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Chapter # 5

INTERFERENCE

Interference:

"The combined effect of two sets of waves on the particles of medium is called interference."

i.e when two similar waves travelling in the same medium in the same direction superpose at a point. The point is disturbed due to the resultant effect of the two waves. The intensity of the resultant wave at that point can be greater or less than the intensity of individual wave. The net wave effect at that point increases or decreases. This effect is called interference. The phenomenon of interference is a wave phenomenon.

Interference is of two types

(i) Constructive Interferene

If the two waves reach a point in phase i.e crest of one wave falls on the crest of the other wave and trough of one wave falls on the trough of the other wave then the two waves reinforce each other and the net wave effect increases i.e the net intensity is greater than the intensity of individual wave. Such an interference is called constructive interference.

(ii) Destructive Interference:

If the two waves reach a point out of phase i.e crest of one wave fa[lls on] the trough of the other, then the two waves cancel each others effect and so the net wave effect decrea[ses] net intensity of ~~individual~~ individual wave

...pe of interference is called "destructive interference".

1. Coherence of Sources:

The phenomenon of interference is a wave phenomenon. It takes place due to superposition of two exactly similar waves. It should be noted that the two waves interfere only if they have phase coherence i.e they come from two coherent sources.

"Two sources are said to be coherent sources if they produce waves of same amplitude, same frequency (wave length) having either no initial phase difference or a constant phase difference."

So coherent sources are those which produce exactly similar waves having either no initial phase difference or a constant phase difference. Coherence is a necessary condition for interference to take place. Two coherent sources undergo no change with time.

Coherence can be obtained for any type of waves. e.g

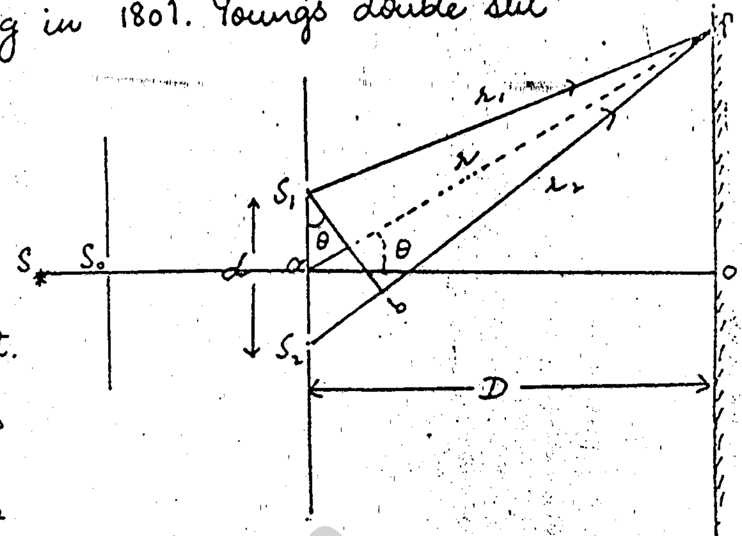
(i) Consider two rods in a water tank being moved up and down by a common source. They dip together to the same depth and also come out together. So they act as coherent sources and the water waves produced by them are coherent waves. i.e they have phase coherence.

(ii) As an example of two coherent sources of light, we can take two small holes or similar slits illuminated by a common source. Because two slits recieve light from the same source, so their phase difference remains constant.

(iii) Two coherent sound waves can be obtained by driving two different loud speakers from the same audio osscillator.

2. Young's Double Slit Experiment.

Hugen,s wave theory of light wave experimentally confirmed by a British Scientist Thomas Young in 1801. Young's double slit experiment is shown in fig.



S is a monochromatic source of light. S_0 is a rectangular slit illuminated by monochromatic light. This light is divided into two parts by slits S_1 and S_2. Now slits S_1 and S_2 act as coherent sources because they are obtained from a single source.

d is the distance between slits S_1 and S_2.

D is the distance of screen and slits S_1 and S_2.

D is >d. Point O is the central point of the screen. Since the waves from S_1 and S_2 cover equal distaces to point O. ∴ point O is a bright point due to constructive interference. Pt. O is called central maxima.

Let us take any point 'P' on the screen at a distance 'y' from the central point O.

r_1 and r_2 are the distances of point P from S_1 and S_2. The path difference between the waves arriving at P is.

$$\text{Path difference} = S_2P - S_1P.$$

To calculate this path difference, we draw a ⊥ S_1b on S_2P such that

$S_1P = bP$.

$$\therefore S_2P = S_2b + bP$$

$$S_2b = S_2P - bP.$$

Now $bP = S_1P$.

$$\therefore S_2b = S_2P - S_1P$$

$\therefore$ Path diff $= S_2 b$ — (1).

From the right angled triangle $\Delta S_1 b S_2$, we have,

$$\frac{S_2 b}{S_1 S_2} = \sin\theta. \qquad \because S_1 S_2 = d.$$

$$\frac{S_2 b}{d} = \sin\theta.$$

$$S_2 b = d \sin\theta.$$

$\therefore$ Equation (1) becomes

$$\boxed{\text{Path diff} = d\sin\theta} \quad \text{— (2).}$$

As $D >> d$ So $\sin\theta \approx \tan\theta$.

$\therefore$ Path diff $= d \tan\theta$.

But from right triangled ΔaOP; $\tan\theta = \frac{y}{D}$.

$$\boxed{\text{Path diff} = d\frac{y}{D}} \quad \text{— (3).}$$

Now for point P to be a bright point the path diff should be equal to integral multiple of λ.

i.e $\frac{dy}{D} = m\lambda$. $\quad m = 0, 1, 2, 3, \ldots$

or $y = \frac{m\lambda D}{d}$.

Where y is the distance of mth bright band from point O. For $m=0$ we get central maxima.

Now for point P tobe a dark point, the path difference should be equal to odd multiple of $\lambda/2$

i.e $\frac{dy}{D} = (2m+1)\frac{\lambda}{2}$.

or $y = (2m+1)\frac{\lambda D}{2d}$.

Where y is the distance of mth dark band from point O. For $m=0$ we get 1st dark band.

Sample Problem- 1

In the double slit experiment, illuminated with mercury vapour lamp, the strong green light of wavelength $\lambda = 546$ nm is visible. The slits are 0.12 mm apart and the screen is 55 cm away. What is the angular position of the first minimum? Of the 10th maximum?

Sol:

$$\lambda = 546\,nm = 546 \times 10^{-9}\,m.$$

$$d = 0.12\,mm = 0.12 \times 10^{-3}\,m.$$

$$D = 55\,cm = 55 \times 10^{-2}\,m.$$

(i) $\theta = ?$ for first minima.

For minima we have;

$$d \sin\theta = (2m+1)\frac{\lambda}{2}.$$

For first minima put $m = 0$

$$\therefore d \sin\theta = \frac{\lambda}{2}.$$

$$\sin\theta = \frac{\lambda}{2d}.$$

$$= \frac{546 \times 10^{-9}}{2 \times 0.12 \times 10^{-3}} = \frac{273}{12} \times 10^{-6}.$$

$$\sin\theta = 2275 \times 10^{-6} = .002275.$$

$$\sin\theta = .0023.$$

$$\theta = \sin^{-1} 0.0023.$$

$$\boxed{\theta = 0.13^\circ} \text{ Ans.}$$

(ii) $\theta = ?$ for 10th maxima.

For maxima we have,

$$d \sin\theta = m\lambda.$$

For 10th maxima put $m = 10$.

$$\therefore d \sin\theta = 10\lambda.$$

$$\sin\theta = \frac{10\lambda}{d}.$$

$$= \frac{10 \times 546 \times 10^{-9}}{0.12 \times 10^{-3}} = \frac{5460}{0.12} \times 10^{-6}.$$

$$\sin\theta = 4.55 \times 10^{-6}.$$

6.

$$\theta = \sin^{-1} 4.55 \times 10^{-6}.$$

$$\boxed{\theta = 2.6^\circ} \text{ Ans}$$

Sample Problem- 2

Sample Problem 2 What is the linear distance on screen C between the adjacent maxima m and $m+1$ of Sample Problem 1?

from S. Prob. 1

$\lambda = 546 \times 10^{-9} m$

$D = 55 \times 10^{-2} m$

$d = 0.12 \times 10^{-3} m.$

Sol:- Distance b/w maxima m and $m+1 = \Delta y = ?$

For mth maxima we have

$$y_m = m \frac{\lambda D}{d}$$

For $(m+1)$th maxima we have,

$$y_{m+1} = (m+1) \frac{\lambda D}{d}.$$

$$\therefore \Delta y = y_{m+1} - y_m$$

$$= (m+1) \frac{\lambda D}{d} - \frac{m \lambda D}{d}$$

$$= \frac{m \lambda D}{d} + \frac{\lambda D}{d} - \frac{m \lambda D}{d}.$$

$$= \frac{\lambda D}{d}$$

$$= \frac{546 \times 10^{-9} \times 55 \times 10^{-2}}{0.12 \times 10^{-3}} = \frac{546 \times 55}{0.12} \times 10^{-8}$$

$$= 2502.5 \times 10^{-6}$$

$$= 2.5025 \times 10^{-3} m.$$

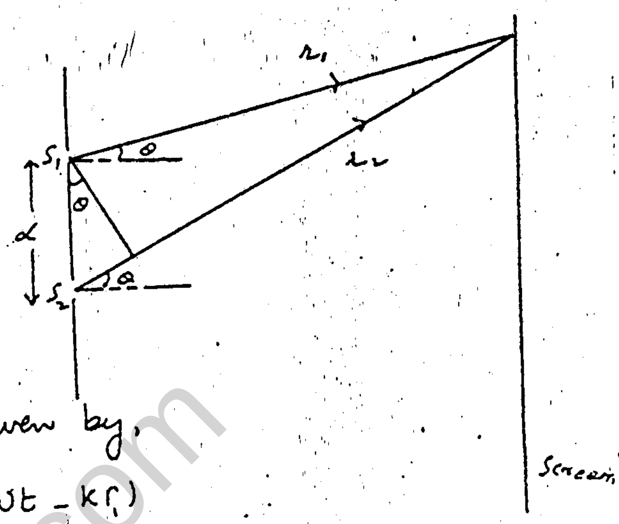
$$= 2.5 \times 10^{-3} m$$

$$\boxed{\Delta y = 2.5 mm.} \text{ Ans.}$$

x ——— x

3 Analytical Treatment of Interference Phenomenon:

Consider two coherent sources S_1 and S_2 giving waves of same angular frequency 'ω'. Let 'd' be the distance b/w S_1 and S_2. Suppose the distance of observation point P on the screen from S_1 be r_1 and r_2 from S_2. It is assumed that r_1 and r_2 are very large as compared to d.



Let the waves reaching P due to coherent sources be given by,

$$\psi_1 = A_1(r_1)\cos(\omega t - kr_1)$$
$$\psi_2 = A_2(r_2)\cos(\omega t - kr_2)$$

Where ψ_1 and ψ_2 are the displacements reaching 'P' and $A_1(r_1)$, $A_2(r_2)$ are the amplitudes reaching there. The total displacement at 'P' is obtained by their superposition

i.e $\psi = \psi_1 + \psi_2$.

$$\therefore \psi = A_1(r_1)\cos(\omega t - kr_1) + A_2(r_2)\cos(\omega t - kr_2)$$
$$= A_1(r_1)\cos\omega t\cos kr_1 + A_2(r_2)\sin\omega t\sin kr_1$$
$$+ A_2(r_2)\cos\omega t\cos kr_2 + A_2(r_2)\sin\omega t\sin kr_2.$$
$$\psi = (A_1(r_1)\cos kr_1 + A_2(r_2)\cos kr_2)\cos\omega t + (A_1(r_1)\sin kr_1 + A_2(r_2)\sin kr_2)\sin\omega t.$$

Let $A_1(r_1)\cos kr_1 + A_2(r_2)\cos kr_2 = A\cos\varphi$ — (i)

and $A_1(r_1)\sin kr_1 + A_2(r_2)\sin kr_2 = A\sin\phi$ — (ii).

Then $\psi = A\cos\varphi\cos\omega t + A\sin\varphi\sin\omega t$.

$$= A[\cos\varphi\cos\omega t + \sin\varphi\sin\omega t]$$

$$\therefore \psi = A\cos(\omega t - \varphi).$$

This equation represents harmonic motion.

The amplitude A is given by squaring and adding (i) and (ii).

$$A^2\cos^2\varphi + A^2\sin^2\varphi = A_1^2(r_1)\cos^2 k r_1 + A_2^2(r_2)\cos^2 k r_2 + 2A_1(r_1)A_2(r_2)\cos k r_1 \cos k r_2$$
$$+ A_1^2(r_1)\sin^2 k r_1 + A_2^2(r_2)\sin^2 k r_2 + 2A_1(r_1)A_2(r_2)\sin k r_1 \sin k r_2.$$

$$A^2(\cos^2\varphi + \sin^2\varphi) = A_1^2(r_1)(\cos^2 k r_1 + \sin^2 k r_1) + A_2^2(r_2)(\cos^2 k r_2 + \sin^2 k r_2)$$
$$+ 2A_1(r_1)A_2(r_2)(\cos k r_1 \cos k r_2 + \sin k r_1 \sin k r_2).$$

$$A^2 = A_1^2(r_1) + A_2^2(r_2) + 2A_1(r_1)A_2(r_2)\cos k(r_2 - r_1) \quad \text{— (iii)}.$$

The phase angle φ is given by dividing (ii) by (i).

$$\tan\varphi = \frac{A_1(r_1)\sin k r_1 + A_2(r_2)\sin k r_2}{A_1(r_1)\cos k r_1 + A_2(r_2)\cos k r_2}.$$

$$\therefore \varphi = \tan^{-1}\left(\frac{A_1(r_1)\sin k r_1 + A_2(r_2)\sin k r_2}{A_1(r_1)\cos k r_1 + A_2(r_2)\cos k r_2}\right).$$

Because $r_1 \approx r_2$ $\therefore A_1(r_1) = A_2(r_2) = A(r)$ where $r = \frac{r_1 + r_2}{2}$ is the mean distance of point 'P' from S_1 and S_2.

Then equation (iii) becomes

$$A^2 = A^2(r) + A^2(r) + 2A^2(r)\cos k(r_2 - r_1).$$
$$A^2 = 2A^2(r) + 2A^2(r)\cos k(r_2 - r_1).$$
$$= 2A^2(r)[1 + \cos k(r_2 - r_1)].$$

$$\because 1 + \cos\theta = 2\cos^2\theta/2.$$

$$\therefore A^2 = 2A^2(r)\left[2\cos^2\frac{k(r_2 - r_1)}{2}\right]$$

$$A^2 = 4A^2(r)\cos^2\frac{1}{2}k(r_2 - r_1) \quad \text{— (a)}.$$

As the intensity of light is propotional to the square of the amplitude, So equation (a) gives intensity at point 'P'.

Equation (a) can also be written as

$$A^2 = 4A^2(r)\cos^2\frac{1}{2}\Delta\varphi \quad \text{— (b)}.$$

Where $\Delta\varphi$ is the phase difference due to the path difference $(r_2 - r_1)$ and is given by

$$\Delta\varphi = k(r_2 - r_1).$$

Now $r_2 - r_1 = d \sin\theta$.

$$\therefore \quad \Delta\varphi = kd\sin\theta \qquad \because k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \Delta\varphi = \frac{2\pi}{\lambda} d\sin\theta.$$

$\therefore$ Phase difference $= \frac{2\pi}{\lambda} \times$ (Path difference)

$\therefore$ Eq. (b) becomes,

$$A^2 = 4A^2(r)\cos^2 \frac{1}{2}\, \frac{2\pi}{\lambda} d\sin\theta.$$

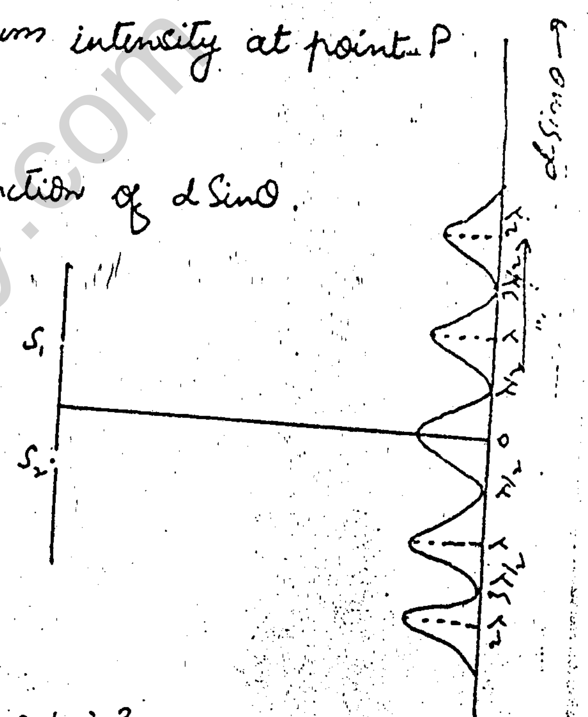
$$A^2 = 4A^2 r \cos^2\left(\frac{\pi}{\lambda} d\sin\theta\right)$$

The intensity at point 'P' is given by

$$I = I_{max} \cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right).$$

Where $I_{max} = 4A^2(r)$ is the maximum intensity at point P for $\theta = 0^\circ$

Fig. shows a plot of intensities as a function of $d\sin\theta$.



Condition of Maxima:

The intensity at P will be maximum when

$$\frac{1}{2}\Delta\varphi = 0, \pi, 2\pi, 3\pi, \ldots$$

$$\frac{1}{2}(kd\sin\theta) = n\pi.$$

$$\frac{1}{2}\,\frac{2\pi}{\lambda} d\sin\theta = n\pi.$$

$$\text{or} \quad d\sin\theta = n\lambda$$

where $n = 0, 1, 2, 3, \ldots$

$$d\sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots$$

Condition of Minima:

The intensity at P will be minimum when

$$\frac{1}{2}\Delta\varphi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\frac{1}{2}(kd\sin\theta) = (2n+1)\,\pi/2.$$

$$\frac{1}{2}\,\frac{2\pi}{\lambda} d\sin\theta = (2n+1)\frac{\pi}{2}.$$

or $d\sin\theta = (2n+1)\frac{\lambda}{2}$ where $n = 0, 1, 2, 3, \ldots$

$$\text{or} \quad d\sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

4. Adding of Electromagnetic Waves (Phasor Method)

We can obtain combined e' ic field of light in double slit experiment by algebraic meth. But this method becomes very difficult when we add more than two waves.

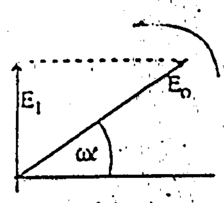
So to overcome this problem, the combined electric field is obtained by phasor method. A phasor is a rotating vector which gives time varying quantities.

A sinusoidal wave disturbance is represented by as,

$$E_1 = E_0 \sin \omega t \quad — \quad (1).$$

This wave can be represented by a rotating vector. For this purpose we take a phasor of magnitude E_0 and rotate it about the origin in anticlockwise direction with angular frequency ω.

Then the disturbance E_1 in equation (1) is represented by the projection of phasor along Y-axis as shown.



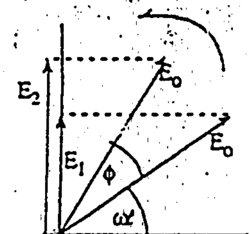
(a).

Now consider a IInd wave of same amplitude but having a phase difference ϕ given as,

$$E_2 = E_0 \sin(\omega t \quad — \quad (2).$$

It is represented by the projection of 2^{nd} phasor along Y-axis as shown,

Now the sum $E = E_1 + E_2$ is obtained by adding the projection of two phasors along y-axis.



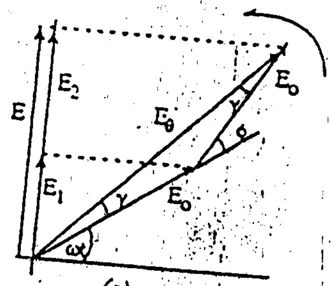
It can be explained more clearly if we redraw the phasors such that tail of one coincide, with the head of the other as shown.

From the fig(c) it is clear that sum

$E = E_1 + E_2$ is equal to projection of phasor of

amplitude E_0 along y-axis.
So from fig(c) we see that projection E can be written as

$$E = E_0 \sin(\omega t - \varphi) \quad (3)$$



(c)

Now in fig(c) the three phasors E_0, E_0 and E_θ form an isosceles Triangle.
Since by mathematics; the sum of two interior angles is equal to the exterior angle.

$$\therefore \varphi = \beta + \beta.$$
$$\therefore \varphi = 2\beta.$$
$$\text{or } \beta = \frac{\varphi}{2}.$$

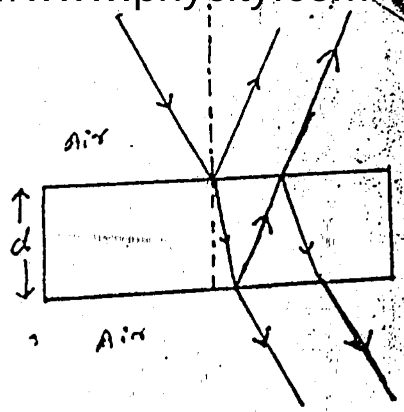
In fig.(c) the length of base of triangle is

$$E_\theta = E_0 \cos\beta + E_0 \cos\beta.$$
$$E_\theta = 2E_0 \cos\beta \quad (4)$$

Where E_θ is the amplitude of the resultant wave. The maximum possible value of the amplitude is $E_\theta = 2E_0$ i.e twice the amplitude of individual wave.

5. Interference in thin films:

Consider a thin film of uniform thickness 'd' and refractive index 'n'. Monochromatic light is made to fall on this film. Light is reflected from the upper and lower surfaces of the film. These rays I and II interfere to give interference fringes.



ray reflected from the lower surface has a longer path than the ray reflected from the upper surface. For normal incidence the ray reflected from the bottom covers a distance 2d greater than the upper ray. In this time the ray reflected from the upper surface undergoes a phase
It is due to the reason that when reflection takes place at the boundary of a denser medium, the crest is reflected as trough and vice versa.

$\therefore$ Equivalent path difference $= 2d - \frac{\lambda_n}{2}$ —(A).

Where λ_n is the wave length in the medium of refractive index n.
Now when light wave passes from one medium (air) to another water both the speed and wavelength change but frequency remains constant.

$\therefore \nu_{air} = \nu_{water}$. $\qquad \nu = \frac{c}{\lambda}$.

$\frac{c}{\lambda} = \frac{V}{\lambda_n}$ —(B).

where c = Velocity of light in air.
and V = Velocity of light in thin film.
λ = Wavelength of light in air.
λ_n = Wavelength of light in thin film.

$\therefore$ From (B) we can write

$\frac{\lambda}{\lambda_n} = \frac{c}{V}$. $\qquad$ But $\frac{c}{V} = n$ (index of refraction).

$\therefore \frac{\lambda}{\lambda_n} = n$.

or $\lambda_n = \frac{\lambda}{n}$.

$\therefore$ Equivalent path difference in (A) becomes

$$\boxed{\text{Equivalent P.d} = 2d - \frac{\lambda}{2n}}$$

$\therefore$ For maxima;

$$\text{Path diff} = m\lambda_n .$$

$$\therefore \quad 2d - \frac{\lambda}{2n} = \frac{m\lambda}{n} .$$

$$2d = \frac{\lambda}{2n} + \frac{m\lambda}{n}$$

$$\text{or } 2d = \frac{1}{n}\left(\frac{\lambda}{2} + m\lambda\right)$$

$$2dn = \left(m + \frac{1}{2}\right)\lambda .$$

$$\text{or} \quad 2dn = (2m+1)\frac{\lambda}{2} \quad \text{---} \; (1).$$

This is the condition for constructive interference. where $m = 0, 1, 2, \ldots$

Now for minima;

$$\text{Path diff} = (2m-1)\frac{\lambda_n}{2} .$$

$$2d - \frac{\lambda}{2n} = (2m-1)\frac{\lambda}{2n} .$$

Multiplying by n on both sides; we get

$$2dn - \frac{\lambda}{2} = (2m-1)\frac{\lambda}{2} .$$

$$2dn = (2m-1)\frac{\lambda}{2} + \frac{\lambda}{2} .$$

$$= \frac{\lambda}{2}(2m - 1 + 1)$$

$$= \frac{\lambda}{2}(2m) .$$

$$2dn = m\lambda \quad \text{---} \; (2). \quad m = 1, 2, 3, \ldots$$

This is the condition of destructive interference. These conditions given by (1) and (2) hold only when media on both sides of the film are same.

Sample Problem-4

A water film ($n = 1.33$) in air is 320 nm thick. If it is illuminated with white light at normal incidence, what colour will it appear to be in reflected light?

Sol:- $n = 1.33$ $d = 320nm.$

for maxima

$$2dn = (2m+1)\frac{\lambda}{2}$$

$$\lambda = \frac{4dn}{2m+1}$$

Putting $m = 0, 1, 2 \ldots$

$$\lambda_0 = \frac{4 \times 320 \times 1.33}{1} = 1702nm \quad (m=0)$$

$$\lambda_1 = \frac{4 \times 320 \times 1.33}{3} = \boxed{567nm} \quad (m=1)$$

$$\lambda_2 = \frac{4 \times 320 \times 1.33}{5} = 340nm \quad m=2.$$

Similarly for minima

$$2dn = m\lambda.$$

$$\lambda = \frac{2dn}{m}.$$

$$\lambda_1 = \frac{2 \times 320 \times 1.33}{1} = 851nm \quad (m=1)$$

$$\lambda_2 = \frac{2 \times 320 \times 1.33}{2} = 426nm \quad (m=2)$$

Only the maxima for $m=1$ lies in the visible region ($400nm \rightarrow 700nm$). Yellow-green light of wavelength $\lambda = 567nm$ will be enhanced by reflected light.

x———x

Sample Problem-5

Lenses are often coated with thin films of transparent substances such as MgF_2($n = 1.38$) to reduce reflection from the glass surface. How thick a coating is needed to produce a minimum reflection at the centre of the visible spectrum? ($\lambda = 550$ nm)

Sol:- $n = 1.38$ (MgF_2), $d = ?$, $\lambda = 550nm.$

Suppose that rays are falling normally.

ie path difference b/w the two rays depends on thickness of air film at the point of incidence. It is equal to twice the thickness of air film at this point. Since the points of equal thickness lie in a circle. So the points of equal. Path difference lie in concentric circles. Thus the fringes obtained in this case are circular. These circular fringes are called "Newton's rings".

The ray of light reflected from the lower surface of the air film (upper surface of glass plate) undergoes a phase change of π or a path difference $\frac{\lambda}{2}$ due to its reflection from denser medium.

So for maxima i.e for constructive interference,

$$2dn = (2m+1)\frac{\lambda}{2}. \qquad \because n = 1 \text{ for air.}$$

$$\therefore 2d = (2m+1)\frac{\lambda}{2}. \quad — (1).$$

and for minima i.e for destructive interference,

$$2d = m\lambda \quad — (2).$$

where $m = 0, 1, 2, 3, \ldots$

$n = 1$ for air.

Let R be the radius of curvature of the convex surface and 'r' the radius of the fringe. Then the thickness of air film is given by,

From the figure

$$\because OB = R.$$

$$R^2 - r^2 = (OA)^2$$

$$\sqrt{R^2 - r^2} = OA.$$

$$d = OB - OA$$

$$d = R - OA.$$

$$d = R - \sqrt{R^2 - r^2}$$

$$d = R - (R^2 - r^2)^{1/2}$$

$$= R - R\left(1 - \frac{r^2}{R^2}\right)^{1/2}.$$

$$= R - R\left(1 - \frac{1}{2}\frac{r^2}{R^2} + \ldots\right)$$

$$= R - R\left(1 - \frac{r^2}{2R^2} + \text{neglegible terms}\right)$$

$$= R - R\left(1 - \frac{1}{2}\cdot\frac{r^2}{R^2}\right).$$

$$= R - R + \frac{r^2}{2R}$$

$$d = \frac{1}{2}\frac{r^2}{R}$$

$$\therefore \quad d = \frac{r^2}{2R}$$

For maxima;

Putting the value of 'd' in ① we get

$$2\left(\frac{r^2}{2R}\right) = (2m+1)\frac{\lambda}{2}.$$

$$r^2 = (2m+1)\frac{\lambda R}{2}$$

$$r^2 = \left(\frac{2m+1}{2}\right)\lambda R$$

$$r^2 = \left(m+\frac{1}{2}\right)\lambda R$$

$$\therefore \quad r = \sqrt{m+\tfrac{1}{2}\,\lambda R}$$

This equation gives the radii of bright rings.

For Minima :-

Putting the value of 'd' in ② we get,

$$2\left(\frac{r^2}{2R}\right) = m\lambda.$$

$$r^2 = m\lambda R;$$

$$\therefore \quad r = \sqrt{m\lambda R}$$

This equation gives the radii of dark rings.

So if λ is known. The value of 'R' can be determined by experimentally measuring 'r'. Conversely we can find λ if we know r and R.

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7. Michelson's Interferometer:

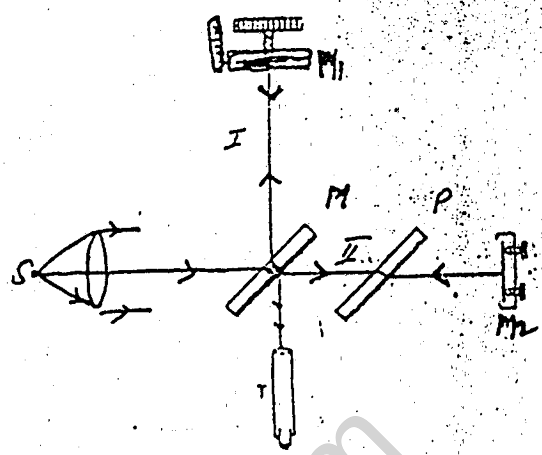
It is a device used to produce interference and also makes certain measurements.

e.g (i) It can be used to measure wavelength of light.

(ii) It can be used to measure Thickness of film.

Construction: It consists of semisilvered glass plate M called

...ter. This plate is semi silvered at the back. A glass plate P having the same thickness as that of M. Two plane mirrors M_1 and M_2 are placed ⊥ to each other. The mirror M_1 is mounted on a carriage and can be moved parallel to itself with the help of micrometer screw.



The plane mirror M_1 and M_2 are provided with levelling screws at their back with the help of which they can be made exactly ⊥ to direction of two beams. The interference bands observed in the telescope T. Fig. shows the arrangement.

Working: Light from a monochromatic source S falls on a semi silvered glass plate M placed at an angle of $\frac{\pi}{4}$ to the incident light. This plate splits up the light into two equal parts due to reflection and transmission. The two parts I and II act as coherent sources because they are derived from the same source 'S'. Ray I is reflected and Ray II is transmitted. A glass plate P having the same thickness as that of M is placed in the path of ray II because ray I passes through the glass thrice while ray II passes only once. So to keep the optical paths equal the plate P is placed in the path of ray II. This IInd plate is called compensating plate. Ray I after reflection from M_1 and transmission through M enters the telescope. The plane mirrors are placed ⊥ to the rays so that they retrace their path. In this way they superpose b/w M and T to produce interference.

Whether we observe a bright or dark band, it depends upon the path difference of two rays.

Now suppose the path difference b/w two rays is zero i.e The two rays cover equal distances before entering the Telescope.
Then they interfere constructively and we observe zeroth bright band.
To change the path difference the mirror M_1 is kept movable and M_2 is fixed.
The following measurements can be made with the interferometer

(i) Measurement of wavelength (λ):

For measuring the wavelength the source of light is monochromatic and M_1 is $\perp$ to M_2. So the fringes obtained are circular.
Now we move the mirror M_1, the fringes go on passing before the eye.
Suppose we are observing zeroth bright band. To see the next bright band, we have to produce a path difference of λ. So we move the mirror M_1 through a distance of $\frac{\lambda}{2}$ because ray II has to cover it twice. So zeroth bright band is replaced by 1st bright band when M_1 is moved through a distance of $\frac{\lambda}{2}$.
So by moving the mirror M_1 through a distance of $\frac{\lambda}{2}$ each time, we find that a bright band is replaced by the next bright band.
Suppose n bright fringes pass before the eye when M_1 is moved through a distance d.

Then

$$d = n\frac{\lambda}{2}$$

$$\lambda = \frac{2d}{n}$$

So knowing d, n, λ can be determined.

(ii) Measurement of thickness:

The plate whose thickness is required is placed in the path of one of the rays. If t is the thickness and [illegible] index of refraction, then path difference introduced $= (n-1)t$.

If M_1 is moved through a distance 'd' then $d = (n-1)t$.
So knowing 'd' and 'n', we can determine the thickness 't' of the plate.

Sample Problem - 7

Yellow light (λ = 589 nm) illuminates a Michelson-interferometer. How many bright fringes will be counted as the mirror is moved through 1.00 cm?

Sol. $\lambda = 589\,nm = 5.89 \times 10^{-9}\,m$, $m = ?$, $P = 1cm = 10^{-2}\,m$.

As we know that

$$P = m\frac{\lambda}{2}.$$

$$m = \frac{2P}{\lambda}$$

$$= \frac{2 \times 10^{-2}}{5.89 \times 10^{-9}} = 3.395 \cdot 10^{-3} \times 10^{7}.$$

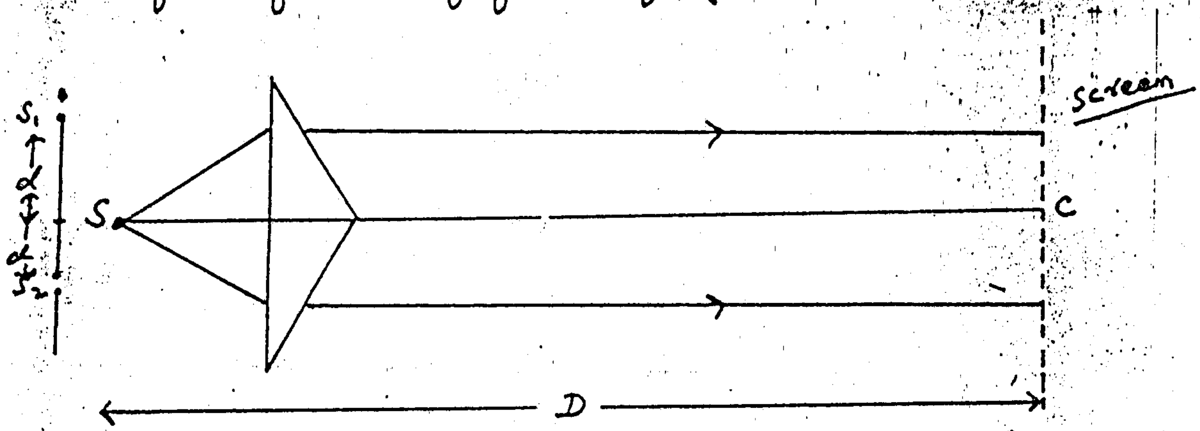
$$= 3.3956 \times 10^{4}.$$

$$\boxed{m = 33956 \text{ fringes}}$$ Ans.

8. Fresnel's Biprism & its use to determine wavelength of light:

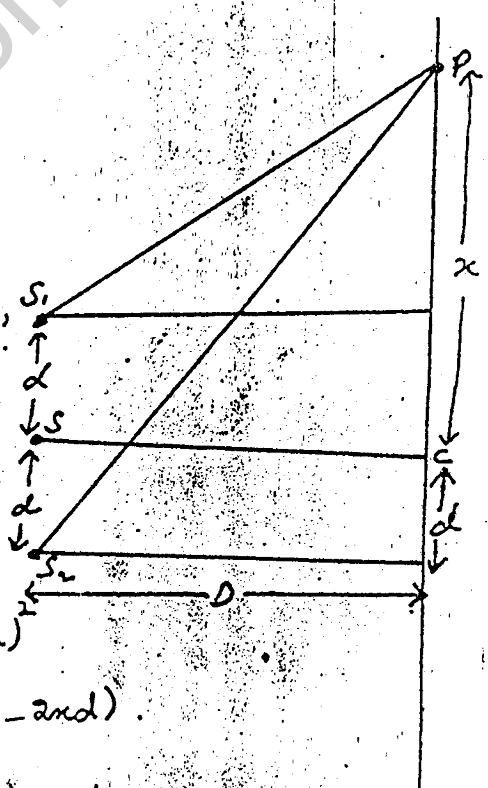
A biprism is a prism whose one angle is little less than 180° and the other two angles are small and equal to each other. It is a combination of two glass prisms with their bases in contact i.e it acts as two prisms placed base to base. It is the best apparatus to produce interference fringes.
Light from a narrow slit ⊥ to the plane of paper and illuminated by a monochromatic source is refracted from the biprism. Such that two virtual images S_1 and S_2 of the slit S are formed. So light appears to be coming from virtual images S_1 and S_2.

These virtual images act as two coherent sources. If a screen is placed infront of them, interference fringes can be obtained.



Let 2d be the distance b/w the two two coherent sources S_1 and S_2. Suppose the screen is placed at a distance D from the slit S. Draw ⊥ SC on the screen. Then C is equidistant from S_1 and S_2. Hence 'C' will be point of maximum intensity.

Let us consider any point P on the screen distant x from C. Join S_1P and S_2P. The two waves have a path difference $S_2P - S_1P$ in reaching 'P'.



Now from fig.

$$(S_2P)^2 = D^2 + (x+d)^2$$

$$(S_1P)^2 = D^2 + (x-d)^2$$

$$(S_2P)^2 - (S_1P)^2 = D^2 (x+d)^2 - (x-d)^2$$

$$= x^2 + d^2 + 2xd - (x^2 + d^2 - 2xd).$$

$$= x^2 + d^2 + 2xd - x^2 - d^2 + 2xd.$$

$$= 4xd.$$

$$(S_2P - S_1P)(S_2P + S_1P) = 4xd.$$

Now $S_2P \simeq S_1P = D$. (for field approximation)

$$\therefore (S_2P - S_1P)(D + D) = 4xd.$$

$$(S_2P - S_1P)\ 2D = 4xd.$$

$$S_2P - S_1P = \frac{4xd}{2D} = \frac{2xd}{D}.$$

Condition for maxima:

The point 'P' will be a bright band if

$$\frac{2nd}{D} = n\lambda \qquad \text{where } n = 0,1,2,3 \ldots$$

$$\text{So } x = \frac{n\lambda D}{2d}$$

This is the distance of nth bright band from 'c'. The distance of $(n-1)$th bright band from 'c' is

$$x' = \frac{(n-1)\lambda D}{2d}.$$

Fringe width "w"

The fringe width or fringe spacing 'w' is defined as,

"The distance between two consecutive bright or dark bands".

So 'w' is given as,

$$w = x - x'$$

$$= \frac{n\lambda D}{2d} - \frac{(n-1)\lambda D}{2d}$$

$$= \frac{\lambda D}{2d}\left[n - (n-1)\right] = \frac{\lambda D}{2d}(n - n + 1)$$

$$\therefore w = \frac{\lambda D}{2d}$$

$$\text{So wavelength } \lambda = \frac{2dw}{D}$$

So knowing w, d and D, we can determine the unknown wavelength λ.

The End.