digital modulation in which a laser is flased on and off at an extremely fast rate. A pulse of light represents number 1 and the absence of light represent 0. Any information can be represented by a particular pattern or code of these 1s and 0s and at the receiving end these are decoded to reconstruct the original information.

Q 10.11: If the source of light signals is not monochromatic, then the light will disperse while propagating through the core of the optical fibre into different wavelengths λ_1, λ_2 and λ_3 etc as shown in fig.

As shown, λ_1 meets the core and cladding at the critical angle and λ_2 and λ_3 are at slightly greater angles. The light paths have thus different lengths. So, the light of different wavelengths reaches the other end of the fibre at different times and the signal received is distorted.

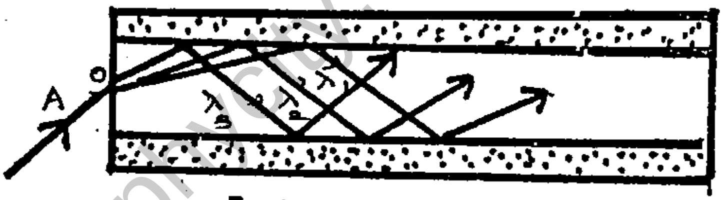

fig.

NUMERICAL PROBLEMS.

P. 10.1:

DATA. Focal length $= f = 5$ cm

Distance of the image from the lens $= q = -25$ cm

(a) Distance of the object from the lens $= p = ?$

(b) Angular magnification $= M = ?$

(c) Angular magnification $= M = ?$ (When image is at infinity)

(P.T.O)

Sol. As $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$
(a)

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{p} = \frac{1}{5} + \frac{1}{25} = \frac{5+1}{25} = \frac{6}{25}$$

$$p = \frac{25}{6} = \boxed{4.2\ cm}$$

(b) Angular magnification of a magnifying glass is;

$$M = 1 + \frac{d}{f} = 1 + \frac{25}{5} = \boxed{6}$$

(c) For image at infinity, the formula becomes;

$$M = \frac{d}{f} = \frac{25}{5} = \boxed{5}$$

As $M = \frac{d}{p}$
$p = f$ (when image is at infinity, object must be at focus)

P.10.2:

DATA. Focal length of objective $= f_o = 96\ cm$
Diameter of objective $= d_o = 12\ cm$
Linear magnification $= M = 24$
(a) Focal length of eyepiece $= f_e = ?$
(b) Diameter of eyepiece $= d_e = ?$

Sol. As $M = \frac{f_o}{f_e}$
(a)

$$\text{or, } f_e = \frac{f_o}{M} = \frac{96}{24} = \boxed{4\ cm}$$

(b) Now the ratio of the diameter of the two lenses should be the same as the ratio of their focal lengths for all the light rays incident on the objective lens to pass through the eye lens. Thus;

$$\frac{d_o}{d_e} = \frac{f_o}{f_e}$$

$$\text{or, } d_e = d_o \times \frac{f_e}{f_o}$$

Putting the values, we have

$$d_e = \frac{12\ cm \times 4\ cm}{96\ cm} = \boxed{0.5\ cm}$$

P.10.3:

DATA. Focal length of objective $= f_o = 20\text{ cm}$
Focal length of eyepiece $= f_e = 5\text{ cm}$
Angular magnification $= M = ?$

Sol. As, $M = \frac{f_o}{f_e} = \frac{20\text{ cm}}{5\text{ cm}} = \boxed{4}$

P.10.4:

DATA. Focal length of objective $= f_o = 100\text{ cm}$
Focal length of eyepiece $= f_e = 5.0\text{ cm}$
(a) Distance of the image $= q_e = ?$
(b) Angular magnification $= M = ?$

Sol (a). $p_e = 5\text{ cm}$, $f_e = 5\text{ cm}$, $q_e = ?$

Using the formula;

$$1/f_e = \frac{1}{p_e} - 1/q_e =$$

$$\frac{1}{q_e} = \frac{1}{p_e} - 1/f_e = \frac{1}{5} - \frac{1}{5} = 0$$

$$q_e = \frac{1}{0} = \boxed{\infty}$$

i.e; For astronomical telescope to be in normal adjustment, the final image always formed at infinity.

(b) As $M = \frac{f_o}{f_e} = \frac{100\text{ cm}}{5\text{ cm}} = \boxed{20}$

P.10.5:

DATA. Distance of object $= p = 3.6\text{ cm}$
Focal length of first convex lens $= f = 3.0\text{ cm}$
Focal length of second convex lens $= f' = 16.0\text{ cm}$
Distance between the lenses $= L = 26\text{ cm}$
Position of the final image $= q' = ?$

Sol. Image position due to first lens is;

$$1/f = \frac{1}{p} + \frac{1}{q}$$

or, $$\frac{1}{q} = 1/f - 1/p = \frac{1}{3} - \frac{1}{3.6}$$

$$\frac{1}{q} = \frac{3.6 - 3}{3 \times 3.6} = \frac{0.6}{10.8}$$

$$q = \frac{10.8}{0.6} = 18\text{ cm} \quad \text{——— (1)}$$

(P.T.O)

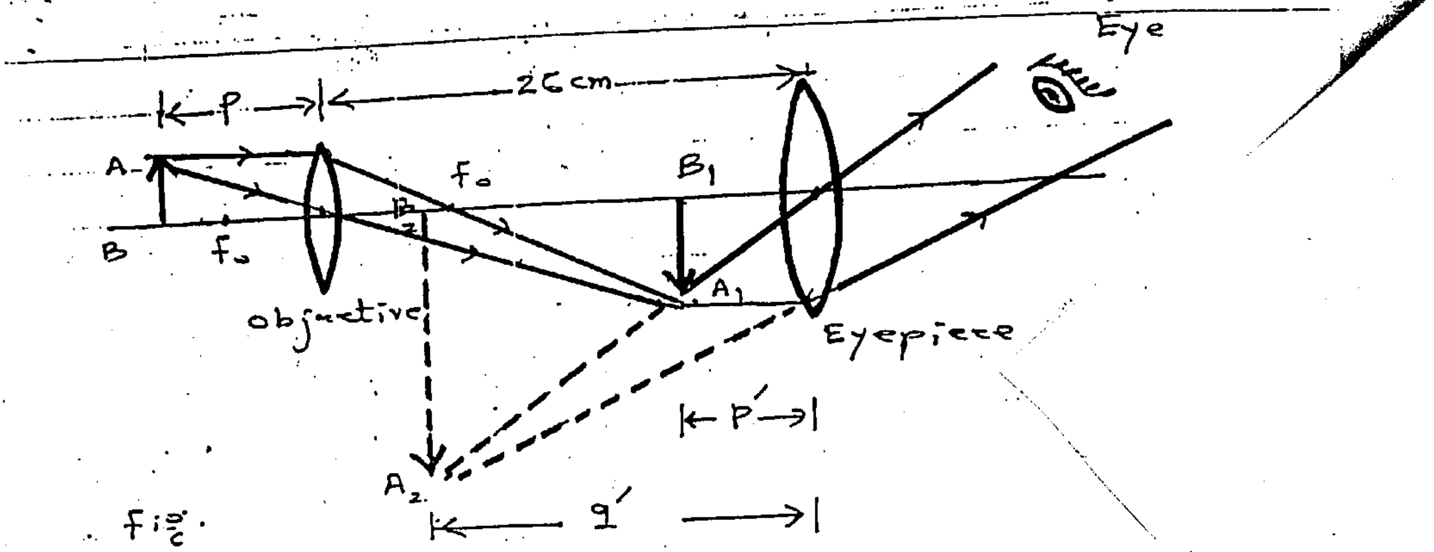

fig.

Hence image A_1B_1 due to first lens is formed 18cm away from it. This image acts as an object P′ for the second lens which is 26cm away from the first lens. Thus distance of object P′ is

$$p' = 26 - 18 = 8 \text{ cm}$$

and $f' = 16\text{cm}$

$q' = ?$

Using;
$$\frac{1}{f'} = \frac{1}{p'} + \frac{1}{q'}$$

$$\frac{1}{q'} = \frac{1}{f'} - \frac{1}{p'} = \frac{1}{16} - \frac{1}{8} = \frac{8-16}{16\times 8} = \frac{-8}{128}$$

or, $q' = \boxed{-16\text{cm}}$

The negative sign indicates that the image is virtual

P.10.6:

DATA. Focal length of objective $= f_o = 1.0$ cm

Focal length of eyepiece $= f_e = 3.0$ cm

Distance of object $= P = 1.2$ cm

Distance of image $= q' = -25$ cm

(a) separation of the lenses $= L = ?$

(b) Magnification $= M = ?$

Sol.(a) Image formed by the objective lens is;

$$\frac{1}{f_o} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f_o} - \frac{1}{p} = \frac{1}{1} - \frac{1}{1.2} = \frac{1.2 - 1}{1\times 1.2} = \frac{0.2}{1.2}$$

or, $q = 6\text{ cm}$ ——— (1)

(P.T.O)

This image will act as an object for the eyepiece, its distance p′ from the eyepiece can be found using;

$$\frac{1}{f_e} = \frac{1}{p'} + \frac{1}{q'}$$

$$\frac{1}{p'} = \frac{1}{f_e} - \frac{1}{q'} = \frac{1}{3} - \left(-\frac{1}{25}\right) = \frac{1}{3} + \frac{1}{25} = \frac{25+3}{75}$$

$$p' = \frac{75}{28} = 2.7\,cm \quad \text{——— (2)}$$

Thus the separation of two lenses is

$$L = q + p' \quad \text{——— (3)}$$

Putting values from eq. (1) and (2), we have

$$L = 6 + 2.7 = \boxed{8.7\ cm}$$

(b) For magnification;

$$M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right) \quad \text{——— (4)}$$

Putting values, we have

$$M = \frac{6}{1.2}\times\left(1 + \frac{25}{3}\right) = 46.7 \simeq \boxed{47}$$

P.10.7:

DATA. Wavelength of Sodium light $= \lambda = 589\,nm = 589\times10^{-9}\,m$

Aperture of the objective $= D = 0.9\,cm = 0.9\times10^{-2}\,m$

(a) Limiting angle of resolution $= \alpha_{min} = ?$

(b) Maximum limit of resolution $= \alpha'_{min} = ?$

Sol.(a) The limiting angle α_{min} is;

$$\alpha_{min} = 1.22\times\frac{\lambda}{D}$$

$$= 1.22\times\frac{589\times10^{-9}}{0.9\times10^{-2}} = \boxed{8\times10^{-5}\ rad.}$$

(b) For max. limit or resolution α'_{min}, the shortest wavelength of the visible spectrum should be used and it is 400 nm for violet colour light.

Thus;

$$\alpha'_{min} = 1.22\times\frac{\lambda}{D}$$

$$= 1.22\times\frac{400\times10^{-9}}{0.9\times10^{-2}} = \boxed{5.42\times10^{-5}\ rad.}$$

(P.T.O)

P.10.8 :

DATA. Magnification of telescope $= M = 5$
Distance between the lenses $= L = 24$ cm
Focal length of objective $= f_o = ?$
Focal length of eyepiece $= f_e = ?$

Sol. Using the rel;

$$L = f_o + f_e$$

$$24 \text{ cm} = f_o + f_e \quad \text{——— (1)}$$

Also;

$$M = f_o/f_e$$

$$5 = \frac{f_o}{f_e} \Longrightarrow f_o = 5f_e \quad \text{——— (2)}$$

Putting this value in eq.(1), we have;

$$24 = 5f_e + f_e$$

$$24 = 6 \cdot f_e$$

$$f_e = \frac{24}{6} = \boxed{4 \text{ cm}}$$

Now

$$f_o = 5f_e = 5 \times 4 = \boxed{20 \text{ cm}}$$

P.10.9 :

DATA. Angle of incidence $= \theta_c = 39^\circ$ (for glass)
Angle of incidence $= \theta_c = ?$ (for water)

Sol. Initially we will find the refractive index 'n' of glass light pipe using the rel;

$$n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 39^\circ} = \frac{1}{0.629} = 1.59 \quad \text{——— (1')}$$

Now for glass water interface using Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{——— (2')}$$

where n_1 = Refractive index for glass $= 1.59$
θ_1 = critical angle $= \theta_c$
$\theta_2 = 90^\circ$ (for total internal reflection)
n_2 = Refractive index for water $= 1.33$

Putting the values in eq.(2), we have

$$1.59 \sin\theta_1 = 1.33 \sin 90^\circ$$

$$\sin\theta_1 = \frac{1.33 \times 1}{1.59} = 0.84$$

$$\theta_1 = \sin^{-1}(0.84) = \boxed{57^\circ}$$

(P.T.O)

P.10.10 :

DATA. Refractive index of core $= n_1 = 1.6$

Refractive index of cladding $= n_2 = 1.4$

(a) critical angle $= \theta_c = ?$

(b) Max. angle of incidence $= \theta = ?$ (for air)

Sol.

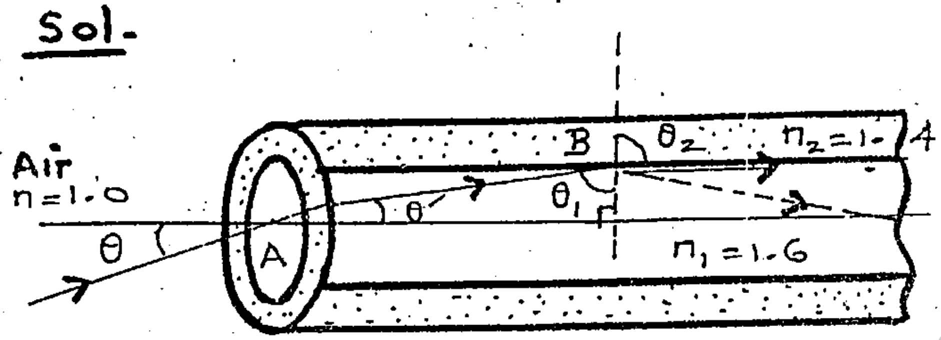

fig.

(a) Using Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{——— (1)} \quad \text{(Ref pt. 'B')}$$

when $n_1 = 1.6$, $n = 1.4$, $\theta_1 = \theta_c$ (the critical angle)

and $\theta_2 = 90^\circ$

So

$$1.6 \sin\theta_c = 1.4 \sin 90^\circ$$

$$\sin\theta_c = \frac{1.4 \times 1}{1.6} = 0.875$$

$$\theta_c = \sin^{-1}(0.875)$$

$$\theta_c = \theta_1 = \boxed{61^\circ \text{ (critical angle)}}$$

(b) From the fig.

$$\theta' = 90^\circ - \theta_c$$

$$\theta' = 90^\circ - 61^\circ = 29^\circ \quad \text{——— (2)}$$

Now for air-optical fibre core interface, using again Snell's law is;

$$n \sin\theta = n_1 \sin\theta' \qquad \because n = 1 \text{ for air (Ref pt. 'A')}$$

$$1 \times \sin\theta = 1.6 \times \sin 29^\circ$$

$$\sin\theta = 1.6 \times 0.485$$

$$\theta = \sin^{-1}(0.776)$$

$$\theta = \boxed{51^\circ}$$

If light beam is incident at the end of the optical fibre at an angle greater than 51°, total internal reflection would not take place.