Nasir Pervaiz Butt
M.Sc. (Physics) M. Phil
Assistant Professor
Govt. College Sargodha.
House No. 338 St. No. 7
Cheema Colony,
Sargodha.

Chapter-3

SOUND

1- Beats:

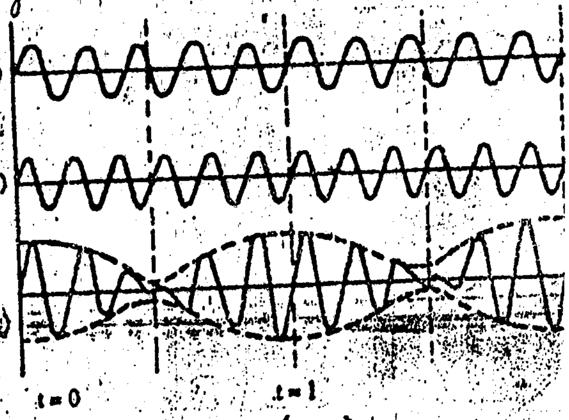
"The periodic alternations in the intensity of sound between minimum and maximum loudness are called beats."

The phenomenon of beats arises due to superposition of two sound waves of same amplitude but of slightly different frequencies.

When two sound waves of same amplitude and having slightly different frequencies superpose each other, they produce a resultant wave whose amplitude varies with time i.e the amplitude rises and falls periodically. These periodic changes in amplitude produce rises and falls in the intensity of sound which we call beats.

In this case we study the superposition of two waves of the same amplitude but of slightly different frequencies at a given point as a function of time.

Let us consider a point in space through which two sine waves are passing. Their frequencies are slightly different.



Due to superposition we get a resultant wave form.

The resultant pressure at a given point

equal to the sum of individual pressures as a function of

rom the fig. we see that the amplitude of the resultant wave is not constant but varies with time. In case of sound waves this variation in amplitude gives rise to variation in loudness called beats.

Let the variation in pressure with time produced by one wave at a given point is given by,

$$\Delta P_1(t) = \Delta P_m \sin \omega_1 t \quad -(i).$$

The variation in pressure produced by the other wave of equal amplitude at the same point is given by,

$$\Delta P_2(t) = \Delta P_m \sin \omega_2 t$$

For simplicity we have taken $\varphi_1 = \varphi_2 = 0$

$\therefore$ By the superposition principle, the resultant pressure is given by,

$$\Delta P(t) = \Delta P_1(t) + \Delta P_2(t).$$

$$\Delta P(t) = \Delta P_m \sin(\omega_1 t) + \Delta P_m \sin \omega_2 t.$$

$$= \Delta P_m (\sin \omega_1 t + \sin \omega_2 t)$$

By trigonometry, we have

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}.$$

$$\sin A + \sin B = 2 \cos \frac{A-B}{2} \sin \frac{A+B}{2}.$$

$$\therefore \Delta P(t) = \Delta P_m \left[2 \cos\left(\frac{\omega_1 t - \omega_2 t}{2}\right) . \sin\left(\frac{\omega_1 t + \omega_2 t}{2}\right)\right]$$

$$\Delta P(t) = 2 \Delta P_m \left[\cos \frac{(\omega_1 - \omega_2)}{2} t . \sin \frac{(\omega_1 + \omega_2)}{2} t \right]$$

Putting $\frac{\omega_1 - \omega_2}{2} = \omega_{amp}$ and $\frac{\omega_1 + \omega_2}{2} = \omega_{av}$.

$$\Delta P(t) = \left[2 \Delta P_m \cos \omega_{amp} t \right] \sin \omega_{av} t. \quad -(ii)$$

The eq. (ii) represents the resultant wave having frequency $\left(\frac{\omega_1 + \omega_2}{2}\right)$ and amplitude $2 \Delta P_m \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t$.

We find that the amplitude depends on time i.e max. value of amplitude $+2\Delta P_m$ and minimum value of amplitude $(-2\Delta P_m)$ both occur

not ... time. In ...

frequency $\left(\frac{\omega_1 - \omega_2}{2}\right)$.

Since intensity is proportional to square of amplitude, so the intensity is maximum both when amplitude is maximum or minimum. So intensity attains maximum value twice in one cycle.

So we conclude that number of beats is twice the number of cycles per second. Hence beat frequency is twice the amplitude frequency.

$$\therefore \quad \omega_{beat} = 2\,\omega_{amp}.$$

$$\omega_{beat} = 2\left(\frac{\omega_1 - \omega_2}{2}\right).$$

$$\omega_{beat} = \omega_1 - \omega_2.$$

As $\omega = 2\pi\nu$.

$$\therefore \quad 2\pi\,\nu_{beat} = 2\pi\nu_1 - 2\pi\nu_2.$$

$$2\pi\,\nu_{beat} = 2\pi(\nu_1 - \nu_2).$$

$$\boxed{\nu_{beat} = \nu_1 - \nu_2}$$

It shows that number of beats produced per second is equal to the difference of the frequencies of the two waves.

2. The Doppler Effect.

"The apparent change in the pitch of sound due to relative motion b/w source and observer is called Doppler Effect."

Consider a source of sound and an observer (listner). It is found that as the source approaches the listener, the pitch of sound is found to be greater than the actual pitch of sound. Similarly if the source moves away from the listner, the pitch is found to decrease. The same happens when source remains at rest and the observer moves towards or away from the source.

This apparent change in the pitch of sound due to motion of source, observer or both is called Doppler effect.

actual frequency of the source i.e it emits waves in one second. Let V be the speed of sound waves. i.e is the distance which the sound waves cover in one second.
If both source and observer are at rest, the first wave emitted by the source reaches the listner in one sec. So ν waves lie over a distance 'V'. The wave length of the sound waves is then given as

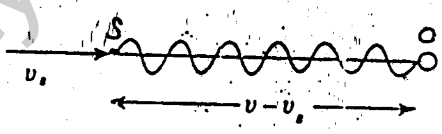
$$\lambda = \frac{V}{\nu}$$

Let us discuss the Doppler effect under the following cases.

Case-1 Source approaching observer at rest:

Suppose the source is moving towards a stationary listner with speed V_s.

When source and listner both are at rest, then ν waves lie over a a distance V as shown in fig 1(a).



But the source moves through distance 'V_s' towards the listner. Thus now ν waves lie over a distance $(V - V_s)$.
Now apparent wave length λ' is decreased and is given by,

$$\lambda' = \frac{V - V_s}{\nu}.$$

So apparent frequency ν' is given by

$$\nu' = \frac{V}{\lambda'}$$

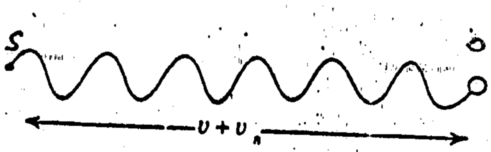
$$\nu' = \left(\frac{V}{V - V_s}\right)\nu \quad \text{--- (1)}$$

This shows that $\nu' > \nu$

So pitch is found to be greater in this case.

Case. 2- Source moves away from observer:

If source moves away from a stationary observer with speed V_s then ν waves lie over a distance $V + V_s$ as shown.
Now the apparent wavelength is given by



$$\lambda' = \frac{V + V_s}{\nu}$$

$\therefore$ the apparent frequency is given by

$$\nu' = \frac{V}{\lambda'}$$

$$\nu' = \frac{V}{\frac{V+V_s}{\nu}}$$

$$\nu' = \left(\frac{V}{V+V_s}\right)\nu \quad \text{— (2)}$$

This shows that $\nu' < \nu$

So pitch is found to be decreased.

Equations (1) and (2) are combined into a single relation as,

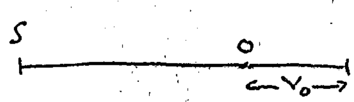
$$\nu' = \left(\frac{V}{V \mp V_s}\right)\nu$$

where $-$ve sign holds for motion towards the observer and the $+$ve sign holds for motion away from the observer.

It should be noted that apparent change in pitch is due to increase or decreases of wavelength due to motion of source.

Case-3 Observer approaching the source at rest

Suppose the observer is moving towards a stationary source with speed V_0. Now the observer recieves more waves in one second. the additional waves recieved by the observer are those which lie over the distance V_0.



So additional waves $= \frac{\nu V_0}{V}$.

These additional waves give additional frequency.

So apparent frequency is given by,

V — $= \nu$

1 — $= \frac{\nu}{V}$

V_0 — $= \frac{\nu V_0}{V}$

$$\nu'' = \nu + \frac{\nu}{V} V_0.$$

$$= \nu\left(1 + \frac{V_0}{V}\right)$$

$$\boxed{\nu'' = \nu\left(\frac{V+V_0}{V}\right)} \quad \text{— 3.}$$

This shows that $\nu'' > \nu$. So pitch is found to be greater.

Case-4. Observer moves away from the source at rest

When observer is moving away from a stationary source with speed V_0, then he recieves less waves in one second. The apparent frequency is given by,

$$\nu'' = \nu - \frac{\nu}{V} V_0$$

$$= \nu\left(1 - \frac{V_0}{V}\right)$$

$$\boxed{\nu'' = \left(\frac{V - V_0}{V}\right)\nu} \quad \text{— (4)}$$

This shows that $\nu'' <$. So pitch is found to be decreased.

Equations (3) and (4) can be combined into a single relation as

$$\nu'' = \left(\frac{V \pm V_0}{V}\right)\nu \quad \text{— (B)}$$

Where +ve sign holds for motion towards the source and −ve sign holds for motion away from the source.

It should be noted that apparent change in pitch takes place because the observer recieve more or less waves per second due to his motion.

Case-5 Both source and observer are in motion

Let us consider the case , both source and observer move towards each other.

As we know that when source moves towards or away from observer at rest then frequency is given by eq.-(A) as,

$$\nu' = \left(\frac{V}{V \mp V_s}\right)\nu$$

If observer also moves towards or away from source in motion then

$$\nu'' = \left(\frac{V \pm V_0}{V}\right)\nu'$$

Putting ν' from the above relation we get,

$$= \left(\frac{343+29}{343}\right)1125 = \frac{372}{343} \times 1125.$$

$$= \frac{372 \times 1125}{343}$$

$$\boxed{\nu' = 1220 \ Hz} \text{ Ans}$$

(c) $V_s = 14.5 \ m/s$, $V_o = 14.5 \ m/s$, $V = 343 \ m/s$, $\nu = 1125 \ Hz$

When source and observer are moving towards each other, then apparent frequency is given by,

$$\nu'' = \left(\frac{V+V_o}{V-V_s}\right)\nu$$

$$= \left(\frac{343+14.5}{343-14.5}\right) \times 1125$$

$$= \left(\frac{357.5}{328.5}\right) \times 1125.$$

$$\boxed{\nu'' = 1224 \ Hz} \text{ Ans.}$$

(d) $V_o = 9 \ m/s$, $V_s = 38 \ m/s$, $\nu = 1125 \ Hz$, $V = 343 \ m/s$.

observer is moving away from source and source is moving towards observer. Then the apparent frequency is given by,

$$\nu''' = \left(\frac{V-V_o}{V-V_s}\right)\nu$$

$$= \left(\frac{343-9}{343-38}\right) \times 1125$$

$$= \frac{334}{305} \times 1125$$

$$\boxed{\nu''' = 1232 \ Hz} \text{ Ans}$$

$$\nu'' = \left(\frac{V \pm V_o}{V}\right)\left(\frac{V}{V \mp V_s}\right)\nu$$

$$\nu'' = \left(\frac{V \pm V_o}{V \mp V_s}\right)\nu \quad \text{— (c)}$$

where $\nu'' = \left(\frac{V + V_o}{V - V_s}\right)\nu$ represents the case when both source and observer are moving towards each other and

$\nu'' = \left(\frac{V - V_o}{V + V_s}\right)\nu$ represents the case when both source and the observer are moving away from each other.

Sample Problem-5

The siren of a police car emits a pure tone at a frequency of 1125 Hz. Find the frequency you would perceive in your car under the following circumstances.

(a) Your car is at rest, police car moving toward you at 29 m/s (65 m/h).

(b) Police car at rest, your car is moving toward it at 29 m/s.

(c) You and the police car moving toward one another at 14.5 m/s.

(d) You are moving at 9 m/s, police car chasing behind you at 38 m/s.

Sol:- (a) $\nu = 1125$ Hz. $V_s = 29$ m/s, $V = 343$ m/s. (~~to~~ learn).

$\nu' = ?$

When source ~~is~~ moves towards observer at rest, then apparent frequency is given by

$$\nu' = \left(\frac{V}{V - V_s}\right)\nu$$

$$= \left(\frac{343}{343 - 29}\right) \times 1125$$

$$= \frac{343 \times 1125}{314}$$

$$\boxed{\nu' = 1229 \text{ Hz}} \quad \text{Ans.}$$

(b) $\nu = 1125$ Hz ; $V_o = 29$ m/s , $V = 343$ m/s , $\nu' = ?$

When observer is moving towards stationary source then apparent frequency is given by,

$$\nu'' = \left(\frac{V + V_o}{V}\right)\nu$$