HEAT

Heat

Heat is energy that flows between the system and its environment by the virtue of a temperature difference between them. Heat is measured in unit of joule or calorie.

Calorie

It is the amount of heat required to raise the temperature of 1 g of water from 14.5^{0}C to 15.5^{0}C.

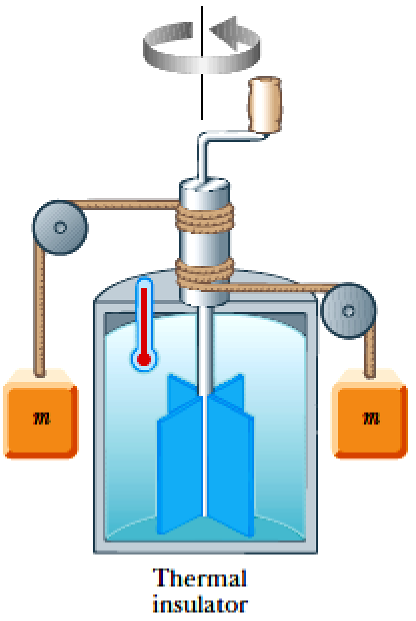
Mechanical Equivalent of Heat

It is defined as:

"The mechanical work required to produce unit quantity of heat"

It is found that 4.2 J of work produces 1 calorie of heat and 4200 J of work produces 1 kilo calorie of heat. It is denoted by J and its value is 4.2 J/Cal. Mathematically, it is described as:

$$J = \frac{W}{Q}$$



Joule's experiment for determining the mechanical equivalent of heat. The falling blocks rotate the paddles, causing the temperature of the water to increase.

Heat Capacity

It is the amount of heat required to raise the temper of substance through 1^{0}C. Mathematically, it is described as:

$$C' = \frac{dQ}{dT}$$

Its SI unit is J/^{0}C or J/K

Specific Capacity (Specific Heat)

It is the amount of heat required to raise the temperature of 1 kg of a substance through 1^{0}C. Mathematically, it is described as:

$$C = \frac{C'}{m}$$

$$C = \frac{1}{m}\frac{dQ}{dT}$$

Its SI unit is J/kg-^{0}C of J/kg-K.

Molar Heat Capacity

It is the amount of heat required to raise the temperature of one mole of a substance through 1^{0}C. Mathematically, it is described as:

$$C = \frac{C'}{n}$$

$$C = \frac{1}{n}\frac{dQ}{dT}$$

Heat of Transformation

The amount of heat per unit mass transferred during a phase change is called heat of transformation or Latent heat. It is denoted by L. Thus total heat transferred in a phase change is describe by the formula:

$$Q = mL$$

Where m is the mass of the sample.

Latent Heat of Fusion

The latent heat during melting or freezing is called latent heat of fusion. It is denoted by L_f.

Latent Heat of Vaporization

The latent heat during boiling or condensing is called latent heat of vaporization.

Internal Energy

It is the sum of all the energies of all the atoms, molecules or ions within a system.

Internal energy is a definite quantity which depends on the state of system only. So it is also called state function. The internal energy E of a system cannot be determined. However its change ΔE can be calculated. Change in internal energy depends only on the initial and final state and not on the path through which it takes place. Thus:

$$\Delta E = E_{final} - E_{initial}$$

Relation between C_P and C_V

Consider a cylinder containing n moles of an ideal gas fitted with a piston. Suppose the piston is fixed i.e., the volume of the gas is constant. When gas is heated at constant volume, all the heat supplied is used only to increase the internal energy of the gas and no work is done, because the volume is constant. So,

$$\Delta E_{int} = Q$$

But $Q = nC_V\Delta t$. Therefore,

$$\Delta E_{int} = nC_V\Delta t \quad \text{-----------} \quad (1)$$

Now consider the same cylinder that is heated at the same pressure i.e., piston is free to move. Now the heat supplied to the gas is given by:

$$Q = nC_P\Delta t \quad \text{-----------} \quad (2)$$

Now this heat supplied partly increases the internal energy and remaining heat is used to do work in moving the piston up:

$$Q = \Delta E_{int} + W$$

Since the work done on the gas during expansion is negative, so

$$Q = \Delta E_{int} - W$$

$$\Delta E_{int} = Q + W \quad \text{-----------} \quad (3)$$

As $W = -P\ \Delta V$

So $W = -nR\ \Delta T$ ----------- (4)

Putting vales of equation (1), (2), (4) in equation (3), we get:

$$nC_V\Delta t \quad = nC_P\Delta t \ \ - nR\,\Delta T$$

$$\Longrightarrow nC_P\Delta t \ \ - nC_V\Delta t \ = \ nR\,\Delta T$$

$$\Longrightarrow n\Delta t(\,C_P - C_V) \ = \ nR\,\Delta T$$

$$\Longrightarrow \ C_P - C_V \ = \ R$$

Thus the difference of C_P and C_V is equal to molar gas constant.

Special Cases

1. For mono-atomic gas:

$$C_V = \frac{3}{2}R$$

And $C_P = C_V + R$

$$\Longrightarrow \ C_P = \frac{3}{2}R + R = \frac{5}{2}R$$

2. For diatomic gas:

$$C_V = \frac{5}{2}R$$

And $C_P = C_V + R$

$$\Longrightarrow \ C_P = \frac{5}{2}R + R = \frac{7}{2}R$$

3. For polyatomic gas:

$$C_V = 3R$$

And $C_P = C_V + R$

$$\Longrightarrow \ C_P = 3R + R = 4R$$

First Law of Thermodynamics

In any thermodynamic process between two equilibrium states i and f, the quantity $Q + W$ has the same value for any path between i and f. This quantity is equal to the change in the value of a state function, called internal energy.

Mathematically, it is written as:

$$Q + W = \Delta E_{int}$$

This equation is known as the first law of thermodynamics.

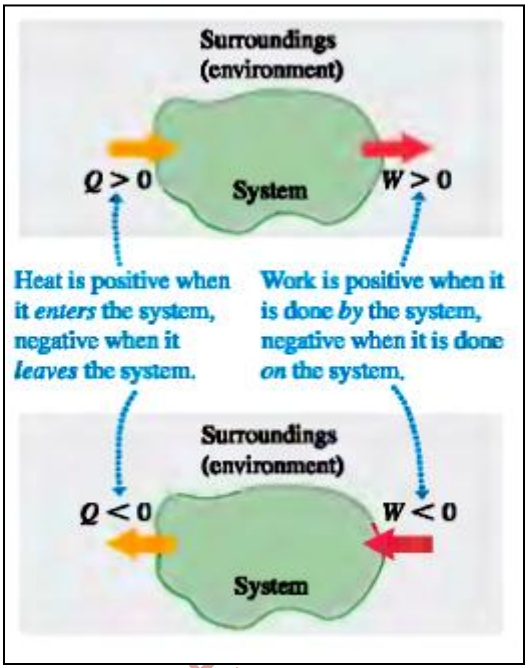
Explanation

To better understand these ideas on a quantitative basis, suppose that a system undergoes a change from an initial state to a final state. During this change, energy transfer by heat Q to the system occurs, and work W is done on the system. As an example, suppose that the system is a gas in which the pressure and volume change from P_i and V_i to P_f and V_f . If the quantity $Q \ + W$ is measured for various paths connecting the initial and final equilibrium states, we find that it is the same for all paths connecting the two states. We conclude that the quantity $Q \ + W$ is determined completely by the initial and final states of the system, and we call this quantity the change in the internal energy of the system. Although Q and W both depend on the path, the quantity $Q \ + W$ is independent of the path.

The mathematical form of 1^{st} law of thermodynamics describes:

- Heat Q absorbed by the system result in increase of internal energy.
- Work done on the gas increases the internal energy of the system.

So, the total increase of internal energy is due to heat absorbed plus work done on the gas. Similarly, when heat is lost by the system or work is done by the system on its surroundings, both Q and W decrease the internal energy of the system.



Applications of First Law of Thermodynamics

Adiabatic Process

It is the process in which no heat can enter or leave the system. This can be achieved by insulating the system completely so that $Q = 0$.

So the first law of thermodynamics becomes:

$$\Delta E_{int} = W$$

So in adiabatic process, the work done on the gas results in increase of internal energy of the gas.

Isothermal Process

It is the process in which temperature of the system remains constant.

So if temperature remains constant, then for ideal gas, the internal energy also remains constant i.e., ΔE_{int} . So the first law of thermodynamics becomes:

$$\Delta E_{int} = Q + W$$

$$\Rightarrow 0 = Q + W$$

Or

$$\Rightarrow Q + W = 0$$

$$\Rightarrow Q = -W$$

This shows that, during isothermal process, if the work W is done on gas, the gas rejects heat to the surroundings. So, the work done on the gas does not remain within the gas but flows out as heat to the surroundings.

Constant Volume Process

If the volume of the gas is kept constant, then no work can be done on the gas. So the first law of thermodynamics becomes:

$$\Delta E_{int} = Q + W$$

$$\Rightarrow \Delta E_{int} = Q + 0$$

Or

$$\Longrightarrow Q = \Delta E_{int}$$

All heat supplied goes to increase the internal energy of the gas.

Cyclic Process

It is a series of processes after which system goes to its initial state.

Figure shows a three step process. It is a cyclic process, because it starts and ends at the same point A. After one cycle, the chance in internal energy is zero i.e., $\Delta E_{int} = 0$.

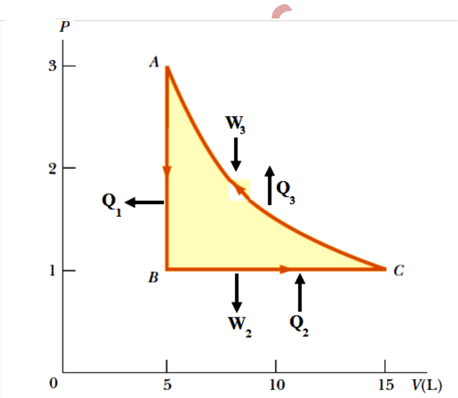
By the first law of thermodynamics:

$$\Delta E_{int} = Q + W$$

$$\Longrightarrow 0 = Q + W$$

or

$$\Longrightarrow Q + W = 0 \text{ (Cyclic Process)}$$



- The process 1 occurs at the constant volume along A-B
- Process 2 occurs at constant pressure along B-C
- Process 3 occurs at constant temperature along C-A.

The total work done is positive because area under the curve 3 (AC) is greater than curve 2 (BC).

Free Expansion

A process in which a gas goes from one side of container to the other half initially evacuated side is called free expansion process.

In this case the gas is initially on one side of the container and when stop cork is open, the gas enters into the evacuated half side. No weight are raised so no work is done. The container is insulated, so the process is adiabatic i.e., $Q = 0 = W$.

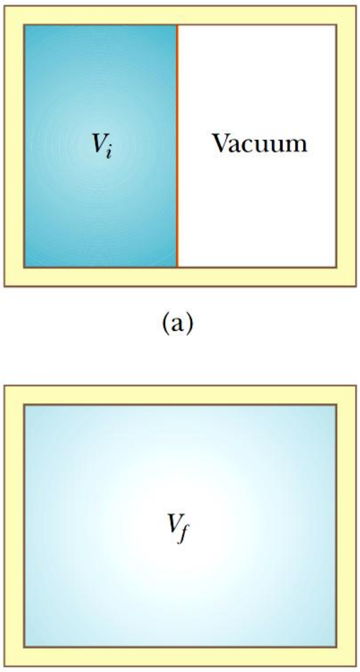
Thus, the first law of thermodynamics becomes:

$$\Delta E_{int} = 0 \quad \text{(Free Expansion)}$$

As the change in internal energy is zero, therefore

$$\Longrightarrow E_{int} = constant$$

So, during the free expansion, the internal energy of the ideal gas remains constant. As the internal energy of an ideal gas only depends upon temperature, so temperature must remain constant during free expansion.



(a)

(b)

In a free expansion, the gas is allowed to expand into a region that was previously evacuated.

Proof of $PV^\gamma = constant$ for an Adiabatic Process

It is the process in which no heat can enter or leave the system. This can be achieved by insulating the system completely so that $Q = 0$.

So the first law of thermodynamics becomes:

$$\Delta E_{int} = W$$

In differential form:

$$dE_{int} = dW \quad \text{-----------} \quad (1)$$

By definition:

$$C_V = \frac{1}{n}\left(\frac{dU}{dT}\right)$$

$$\Longrightarrow C_V = \frac{1}{n}\left(\frac{dE_{int}}{dT}\right) \qquad \therefore U = E_{int}$$

$$\Longrightarrow dE_{int} = nC_V dT \quad \text{-----------} \quad (2)$$

As the work done on the system of gas is described as:

$$dW = -P\,dV$$

$$\Longrightarrow P\,dV = -dW$$

$$\Longrightarrow P\,dV = -dE_{int} \qquad \therefore From\ equation\ (1)$$

$$\Longrightarrow P\,dV = -nC_V dT \quad \text{-----------} \quad (3)$$

From ideal gas equation:

$$PV = nRT$$

In differential form, it becomes:

$$d(PV) = d(nRT)$$

$$\Longrightarrow P\,dV + VdP = nR\,dT$$

$$\Longrightarrow VdP = nR\,dT - P\,dV$$

$$\Longrightarrow VdP = nR\,dT - (-nC_V dT)$$

$$\Longrightarrow VdP = nR\,dT + nC_V dT$$

But $nC_P dT = nR\,dT + nC_V dT$

$$\Longrightarrow VdP = nC_P dT \quad \text{-----------} \quad (4)$$

Dividing equation (3) and (4), we get:

$$\frac{VdP}{P\,dV} = -\frac{nC_P dT}{nC_V d}$$

$$\Longrightarrow \frac{VdP}{P\,dV} = -\frac{C_P}{C_V}$$

$$\Longrightarrow \frac{VdP}{P\,dV} = -\gamma$$

$$\Longrightarrow \frac{dP}{P} = -\gamma\frac{dV}{V}$$

Integrating both sides:

$$\int_{P_i}^{P_f} \frac{dP}{P} = -\gamma \int_{V_i}^{V_f} \frac{dV}{V}$$

$$|\ln\ P|_{P_i}^{P_f} = -\gamma |\ln V|_{V_i}^{V_f}$$

$$\Rightarrow \ln P_f - \ln P_i = -\gamma[\ln V_f - \ln V_i]$$

$$\Rightarrow \ln \frac{P_f}{P_i} = -\gamma \ln \frac{V_f}{V_i}$$

$$\Rightarrow \ln \frac{P_f}{P_i} = \ln \left(\frac{V_f}{V_i}\right)^{-\gamma}$$

$$\Rightarrow \ln \frac{P_f}{P_i} = \ln \left(\frac{V_i}{V_f}\right)^{\gamma}$$

$$\Rightarrow \frac{P_f}{P_i} = \frac{{V_i}^{\gamma}}{{V_f}^{\gamma}}$$

$$\Rightarrow P_i{V_i}^{\gamma} = P_f{V_f}^{\gamma}$$

Or $P\ V^{\gamma} = constant$

Show that adiabat is stepper than Isotherm

Adiabatic process is represented by the equation:

$$P\ V^{\gamma} = constant$$

Taking derivative on both sides:

$$d(P\ V^{\gamma}) = 0$$

$$\Rightarrow V^{\gamma} dP + P\ d(V^{\gamma}) = 0$$

$$\Rightarrow V^{\gamma} dP + P\ \gamma V^{\gamma-1} dV = 0$$

$$\Rightarrow V^{\gamma} dP = -P\ \gamma V^{\gamma-1} dV$$

$$\Rightarrow \frac{dP}{dV} = -\frac{\gamma P V^{\gamma-1}}{V^{\gamma}}$$

$$\Rightarrow \frac{dP}{dV} = -\gamma P V^{\gamma-1-\gamma} = -\gamma P V^{-1}$$

$$\Rightarrow \frac{dP}{dV} = -\gamma \frac{P}{V} \qquad ------- \quad (1)$$

Here $\frac{dP}{dV}$ gives the slope of adiabat.

Now isothermal process is described by the formula:

$$PV = constant$$

Taking differentials on both sides, we get:

$$d(PV) = 0$$

$$\Rightarrow V\ dP\ +\ P\ dV = 0$$

$$\Rightarrow V\ dP = -P\ dV$$

$$\Rightarrow \frac{dP}{dV} = -\frac{P}{V} \qquad ------- \quad (2)$$

Here $\frac{dP}{dV}$ gives the slope of isotherm

Thus from (1) and (2), it can be seen that adiabat is steeper than isotherm.

The Transfer of Heat

Heat energy can be transferred from one place to the other by three ways:

- Conduction
- Convection
- Radiation

Conduction

In process of conduction, the heat energy is transferred from one point to the other due to vibrations and collisions of molecules

The method of transfer of heat is prominent in solids (metallic solids) and small in liquids and negligible in gases.

If one end of rod is heated, the K.E of the molecules at the hot end is increased so they vibrate with larger amplitude. So they collide with neighboring molecules and give same energy to them. This process goes on and the energy is transferred from molecule to molecule down the rod. Thus it can be said that conduction of heat is due to vibration and collisions of molecules.

Factors on which conduction depends

Consider a rod of length L. the two ends of the rod are at temperature T_1 and T_2 respectively and the cross-section of the rod is A. as the two ends of the rod are at two different temperatures, the heat will flow from hot end to cold end through the rod. The heat which is conducted through the rod depends upon:

i. Area of cross-section A of the rod. Larger te area, greater will he the flow of heat i.e.,

$$Q \propto A$$

ii. The temperature difference:

The greater the temperature difference between the two ends, greater will be the flow of heat i.e.,

$$Q \propto (T_1 - T_2)$$

iii. The time of conduction:

More heat will flow in longer time i.e.,

$$Q \propto t$$

iv. The length of the rod:

Longer the rod, smaller will be the amount of heat across the rod i.e.,

$$Q \propto \frac{1}{L}$$

Combining all factors, we get:

$$Q \propto \frac{A(T_1 - T_2)t}{L}$$

$$\frac{Q}{t} \propto \frac{A(T_1 - T_2)}{L}$$

Where $\frac{Q}{t}$ is the rate of flow of heat denoted by H.

$$H \propto \frac{A(T_1 - T_2)}{L}$$

$$H = \frac{KA(T_1 - T_2)}{L}$$

Where K is called thermal conductivity. It depends upon the nature of the material of rod.

If T_1 and T_2 are denoted as T_H and T_L then

$$H = \frac{KA(T_H - T_L)}{L}$$

$$\Rightarrow H = \frac{A(T_H - T_L)}{\left({}^{L}/_{K}\right)}$$

$$\Rightarrow H = \frac{A(T_H - T_L)}{R}$$

Where $\frac{L}{K} = R$ is called thermal resistance or R-value. So for good thermal conductors, the R-value is small and vice versa.

If we have a small rod of length dx and temperature dT then rate of flow of heat is given as:

$$H = -KA\frac{dT}{dx}$$

The negative sign shows that heat flows in the direction of decreasing $\frac{dT}{dx}$, where $\frac{dT}{dx}$ is called temperature gradient.

Convection

It is the process in which heat is transferred from one place to another by actual movement of the molecule.

As the molecules can move only in liquids and gases (fluids), so fluids are heated by this process. In solids, heat can't be transferred by convection because molecules can't leave their position.

Factors

Transfer of heat by convection depends on:

i. Temperature difference between the source and the place to which heat is transferred
ii. Nature of the fluid
iii. Density, viscosity, specific heat and the thermal conductivity of the fluid
iv. Velocity of the fluid

Radiation

It is the process in which heat is transferred from one place to another place in the form of waves without a material medium.

Since there is no medium between the sun and the earth. Heat reaches the earth from sun by radiation. Heat waves are called thermal radiation. Thermal radiation belong to the electromagnetic spectrum of radiation. Both thermal radiation and light have the similar properties and obey identical laws.