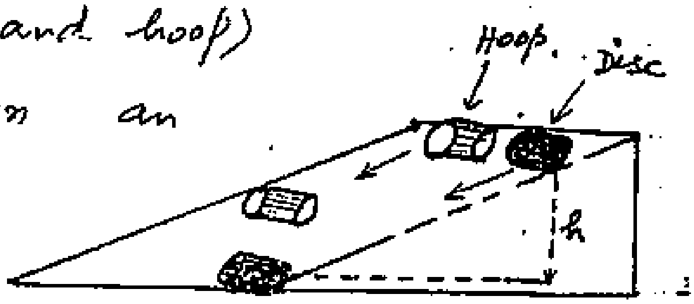


fig-12.

If no energy is lost against friction, the total K.E. of the disc or hoop on reaching the bottom of the incline must be equal to its P.E. at the top i.e.

$$P.E. = K.E_{tran} + K.E_{rot}$$

or $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \longrightarrow (4)$$

For Disc:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{2mv^2 + mv^2}{4} = \frac{3}{4}mv^2$$

or $$\cancel{m}gh = \frac{3}{4}\cancel{m}v^2, \quad v^2 = \frac{4}{3}gh$$

$$\therefore \quad v = \sqrt{\frac{4}{3}gh} \longrightarrow (5)$$

For Hoop:-

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $$\cancel{m}gh = \cancel{m}v^2, \quad v^2 = gh$$

$$\therefore \quad v = \sqrt{gh} \longrightarrow (6)$$

Example 5.5:- A disc without slipping rolls down a hill of height 10 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom?

Sol.:- As given: $h = 10m, v = ?$

$$\therefore \quad v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}\times 9.8\times 10}$$

$$\therefore \quad v = 11.4 \text{ m/s.}$$

ARTIFICIAL SATELLITES :-

"Artificial satellites" are objects that orbit around the earth. Many man-made satellites are orbiting around the earth in circular orbits. They are used for different purposes. They are put into orbits by rockets and are held in orbits by gravitational pull of the Earth. once in orbits, they do not need any propelling force to keep them there.

The low flying Earth satellites have acc. = 9.8 m/s^2, towards the Centre of the Earth. If they do not, they would fly off in a straight line tangent to the Earth.

Critical Velocity :-

Defi. :- "The minimum velocity necessory to put a satellite into the orbit is called the critical velocity."

Equation of critical velocity :-

consider a satellite is moving in a circle close to earth, the centripetal force acting on it is given by:

$$F_c = \frac{mv^2}{R} \longrightarrow (1)$$

where v = orbital velocity of the satellite

R = Radius of Earth = 6.4×10^6 m,

This centripetal force is supplied by the gravitational force and is given by:

$$F_g = mg \longrightarrow (2)$$

$$\because \quad F_c = F_g$$

$$\therefore \quad \frac{\cancel{m}v^2}{R} = \cancel{m}g$$

$$\frac{v^2}{R} = g \quad \Rightarrow \quad v^2 = gR$$

or $\quad v = \sqrt{gR}$

$$= \sqrt{9.8 \times 6.4 \times 10^6} = 7.9 \times 10^3 \text{ m/s}$$

$$\therefore \quad V = 7.91 \text{ Km/s}.$$

which is the numerical value of critical velocity.

Time period :- "It is the time required by the satellite to complete one revolution around the Earth".

Since velocity of a satellite in an orbit is constant, so using: $\quad s = vt$, $\quad t = \frac{s}{v}$

or $\quad T = \frac{2\pi R}{v} = \frac{2 \times 3.14 \times 6.4 \times 10^6}{7.91 \times 10^3}$

$$= 5075 \text{ s} = 84.58 \simeq 84.6 \text{ minutes}.$$

$$\therefore \quad T = 84.6 \text{ minutes}.$$

If, however, a satellite in a circular orbit is at an appreciable distance 'h' above the Earth's surface, we must take into account

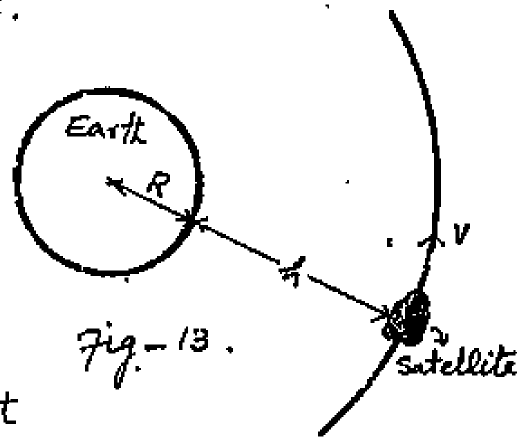


Fig-13.

the experimental fact that the gravitational acc:

decreases inversely as the square of the distance from the centre of Earth ($g \propto \frac{1}{R^2}$).

The higher the satellite, the slower will the required speed and longer it will take to complete one revolution around the earth. Close orbiting satellites orbit the Earth at a height of about 400 km.

Global Positioning System :-

Twenty four close orbiting satellites ($h = 400$ km) form the Global positioning System. With the help of this system, An Airline pilot, a sailor or any other person can now use a pocket size instrument or mobile phone to find his position on Earth's surface within 10 m accuracy.

REAL AND APPARENT WEIGHT :-

(1) **Weight** :- "The force with which Earth attracts an object towards its centre is called weight."

(2) **Real Weight** :- "The weight of an object due to the gravitational pull of the Earth on the object is called real weight."

It is usually denoted by "W" or "mg". Its value depends upon the value of 'g'. If $g = 0$, $W = 0$. It is a vector quantity and its unit is 'N'.

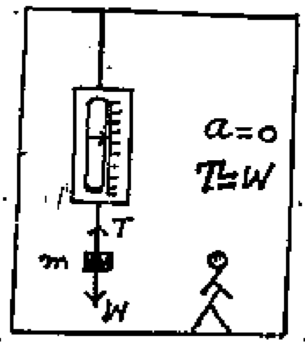
(3) **Apparent Weight** :- "The weight measured by a spring balance is called apparent weight." It is equal to the tension in the spring or string. It is denoted by "W′" or "T". Its value depends upon magnitude and direction of motion of the body.

Apparent weight of an object in an elevator (or lift) :-

Consider an object of mass 'm' tied to a spring balance that is attached to the ceiling of a lift. The reading on the spring balance shows the tension in the string, denoted by 'T' and is called apparent weight of the object. Its value depends upon the acc. of the lift. We shall discuss it in following four cases:

(i) When lift is at rest :-

When the lift is at rest or moving with uniform velocity, then according to second law of motion the acc. '$a=0$' and hence net force is zero.



Lift At rest

Fig-14(a)

If 'W' is the gravitational force (Real weight) acting on it and 'T' is tension in the string, then we have:

$$F_{net} = T - W$$

$\because F_{net} = 0$, $\quad 0 = T - W \quad$ or $\quad T = W = mg.$

$\therefore$ Apparent wt. = Real wt.

(ii) When the lift is moving upwards :-

When the lift (or elevator) is moving upwards with an acc. a, then upward tension is greater than downward force (fig.-14(b)).

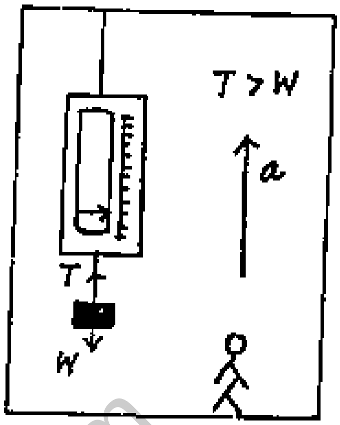


fig-14(b) Lift moving up.

i.e. $F_{net} = T - W$, $F_{net} = ma$ (2nd law of motion).

$\Rightarrow \quad T - W = ma$

or $\quad T = W + ma = m(g+a).$

$\therefore$ App. wt. > Real wt. by an amount 'ma'.

(iii) When the lift is moving downwards :-

When the lift and hence, the object is moving downwards with an acc. 'a' (fig.-14(c)), then we have :

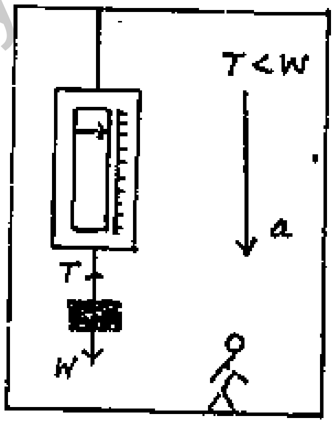


fig-14(c) Lift moving down.

$F_{net} = W - T$, $F_{net} = ma$ (2nd law of motion).

$\Rightarrow \quad W - T = ma$, or $T = W - ma = mg - ma$

or $\quad T = m(g-a) \longrightarrow *$

$\therefore$ App. wt < Real wt by an amount 'ma'.

(IV) When the lift falls freely under gravity :-

When the lift is falling freely under gravity, then : $a = g$, and hence; $T = W - mg$, or from $*$ eq. :- $\quad T = m(g-g) = 0$

$\therefore \quad T = $ App. wt. $= 0$

Thus the apparent wt. of the object shown by the scale will be zero i.e. condition of weightlessness.

It is clear from the above discussion that the app. wt. of the object is not necessarily equal to its true or real wt. in an accelerating system.

WEIGHTLESSNESS IN SATELLITES AND GRAVITY FREE SYSTEM :-

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is, whether it is falling under the force of attraction of Earth, the Sun or some distant star.

"An Earth's satellite is freely falling object".

This statement may be surprising at first, but it is easily seen to be correct from the following example:

Consider the behaviour of a projectile shot parallel to the horizontal surface of the earth in the absence of air friction. If the projectile is thrown at successively larger speeds, then during its free fall to the Earth, the curvature of the path decreases with increasing horizontal speeds. If the object is thrown fast enough parallel to the Earth, the curvature of its path will match the curvature of the earth as shown in fig-15. In this Case the space ship will simply circle the Earth.

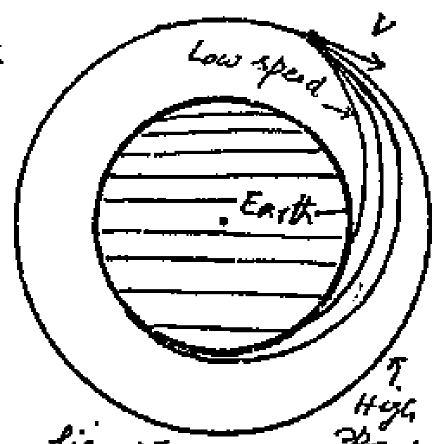


fig-15.

The space ship is accelerating towards the Centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply "g" i.e. the free fall acceleration. In fact, the space ship is falling towards the Centre of the Earth at all the times, but the curvature of the earth prevents the space ship from hitting. Since the space ship is in free fall, all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellites. Such a system is called "Gravity Free system."

ORBITAL VELOCITY :-

Defi. :- "When a body is moving in an orbit (circular or elliptical), its motion is called orbital motion and its velocity is called "orbital velocity."

Explanation :- The Earth and all other planets in our solar system revolve around the sun in nearly circular paths. The artificial satellites launched by men also adopt nearly circular paths around the Earth. This type of motion is called orbital motion.

Equation of orbital velocity :-

Consider the fig-16, in which an artificial satellite is circling around the Earth. Let the mass of satellite is 'm_s' and 'v' is its orbital speed. The mass of Earth is 'M' and 'r' represents the radius of the orbit. A Centripetal force "$\frac{m_s v^2}{r}$" is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite i.e. "$\frac{GMm_s}{r^2}$". Equating the two forces we get :

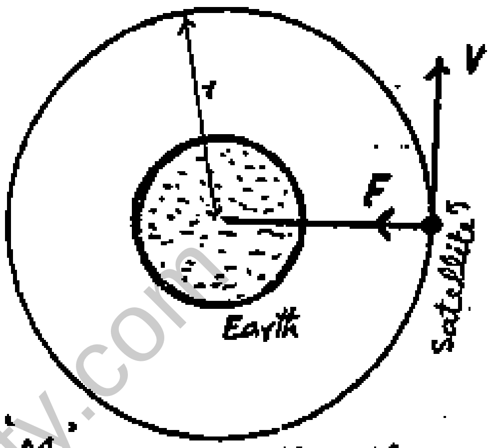


Fig.-16.

$$\frac{\cancel{m_s} v^2}{\cancel{r}} = \frac{GM\cancel{m_s}}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$

or $$v = \sqrt{\frac{GM}{r}}$$

The above eq. shows that the orbital speed is independent of the mass of a satellite. Thus any satellite orbiting at a distance 'r' from Earth's centre must have the orbital speed as given by above equation. Any speed less than this will bring the satellite tumbling back to Earth.

Example 5.6:- An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of Earth $M = 6\times10^{24}$ kg, and its radius $R = 6400$ km.

Sol. :- Given that: $M = 6\times10^{24}$ kg, $R = 6400$ km

$h = 384000$ km, $\therefore$ $r = R + h = 6400 + 384000 = 390400$ km

using eq. $v = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{6.67\times10^{-11} \times 6\times10^{24}}{390400\times10^{3}}}$

$v = 1012$ m/s $= 1.012$ km/s.

Also $T = \dfrac{2\pi r}{v} = \dfrac{2\times3.14\times390400\times10^{3}}{1012} \times \dfrac{1}{24\times60\times60}$ days

$\therefore$ $T = 28.1$ days.

ARTIFICIAL GRAVITY :-

Defi. :- "The gravitational acceleration produced in a space craft by rotating it about its own axis is called artificial gravity."

Explanation :- Astronauts with respect to their space crafts (or satellites) are in the state of weightlessness. The weightlessness creates a lot of problems for the astronauts while performing their duties in the space crafts. To get rid of this difficulty an "artificial gravity" is created in the space ship by rotating it around its own axis. The astronauts then are pressed towards the outer rim and exerts a force on the floor of the space ship in much the same way as on the Earth.

Expression for frequency :-

Consider a space craft of the shape as shown in fig-17. The outer radius of the space craft is 'R' and it rotates around its own central axis with angular speed 'ω', then its angular acc. 'a_c' is :

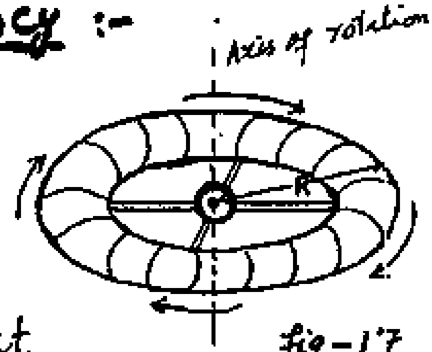


fig-17.

$$a_c = R\omega^2 \longrightarrow (1)$$

$a_c = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = \frac{R^{\cancel{2}}\omega^2}{\cancel{R}}$

$\therefore a_c = R\omega^2$

But $\omega = \frac{2\pi}{t}$, where t is the period of revolution of space craft.

Hence : $a_c = R\left(\frac{2\pi}{t}\right)^2 = R\left(\frac{4\pi^2}{t^2}\right)$

As $f = \frac{1}{t}$, $\therefore a_c = R \times 4\pi^2 \times \frac{1}{t^2} = R \times 4\pi^2 f^2$

or $f^2 = \frac{a_c}{4\pi^2 R}$ or $f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$

As the force of gravity provides the required centripetal acc., $\therefore a_c = g$

Hence $f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \longrightarrow (2)$

when the space-ship rotates with this frequency, the artificial gravity like Earth is provided to the in-habitants of the space ship.

GEOSTATIONARY ORBITS :-

Geo-Stationary Satellite :- "A satellite whose orbital motion is synchronized with the rotation of the earth is called geo-synchronous or geo-stationary satellite".

A geo-synchronous satellite remains always over the same point on the equator as the Earth spins on its axis. Such a satellite is very useful for worldwide communication, weather observation, navigation, and other military uses.

Geo-Stationary orbit :- "A geo-stationary orbit is such an orbit in which an artificial satellite remains at rest with respect to Earth."

Expression for geo-Stationary orbital radius:-

We know that the orbital speed necessory for the circular orbit of a satellite is given by:

$$v = \sqrt{\frac{GM}{r}} \longrightarrow (1)$$

where 'r' is the distance of satellite from the centre of eath, 'M' is mass of earth and 'G' is gravitational constant.

But this speed must be equal to the avg. speed of the satellite in one day i.e.

$$v = \frac{S}{t} = \frac{2\pi r}{t} \longrightarrow (2)$$

where 't' is the period of revolution of the satellite, that is equal to one-day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the earth rotates in one day and the satellite will revolve around the earth in one day, the satellite at

"A" will always stay over the same pt. "A" on the earth as shown in fig-18.

Comparing eq. (1) and (2) we have:

$$\frac{2\pi r}{t} = \sqrt{\frac{GM}{r}}$$

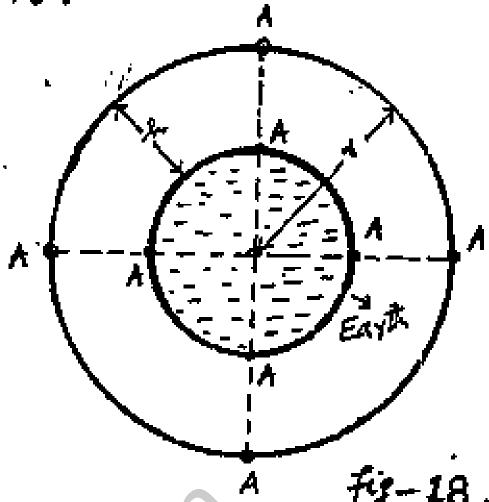


fig-18.

squaring both sides

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM}{r}$$

By cross-multiplication:

$$4\pi^2 r^3 = GMt^2$$

or

$$r^3 = \frac{GMt^2}{4\pi^2} \Rightarrow r = \left[\frac{GMt^2}{4\pi^2}\right]^{1/3}$$

$$\therefore \quad r = \left[\frac{GMt^2}{4\pi^2}\right]^{1/3} \longrightarrow (3)$$

The above eq. (3) gives the orbital radius of a geo-stationary satellite. Putting values we get:

$$r = \left[\frac{6.67\times10^{-11}\times 6\times10^{24}\times(24\times60\times60)^2}{4\times(3.14)^2}\right]^{1/3} = 4.23\times10^4\,km$$

$$\therefore \quad r = 4.23\times10^4\ km.$$

which is the orbital radius measured from the Centre of the earth, for a geo-stationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. This height above the equator is 36,000 km!

COMMUNICATION SATELLITES:-

Geo-stationary satellites are used for communication purposes. A satellite communication system can be set up by placing several geo-stationary satellites in orbit over different points on the surface of the Earth. One such satellite covers 120 of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites as shown in fig.-19.

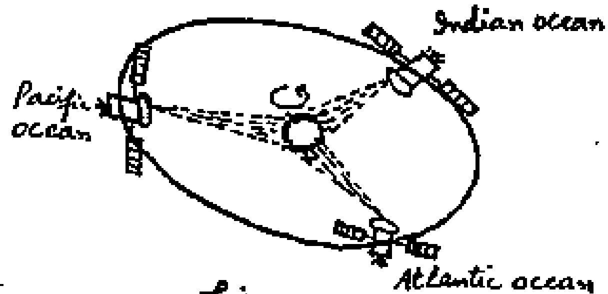


fig.-19.

Since these geo-stationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and re-transmit the signals is provided by large solar cell panels fitted on these satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries. You can also pick up the signal from the satellite using a dish antenna on your house. The largest satellite system is managed by 126 countries, International Telecommunication Satellite organization (INTELSAT). An INTELSAT-VI satellite operates at microwave frequencies of 4, 6, 11 and 14 GHz, and has a capacity of 30,000 two way telephone circuits plus three T.V. channels.

Example 5.7 :- Radio and T.V. signals bounce from a synchronous satellite. This satellite circles the Earth once in 24-hrs. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate. (a) What is the orbital radius for a synchronous satellite? (b) What is its speed?

Sol. :- (a) We know that: $G = 6.67\times10^{-11}\ Nm^2kg^{-2}$,

$$M = 6\times10^{24}\ kg,\quad t = 24\times60\times60\ s = 86400\ s$$

As : $r = \left[\dfrac{GMt^2}{4\pi^2}\right]^{1/3}$, putting values, we get:

$$r = \left[\frac{6.67\times10^{-11}\times6\times10^{24}\times(86400)^2}{4\times(3.14)^2}\right]^{1/3} = 4.23\times10^{7}\ m$$

$$\therefore\ r = 4.23\times10^{7}\ m/s.$$

(b) $\because\ v = \dfrac{2\pi r}{t} = \dfrac{2\times3.14\times4.23\times10^{7}}{86400} = 3.07\times10^{3}\ m/s$

or $v = 3.1\ km/s$.

NEWTON's AND EINSTEIN's VIEWS OF GRAVITATION :-

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, consider the space as a thin rubber sheet. If a heavy weight is hung from it, it curves as shown in fig-20(a). The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (a curved line equivalent to straight lines in plane geometry), in curved space time. Thus, a body at rest or moving slowly near the great mass (fig-20(a)) would follow a geodesic towards the body.

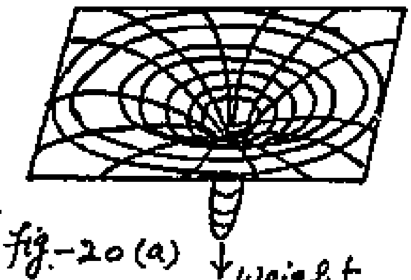


fig-20(a) "Rubber sheet analogy for curved space time".

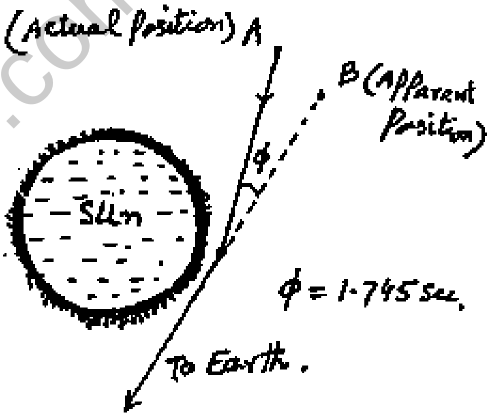


fig-20(b) Bending of starlight by the Sun.

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity but he could not explain why gravity follow an inverse square law. Einstein's theory also says that gravity follows an inverse square law (except in strong gravitational fields), but it tells us why this should be so. That is why Einstein's theory is better than Newton's theory.

Einstein inferred that if gravity and acceleration:

are precisely equivalent, gravity must bend light, by a precise amount that could be calculated. This was not entirely a startling suggestion: Newton's theory, based on the idea of light as a stream of tiny particles, also suggested that a light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exactly twice as great as it is according to Newton's theory.

When the bending of starlight caused by the gravity of the Sun was measured during a solar eclipse in 1919, (fig-20(b)) and found to match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

QUESTIONS :

Note :- The answers to the questions at the end of the Chapter are given below. For questions see last page.

Q.5.1 :-

Tangential velocity	Angular velocity
(1) It is the linear velocity of a particle moving along a curve or a circle directed along the tangent at any pt. on the curve.	(1) It is the rate of change of angular displacement of a particle moving along a circular path.
(2) It is denoted by v_T.	(2) It is denoted by 'ω'.
(3) Its formula is : $v = r\omega$. Also: $v = \frac{s}{t}$.	(3) Its formulae are: $\omega = \frac{\Delta\theta}{\Delta t}$ Also: $\omega = \frac{v}{r}$.
(4) Its unit is m/s.	(4) Its unit is rad/s.