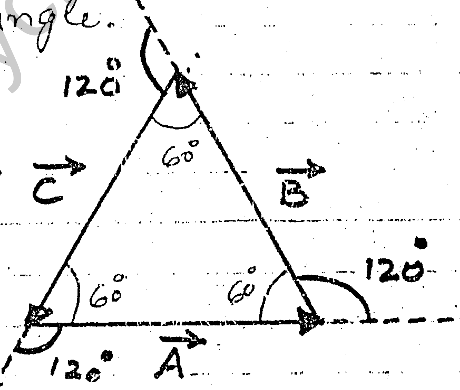
QUESTIONS

Question 2.1 :- Define the terms
(i). Unit Vector (ii). Position Vector (iii). Component of a Vector.

Answer :- See Theory

Question 2.2 :- The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

Answer :- The resultant vector of three vectors of equal magnitudes is equal to zero, when they are represented by the three adjacent sides of an equilateral triangle.
(In such triangles, all the sides are equal in magnitudes and angles inside it are of 60° each).
By using head to tail rule, tail of $\vec{C}$ coincides with head of vector $\vec{A}$. So magnitude of vector



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = \vec{O}$$

$$|\vec{R}| = \text{Zero}$$

Orientation :- From figure it can be seen that the orientation between $\vec{A}$ and $\vec{B}$ is 120° and between $\vec{B}$ and $\vec{C}$ is also 120°. Same for $\vec{C}$ and $\vec{A}$

Question 2.3 :- Vector $\vec{A}$ lies in XY-plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite signs?

Answer :- Consider a vector $\vec{A}$ lying in Q-I represented by a line $\vec{OP}$ and making an angle 'θ' with x-axis.

Q-I

(a) - It will have both of its rectangular components negative when it lies in quadrant III. So it is oriented at an angle of $180° + \phi$ with +ve x-axis as shown in figure (a)

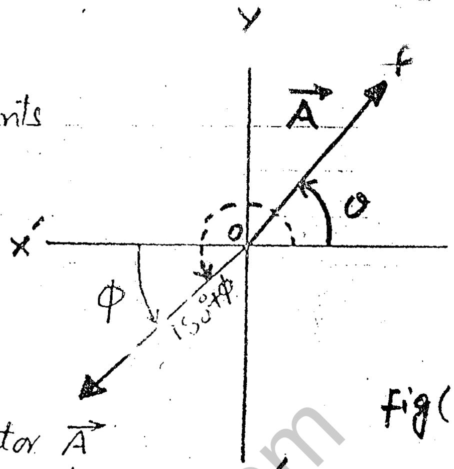


Fig (a)

(b). The components of vector $\vec{A}$ will have opposite signs when it lies in Q-II or in Q-IV.

(i). In Q-II :-
When vector $\vec{A}$ is oriented with $180° - \phi$. It will have its components with opposite signs i.e
$A_x = -ve$ and $A_y = +ve$.

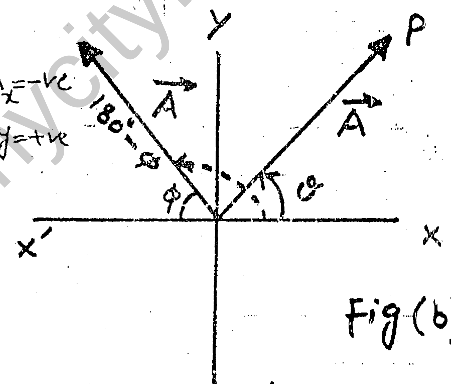
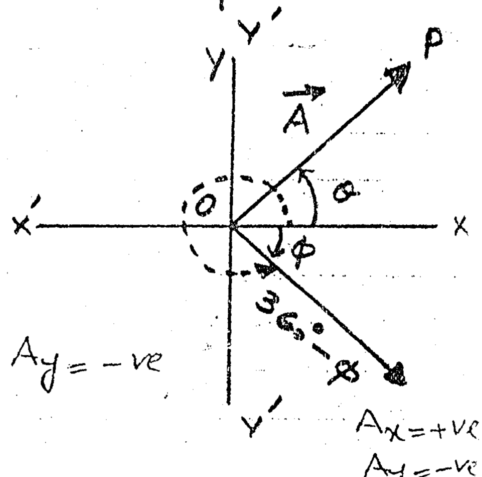


Fig (b)

(ii) In Q-IV
When vector $\vec{A}$ is oriented at an angle of $360° - \phi$, then it will have again its components with opposite signs.

In this case
$A_x = +ve$ and $A_y = -ve$



Question 2.4 :- If one of the components of a vector is not zero, can its magnitude be zero? Explain.

Answer :- No, a vector cannot have zero magnitude in this case.

If A_x, A_y are the components

Then $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Let $A_x = 0$ $\quad A = \sqrt{0^2 + A_y^2} = A_y \neq 0$

or $A_y = 0$ $\quad A = \sqrt{A_x^2 + 0^2} = A_x \neq 0$

This shows that if one of the component of a vector is not zero, then magnitude of vector cannot be zero.

Question:- 2.5:- Can a vector have a component greater than the vector's magnitude?

Solution:- No, a vector cannot have a component greater than its magnitude.

As we have $\quad A_x = A\cos\theta$

A_x will attain its maximum value when $\cos\theta$ will have max. value One i-e $\cos 0^\circ = 1$

$\therefore \quad A_x = A = $ Maximum value

Similarly $\quad A_y = A\sin\theta$

A_y will attain its maximum value when $\sin\theta$ will have max. value One i-e $\sin 90^\circ = 1$

$\therefore \quad A_y = A\sin 90^\circ = A = $ Max. value

Thus it can be seen that a component can have maximum value equal to the magnitude of a vector.

Question 2.6:- Can the magnitude of a vector have negative value?

Solution:- No, a vector cannot have negative magnitude. Because magnitude of a vector is independent of its direction.

If we have a vector "$-\vec{A}$" then

$\vec{A} = |\vec{A}|(-\hat{A})$.

Negative sign will indicate the opposite direction of the vector $\vec{A}$. $|\vec{A}|$ = Magnitude will always be positive.

Negative magnitude is a meaningless idea.

Question 2.7 :- If $\vec{A} + \vec{B} = \vec{0}$, What can you say about the components of the two vectors?

Answer :- If $\vec{A} + \vec{B} = 0$ then

$$\vec{A} = -\vec{B}$$

We can write interms of their rectangular components

$$A_x\hat{i} + A_y\hat{j} = -(B_x\hat{i} + B_y\hat{j}).$$

$$A_x\hat{i} + A_y\hat{j} = -B_x\hat{i} - B_y\hat{j}$$

By comparing the coefficients of $\hat{i}$ and $\hat{j}$, we get

$$A_x = -B_x$$

and $$A_y = -B_y.$$

It means that if sum of the two vectors is zero, then their respective rectangular components will be of the same magnitude but in opposite direction.

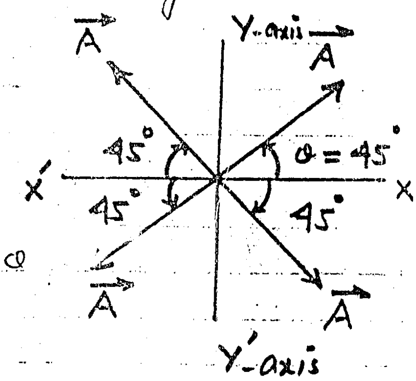
Question 2.8 :- Under what circumstances would a vector have components that are equal in magnitude?

Answer :- If A_x and A_y are the components of a vector $\vec{A}$ which makes an angle 'θ' with x-axis, Then

$$A_x = A\cos\theta$$

and $$A_y = A\sin\theta$$

As given



$$A_x = A_y$$

$$A\cos\theta = A\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \pm 1$$

$$\tan\theta = \pm 1$$

$$\theta = \tan^{-1}(\pm 1)$$

$$\theta = 45^\circ \text{ with } x\text{-axis}$$

or $$= 45^\circ \text{ with } -x\text{-axis}$$

So, for those angles for which $\tan\theta$ becomes equal to ± 1, then the magnitude of rectangular comp. of a vector will be equal to each other

$$\because \sin 45^\circ = \cos 45^\circ = 0.707$$

Question 2.9:- Is it possible to add a vector quantity to a scalar quantity?

Answer:- No, it is not possible to add a vector quantity to a scalar quantity, because they are totally different things. Scalars have only the magnitude but vectors have both the magnitude and direction. Vectors are added only with vectors.

Question 2.10:- Can you add zero to a null vector?

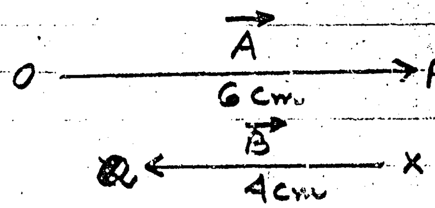
Answer:- No, we cannot add zero to a null vector because scalars cannot be added into vectors. Vectors are added only with vectors.

Question 2.11:- Two vectors have unequal magnitudes can their sum be zero? Explain.

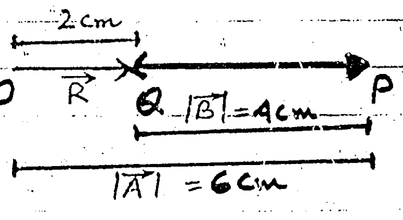
Answer:- No, two vectors having unequal magnitude cannot have resultant zero.

Consider two vectors $\vec{A}$ and $\vec{B}$ having magnitudes 6cm and 4cm respectively. The resultant vector attains its minimum value when these are antiparallel to each other.

Adding these two vectors by head-to-tail rule



$$\vec{OP} + \vec{PQ} = \vec{OQ}$$
$$\vec{A} + \vec{B} = \vec{R}$$
$$|\vec{R}| = |\vec{OQ}| = 6\text{ cm} - 4\text{ cm}$$
$$= 2 \neq 0$$

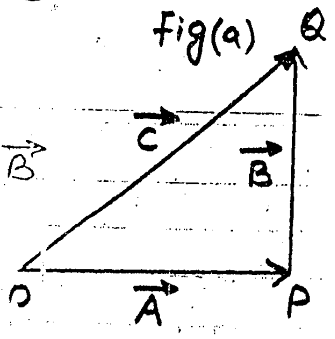


Question 2.12:- Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

Answer:- Consider two vectors $\vec{A}$ and $\vec{B}$ which have the same magnitudes but are perpendicular to each other.

Adding them by Head-to-tail rule, then $\vec{C}$ be their resultant vector i-e

$$\vec{C} = \vec{A} + \vec{B}$$



Fig(a)

Now finding -ve of vector $\vec{B}$, and $-\vec{B}$ is added into $\vec{A}$ by head-to-tail rule, $\vec{D}$ be their resultant vector as shown in figure (b)

i-e $\vec{D} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

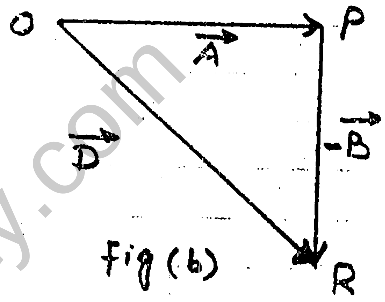


Fig (b)

Combining both the figures.

It is clear from the figure (c) that

- $\overline{OP}$ is common in both the triangles ΔOPQ and ΔOPR.
- $PQ = PR = B$.
- $\angle O\hat{P}Q = \angle OPR = 90^\circ$

$\therefore$ $\Delta OPQ \cong \Delta OPR$

and $\angle P\hat{O}Q = \angle P\hat{O}R = 45^\circ$

We get

$$\angle Q\hat{O}R = \angle P\hat{O}Q + \angle POR$$

$$= 45^\circ + 45^\circ$$

$$\angle Q\hat{O}R = 90^\circ$$

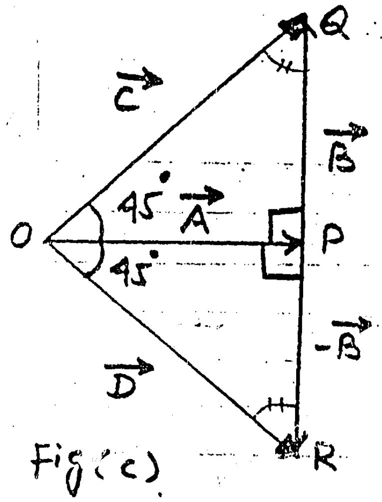


Fig (c)

Hence resultant vector of sum and differenc of two perpendicular vectors of equal magnitudes are also perpendicular to each other

For magnitudes of $\vec{C}$ and $\vec{D}$, using formula of magnitude for rectangular components i-e

$$|\vec{C}| = \sqrt{A^2 + B^2} \quad \text{and}$$

$$|\vec{D}| = \sqrt{(A)^2 + (-B)^2}$$

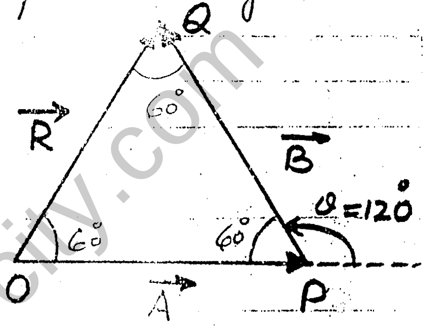
$$= \sqrt{A^2 + B^2}$$

$$\therefore \quad |\vec{C}| = |\vec{D}|$$

So they are also equal in magnitudes.

Question 2.13 :- How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude?

Answer :- Consider an equilateral triangle ΔOPQ. In such triangle, all the sides are equal and all the angles inside it are of 60° each. If two vectors $\vec{A}$ and $\vec{B}$ are represented by the two sides $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ respectively of ΔOPQ as shown in figure. Then by head-to-tail $\overrightarrow{OQ}$ will be resultant vector



$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$
$$\vec{R} = \vec{A} + \vec{B}.$$

Since $OP = PQ = OQ$

$$|\vec{A}| = |\vec{B}| = |\vec{R}|$$

Then it can be seen from figure that orientation between $\vec{A}$ and $\vec{B}$ is 120°.

Question 2.14 :- The two vectors to be combined have magnitudes 60N and 35N. Pick the correct answer from those given below and tell why is it the only one of three that is correct.

Answer :- As we have $F_1 = 60N$ and $F_2 = 35N$. Resultant of these two forces will attain its maximum value when both are parallel to each other and minimum when both are anti-parallel. ~~to~~

For Parallel $F = F_1 + F_2 = 60 + 35 = 95N$

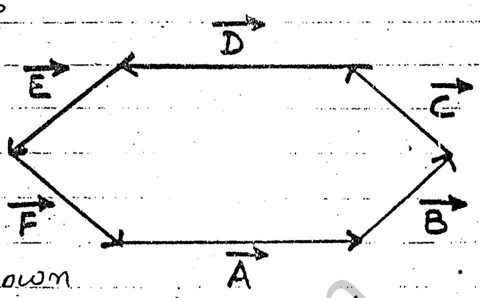
For Anti-parallel $F' = F_1 - F_2 = 60 - 35 = 25N$

So the range of resultant is lying between 25 to 95N.

Therefore the correct answer is '70N'. It is lying within the range, Other Two 100N and 20N are out of range.

Question 2.15 :- Suppose the sides of a closed polygon represent vector arranged head to tail. What is the sum of these vectors?

Answer :- If the sides of a closed hexagon represent vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$ and $\vec{F}$ and they are added by using head to tail rule as shown in the figure. There resultant vector will have zero magnitude. Because head of the last vector $\vec{F}$ coincides with the tail of the first vector $\vec{A}$.



$$\therefore \quad \vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = \vec{0}$$

$$R = 0$$

Question 2.16 :- Identify the correct answers.

(i) :- Two ships X and Y are travelling in different directions at equal speeds. The actual direction of motion of 'X' is due north but to an observer on Y, the apparent direction of motion of X is north - east. The actual direction of motion of Y as observed from the shore will be.

(A). EAST (B) WEST

(C). SOUTH - EAST (D) SOUTH - WEST

(ii) :- A horizontal force F is applied to a small object 'P' of mass 'm' at rest on a smooth plane inclind at an angle 'θ' to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is

(a) $F\cos\theta - mg\sin\theta$ (b) $F\sin\theta - mg\cos\theta$

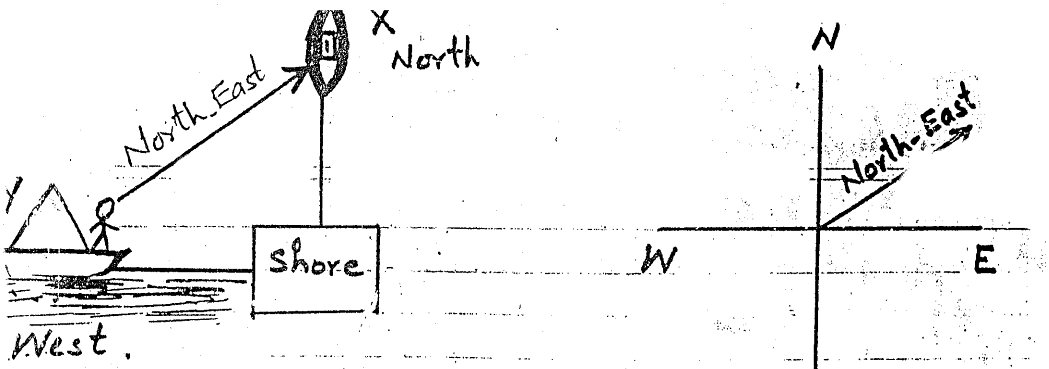
(c) $F\cos\theta + mg\cos\theta$ (d) $F\sin\theta + mg\sin\theta$

(e) $mg\tan\theta$.

Answer :-

(i)- The best option is (B) i-e West.

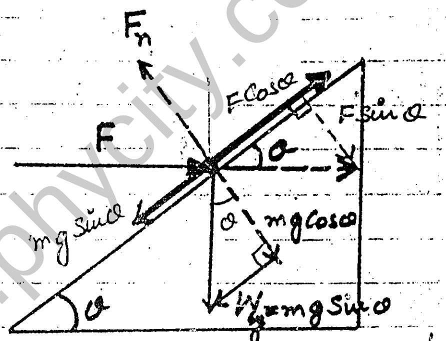
As the ship 'X' is moving from shore towards North.



West.

Then according to observer on the ship Y, the ship 'X' is moving towards North-east, therefore ship 'Y' is moving towards west w.r. to shore.

(ii)



Weight is resolved into two components mg Cos θ and mg Sin θ. Also force F is resolved into two components F Cos θ and F Sin θ. Both mg Cos θ and F Sin θ are acting opposite to the normal reaction of the plane so $F_n = mg\cos\theta + F\sin\theta$ Balance each other.

It can be seen from the figure that two forces F Cos θ and mg Sin θ are acting on the body along the plane in opposite directions. Since body is moving in upward direction under the action of force, so $F\cos\theta > mg\sin\theta$. The net force acting on the mass 'm' in upward direction is

$$F_{net} = F\cos\theta - mg\sin\theta$$

Therefore the best option is (a).

Question 2.17:- If all the components of vectors $\vec{A_1}$ and $\vec{A_2}$ were reversed, how would this alter $\vec{A_1} \times \vec{A_2}$?

Answer:- Consider two vectors $\vec{A_1}$ and $\vec{A_2}$ making an angle 'θ' with each other as shown in figure (a).

$$\vec{A_1} \times \vec{A_2} = A_1 A_2 \sin\theta \, \hat{n} \quad \text{——— (1)}$$

Then according to Right hand rule, $\hat{n}$ is indicating the direction of $\vec{A_1} \times \vec{A_2}$ which is in upward direction.

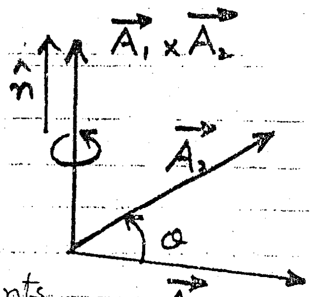


Fig (a).

If we reverse all the components $\vec{A_1}$ and $\vec{A_2}$ vectors. Then these vectors are also reversed as shown in the figure (b).

$$-\vec{A_1} \times -\vec{A_2} = A_1 A_2 \sin\theta \, \hat{n} \longrightarrow (2)$$

Then according to Right hand rule, $\hat{n}$ is indicating the direction of $-\vec{A_1} \times -\vec{A_2}$ which is again in upward direction.

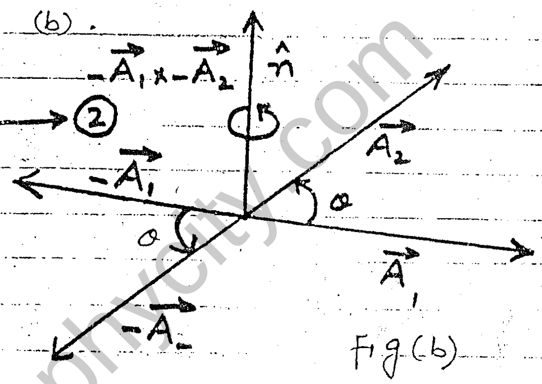


Fig (b)

So by comparing eq (1) and eq (2).

$$\vec{A_1} \times \vec{A_2} = -\vec{A_1} \times -\vec{A_2}$$

Hence, the vector product of two vectors remains unchanged even by reversing all the components of vectors.

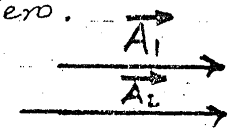
Question 2.18 :- Name the three different conditions that could make $\vec{A_1} \times \vec{A_2} = 0$

Answer :- As $\vec{A_1} \times \vec{A_2} = 0$

(a). Either $A_1 = 0$ or $A_2 = 0$

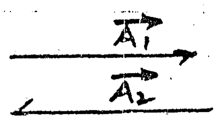
(b). If both the vectors $\vec{A_1}$ and $\vec{A_2}$ are parallel to each other i.e $\theta = 0°$

$\therefore \; \vec{A_1} \times \vec{A_2} = A_1 A_2 \sin 0° \, \hat{n} =$ Zero.



(c). When $\vec{A_1}$ and $\vec{A_2}$ are antiparallel to each other i-e $\theta = 180°$

$\vec{A_1} \times \vec{A_2} = A_1 A_2 \sin 180° \, \hat{n} =$ Zero.



Question 2.19 :- Identify true or false statements and explain the reason.

(a)- A body in equilibrium implies that it is not moving nor rotating.

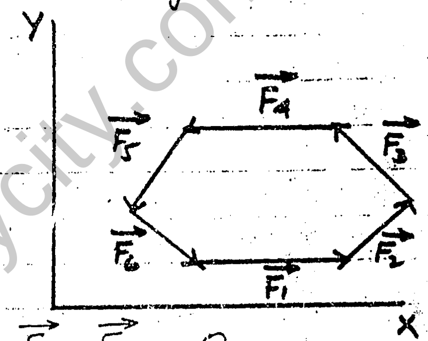
b)- If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Answer :-

(a)- It is a false statement. A body is said to be in equilibrium when it is at rest or moving with uniform linear velocity or rotating with uniform angular velocity.

(b)- It is a true statement.

If all the forces acting on the body lie in a common plane. Such forces are said to be coplanar. Therefore along a closed polygon,



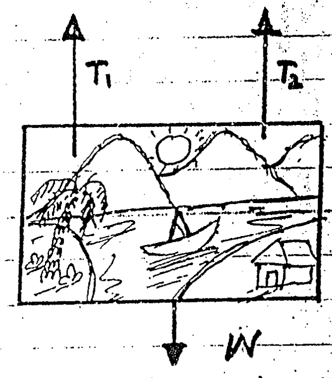
$$\Sigma \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} + \vec{F_6} = 0$$

and $\Sigma \vec{\tau} = 0$

Therefore the body is in equilibrium.

Question 2.20 :- A picture is suspended from a wall by two strings. Show by diagram, the configuration of the strings for which the tension in the strings will be minimum.

Answer :- When a picture is suspended from a wall by two strings as shown in figure. This configuration of the strings shows that the tension in the strings will be same and equal to the weight of picture.



Applying first condition of equilibrium

$T_1 = T_2 = T$

$$\Sigma F = 0$$

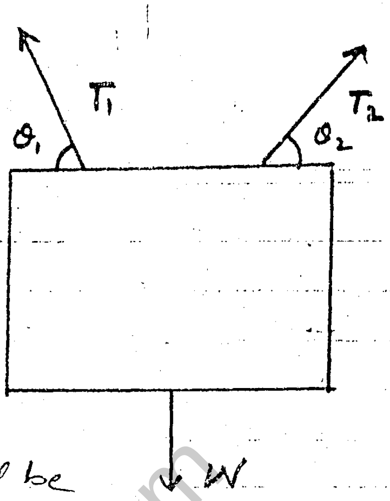
$$T + T - W = 0$$

$$2T = W$$

$$T = \frac{W}{2}$$

In this case the tension in both the string will be min.

In other case when the tensions T_1 and T_2 are making any angles θ_1 and θ_2, then after resolving the tensions in their rectangular components and by applying first condition of equilibrium

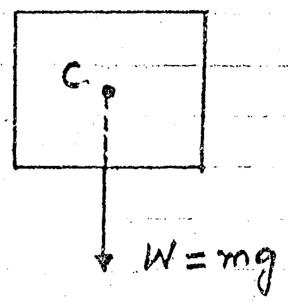


$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

Then tension in the string will be greater than the above case.

Question 2.21: Can a body rotate about its centre of gravity under the action of its weight?

Answer :- No, a body cannot rotate about its centre of gravity under the action of its weight. Because, weight always acts at the centre of gravity. In this case line of action of the force (weight) passes through centre of gravity of the body, so



moment arm of the force = l = 0

Force = Weight = W

Then

Torque = (Force)(Moment Arm)

$$\tau = (W)(0)$$

$$\tau = \text{Zero}$$

No, turning effect is produced. Therefore a body cannot rotate about its centre of gravity under the action of its weight.