

Chapter - 25

1.

Basic Physics, Part

HEAT

1 Fundamental Concepts

(i) Heat:

"Heat is a form of energy associated with molecular motion."

or

"Heat is energy that flows b/w a system and its environment by virtue of a temp. difference. b/w them."

Unit of Heat

(i) Joule (ii) Calorie (iii) BTU

Calorie:

It is the amount of heat required to raise the temp. of 1g of water from 14.5°C to 15.5°C.

$$1 \text{ Cal} = 4.186 \text{ J.}$$

$$1 \text{ Btu} = 1055 \text{ J.}$$

Mechanical Equivalent of Heat:

It is defined as;

"The mechanical work required to produce unit quantity of heat."

It is found that 4.2 Joule of work produces 1 Calorie of heat and 4200 J work produces 1 Kilo Calorie of heat. It is denoted by J. So its value is 4.2 J/cal or 4200 J/K Cal.

It is expressed as

$$J = \frac{W}{Q}$$

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Heat Capacity

It is the amount of heat required to raise the temp. of a substance (system) through 1°C .

Mathematically it is written as

$$C = \frac{dQ}{dT}$$

Its S.I unit is $\text{J/}^{\circ}\text{C}$ or J/K .

It is characteristic of a particular object.

Specific Capacity (Specific Heat)

It is the amount of heat required to raise the temp. of 1 kg of a substance through 1°C . OR It is the heat capacity per unit mass.

Mathematically it is written as

$$c = \frac{C}{m}$$

$$c = \frac{1}{m} \frac{dQ}{dT}$$

Its S.I unit is $\text{J/kg}^{\circ}\text{C}$ or $\text{J/kg}\cdot\text{K}$.

Molar Heat Capacity (Molar Specific Heat)

It is the amount of heat required to raise the temp. of one mole of a substance through 1°C .

Mathematically it is written as

$$C_m = \frac{C}{n} = \frac{1}{n} \frac{dQ}{dT}$$

Its S.I unit is $\text{J/mole}^{\circ}\text{C}$ or $\text{J/mole}\cdot\text{K}$.

Heat Capacity (at constant Pressure)

It is the amount of heat required to raise the temp. of a gas through 1°C at const. pressure. It is given as

$$C_p = \left(\frac{dQ}{dT} \right)_p = \left(\frac{dU + PdV}{dT} \right)_p$$

Molar Heat Capacity (at const. Pressure)

It is the amount of heat required to raise the temperature of one mole of a gas through 1°C at const. pressure.

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It is given as:

$$C_p = \frac{C_p}{n}$$

$$= \frac{1}{n} \left(\frac{dQ}{dT} \right)_p = \frac{1}{n} \left(\frac{du + PdV}{dT} \right)_p$$

Heat Capacity at constant Volume:

It is the amount of heat required to raise the temp. of one mole of a gas, through 1°C at const volume.

It is given as

$$C'_v = \left(\frac{dQ}{dT} \right)_v = \left(\frac{du + PdV}{dT} \right)_v$$

$$\therefore C'_v = \left(\frac{du}{dT} \right)_v = \left(\frac{\partial u}{\partial T} \right)_v$$

Molar Heat Capacity (at constant vol.)

It is the amount of heat required to raise the temp. of one mole of a gas through 1°C at const volume. It is given as

$$C_v = \frac{C'_v}{n}$$

$$= \frac{1}{n} \left(\frac{dQ}{dT} \right)_v = \frac{1}{n} \left(\frac{du + PdV}{dT} \right)_v$$

$$\therefore dV = 0 \therefore PdV = 0.$$

$$\therefore C_v = \frac{1}{n} \left(\frac{du}{dT} \right)_v = \frac{1}{n} \left(\frac{\partial u}{\partial T} \right)_v$$

Heat of Transformation (Latent heat)

The amount of heat per unit mass transferred during a phase change is called the heat of transformation or latent heat. It is denoted by L.

The total heat transferred in a phase change is then

$$Q = mL$$

where 'm' is the mass of sample that changes phase

$$\frac{1 \text{ kg}}{\text{m}^3} = \frac{L}{\text{mL}}$$

$$Q = mL$$

Latent Heat of Fusion (L_f):

"The latent heat during melting or freezing is called latent heat of fusion." It is denoted by L_f .

Latent Heat of Vaporization (L_v):

"The latent heat during boiling or condensing is called latent heat of vaporization." It is denoted by L_v .

Internal Energy:

"It is the sum of all the energies of all the atoms, molecules or ions within a system."

Internal energy is a definite quantity which depends on the state of system only. So it is also called state function. The internal energy E of a system cannot be determined. However its change ΔE can be calculated.

Change in internal energy depends only on the initial and final state and not on the path through which takes place.

Internal energy E of a system is also a state function and depends only on the initial and final states of the system.

$$\text{Thus } \Delta E = E_{\text{final}} - E_{\text{initial}}$$

Relation b/w " C_p " and " C_v "

Consider a cylinder containing n moles of an ideal gas fitted with a piston. Suppose the piston is fixed. So that volume is const.

When gas is heated at const volume, all the heat supplied is used only to increase the internal energy of the gas and no work is done because volume is const.

$$\text{So } \Delta E_{\text{int}} = Q \quad (i)$$

$$\text{But } Q = nC_v \Delta T \quad (ii)$$

From (i) and (ii),

$$\Delta E_{\text{int}} = nC_v \Delta T \quad (A)$$

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Now consider the same cylinder heat at const pressure i.e. piston is free to move. Now the heat supplied to the gas is given by

$$Q = nC_p \Delta T \quad (B)$$

Now this heat supplied partly increases the internal energy and the remaining heat is used to do work in moving the piston up

$$\therefore Q = \Delta E_{int} + W$$

Because the change in internal energy is same both at const volume and at const pressure.

Now since work done on the gas during expansion is -ve

$$\therefore Q = \Delta E_{int} - W$$

$$\text{or } \Delta E_{int} = Q + W \quad (C)$$

$$\text{Now } W = -P \Delta V$$

$$W = -nR \Delta T \quad (D)$$

\therefore Putting (A), (B) and (D) in (C) we get

$$nC_v \Delta T = nC_p \Delta T - nR \Delta T$$

$$nC_p \Delta T - nC_v \Delta T = nR \Delta T$$

$$n \Delta T (C_p - C_v) = nR \Delta T$$

$$\therefore \boxed{C_p - C_v = R}$$

Thus difference of C_p and C_v is equal to molar gas const

(i) For monoatomic gas

$$C_v = \frac{3}{2} R$$

$$\therefore C_p = C_v + R$$

$$C_p = \frac{3}{2} R + R = \frac{5}{2} R$$

(ii) For diatomic gas $C_v = \frac{5}{2} R$

$$\therefore C_p = C_v + R = \frac{5}{2} R + R = \frac{7}{2} R$$

(iii) For polyatomic gas

$$C_v = 3R$$

$$\therefore C_p = C_v + R = 3R + R = 4R$$

Sample Problem-1

Sample Problem 1 A copper sample whose mass m_c is 75 g is heated in a laboratory oven to a temperature T_c of 312°C. The copper is then dropped into a glass beaker containing a mass m_w (= 220 g) of water. The effective heat capacity C_b of the beaker is 190 J/K. The initial temperature T_w of the water and the beaker is 12.0°C. What is the common final temperature T_f of the copper, the beaker, and the water?

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Solution

Hot Cu.

$$m_c = 75g = 0.075kg$$

$$C_c = 387 J/kgK$$

$$T_i = 312^\circ C = 585K$$

$$T_f = ?$$

$$Q_1 = C_c m_c \Delta T$$

$$= 387 \times 0.075 (T_i - T_f)$$

$$Q_1 = 387 \times 0.075 (585 - T_f)$$

glass beaker

$mgC_g =$ effective heat capacity of beaker

$$\therefore C'_b = mgC_g$$

$$C'_b = 190 J/K$$

$$T_i = 12^\circ C = 285K$$

$$T_f = ?$$

$$Q_{g1} = C'_b (T_f - T_i)$$

$$Q_{g1} = 190 (T_f - 285)$$

Cold Water

$$m_w = 220g = 0.22kg$$

$$C_w = 4190 J/kgK$$

$$T_i = 285K$$

$$T_f = ?$$

$$Q_{g2} = C_w m_w \Delta T$$

$$= 4190 \times 0.22 (T_f - T_i)$$

$$Q_{g2} = 4190 \times 0.22 (T_f - 285)$$

According to law of heat exchange

$$Q_{lost} = Q_{gained}$$

$$Q_1 = Q_{g1} + Q_{g2}$$

$$387 \times 0.075 (585 - T_f) = 190 (T_f - 285) + 4190 \times 0.22 (T_f - 285)$$

$$29.025 (585 - T_f) = 190 (T_f - 285) + 921.8 (T_f - 285)$$

$$29.025 \times 585 - 29.025 T_f = 190 T_f - 190 \times 285 + 921.8 T_f - 921.8 \times 285$$

$$16979.625 - 29.025 T_f = 190 T_f - 54150 + 921.8 T_f - 262713$$

$$333892.625 = 1140.825 T_f$$

$$T_f = 292.63 K$$

$$t_f = 292.63 - 273$$

$$t_f = 19.6^\circ C \quad \text{Ans.}$$

Sample Problem 2

Sample Problem 2 A family enters a winter vacation cabin that has been unheated for such a long time that the interior temperature is the same as the outside temperature ($0^\circ C$). The cabin consists of a single room of floor area 6 m by 4 m and height 3 m. The room contains one 2-kW electric heater. Assuming that the room is perfectly airtight and that all the heat from the electric heater is absorbed by the air, none escaping through the windows or furniture, how long after the heater is turned on will the air temperature reach the comfort level of $21^\circ C$ ($70^\circ F$)?

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Solution

$$\Delta T = 21^\circ\text{C} = 21\text{K}$$

$$\text{Volume of room} = 6\text{m} \times 4\text{m} \times 3\text{m} = 72\text{m}^3$$

$$V = 72000\text{ litre}$$

$$1\text{ lit} = 10^{-3}\text{m}^3$$

$$10^3\text{ lit} = 1\text{m}^3$$

$$\text{Power of heater} = 2\text{KW} = 2000\text{ Watt}, C_v = 20.8\text{ J/mol}\cdot\text{K}$$

Time $t = ?$ (Time after which room's temp will increase from 0°C to 21°C)

Suppose room contains oxygen and nitrogen behaving like ideal diatomic gas.

Let us consider n moles of ideal diatomic gas in the room

$$\therefore n = \frac{72000}{22.4} = 3.2 \times 10^3\text{ mol.}$$

As room is air tight \therefore Volume is const.

\therefore Heat supplied at const volume is given by

$$Q = nC_v\Delta T.$$

$$= 3.2 \times 10^3 \times 20.8 \times 21.$$

$$Q = 1.4 \times 10^6\text{ J}$$

Now power is given as

$$P = \frac{\text{Energy}}{\text{Time}}$$

$$P = \frac{Q}{t}$$

$$t = \frac{Q}{P} = \frac{1.4 \times 10^6}{2000}$$

$$= \frac{1.4}{2} \times 10^3 = 0.7 \times 10^3$$

$$= 700\text{ gm.}$$

$$= \frac{700}{60} = 11.6$$

$$\boxed{t = 12\text{ min}} \quad \text{Ans.}$$

\therefore Temp. of room will increase from 0°C to 21°C in about 12 min.

Sample Problem-3

Sample Problem 3 The internal structure of hydrogen shows a series of discrete excited states, the first such state being at an energy of $E = 10.2\text{ eV}$ above the lowest (ground) state. At what temperature would the average translational kinetic energy be equal to the energy of the excited state?

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Solution: Energy of excited state = $E = 10.2 \text{ eV}$

$T = ?$ $k = 8.6 \times 10^{-5} \text{ eV/K}$

As by equipartition energy theorem, we have //

$$E = \frac{3}{2} kT$$

$$\therefore T = \frac{2E}{3k} = \frac{2 \times 10.2}{3 \times 8.6 \times 10^{-5}} = 7.9 \times 10^4 \text{ K}$$

$T = 7.9 \times 10^4 \text{ K.}$ Ans.

First Law of Thermodynamics:

Statement:-

(i) In any Thermodynamic process b/w equilibrium states i and f the quantity $Q+W$ has the same value for any path b/w i and f. This quantity is equal to the change in the value of a state function called the internal energy.

Mathematically, it is written as

$$\Delta E_{int} = Q + W. \quad (1)$$

It should be noted that when heat Q is absorbed by a system. It is taken as +ve and this heat absorbed increase the internal energy of the gas.

Also when work is done on a gas the internal energy increases. So both heat added to the system and work done on the system increase internal energy of the system. So Q and W are taken as

+ve. So total increase in internal energy due to heat absorbed plus work done on a system.

So whenever some interaction of a system takes place with its surroundings, the entire change in internal energy ΔE_{int} is due to Q and W

$$\Delta E_{int} = Q + W$$

This is called first law of thermodynamics.

When heat is lost by the system or work is done by the system on its surroundings, both Q and W decrease the internal energy of a system.

So we have following sign conventions

- (i) "Q" is +ve when it is surrounded by the gas
- (ii) "W" is +ve when work is done on the system.
- (iii) ΔE is +ve when increases (iv) ΔE is -ve when decreases.
- (v) R is -ve when it is rejected by the system
- (vi) W is -ve when it is done by the system.

If a system changes its states from i to f along different paths, then the quantities Q, W have diff. values along diff. paths but the quantity (Q+W) have the same value. So the quantity (Q+W) independent of path is called internal energy. It depends on initial and final states. Also it has definite value for each state hence it is a state function. The internal energy is such a state function whose value can't be calculated but its change can only be determined.

If a system undergoes a small change i.e. small amount of heat dQ changes & small work dW is done then 1st law of thermodynamics becomes $dE_{int} = dQ + dW$. Here dQ & dW are not exact differential because they don't depend on state but dE_{int} is exact diff because it depends on state.

Application of 1st Law of Thermodynamics

We shall apply first law of thermodynamics to explain the following process.

Adiabatic Process:

It is the process in which no heat can enter or leave the system.

This process can be achieved by insulating the system completely from its surroundings. So that $Q = 0$

So the first law of thermodynamics becomes

$$\Delta E_{int} = Q + W$$

$$\Delta E_{int} = W$$

In the differential form

$$dE_{int} = dW \quad (1)$$

Now we define the relation b/w P and V for an adiabatic process performed on an ideal gas

By definition

$$C_v = \frac{1}{n} \left(\frac{du}{dt} \right)$$

$$u = E_{int}$$

$$\therefore C_v = \frac{1}{n} \frac{dE_{int}}{dt}$$

$$\therefore dE_{int} = nC_v dT \quad (2)$$

Now the work done by the system is given by

$$dw = -PdV$$

$$\text{or } PdV = -dw$$

$$PdV = -dE_{int}$$

$$PdV = -nC_v dT \quad (3) \text{ using eq. (2)}$$

Now ideal gas eq. has become

$$PV = nRT$$

In differential form, it becomes

$$d(PV) = nR(dT)$$

$$Pv + v dP = nRdT$$

$$VdP = nRdT - PdV$$

$$VdP = nRdT - (-nC_v dT) \quad \text{using (3)}$$

$$\therefore VdP = nRdT + nC_v dT$$

$$\text{But } nC_v dT + nRdT = nC_p dT$$

$$\therefore VdP = nC_p dT \quad (4)$$

Dividing (4) by (3) we get

$$\frac{VdP}{PdV} = -\frac{nC_p dT}{nC_v dT}$$

$$\frac{VdP}{PdV} = -\frac{C_p}{C_v}$$

$$\frac{VdP}{PdV} = -\gamma$$

$$\text{But } \frac{C_p}{C_v} = \gamma$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

Integrating both sides b/w initial state i and final state f. we get

$$\int_{P_i}^{P_f} \frac{dP}{P} = -\gamma \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\ln P_f - \ln P_i = -\gamma (\ln V_f - \ln V_i)$$

$$\ln \frac{P_f}{P_i} = -\gamma \ln \frac{V_f}{V_i}$$

$$\ln \frac{P_f}{P_i} = \ln \left(\frac{V_f}{V_i} \right)^{-\gamma} = \ln \left(\frac{V_i}{V_f} \right)^{\gamma}$$

$$\frac{P_f}{P_i} = \frac{V_i^{\gamma}}{V_f^{\gamma}}$$

$$\text{or } P_i V_i^{\gamma} = P_f V_f^{\gamma} \quad (5)$$

$$\text{or } \boxed{PV^{\gamma} = \text{Const}} \quad (6)$$

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Equations (5) and (6) represent the relation b/w pressure and volume of an ideal gas in an adiabatic process
 In terms of temperature eq. (6) can be written as

$$P V^\gamma \frac{V}{V} = \text{const}$$

$$P V^{\gamma-1} V = \text{const}$$

$$(PV) V^{\gamma-1} = \text{const}$$

For ideal gas $PV = nRT$

$$\therefore (nRT) V^{\gamma-1} = \text{const}$$

$$\therefore T V^{\gamma-1} = \frac{\text{const}}{nR}$$

$$nR = \text{const}$$

$$T V^{\gamma-1} = \text{const}$$

$$\text{or } T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\text{or } T_f = T_i \left(\frac{V_i^{\gamma-1}}{V_f^{\gamma-1}} \right)$$

$$\boxed{T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1}}$$

(ii) Isothermal Process:

It is the process in which temperature of the system remains constant.

So if temperature remains const then for ideal gas internal energy also remains const.

$$\therefore \Delta E_{\text{int}} = 0$$

So first law of thermodynamics becomes:

$$\Delta E_{\text{int}} = Q + W$$

$$0 = Q + W$$

$$\text{or } Q + W = 0$$

If the work W is done on the gas then the gas rejects heat Q to the surroundings. So the work done on the gas does not remain within the gas but flows out as heat to the surroundings.

(iii) Constant Volume Process:

If the volume of gas is kept const then no work can be done.

So by the first law of thermodynamics becomes,

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$$\Delta E_{int} = Q + W$$

$$\Delta E_{int} = Q + 0$$

$$Q = \Delta E_{int}$$

∴ All heat supplied goes to increase the internal energy of the gas.

(iv) Cyclic Process:

It is a series of processes after which system goes to its initial state

Fig shows a Three-step process. because the process starts and ends at the same point A.

∴ After one cycle change in internal energy is zero.

$$\therefore \Delta E_{int} = 0$$

∴ By first law of thermodynamics

$$\Delta E_{int} = Q + W$$

$$Q + W = 0 \quad (\text{Cyclic process}).$$

(i) The process 1 occurs at const

volume along A-B.

(ii) Process 2 occurs at const

pressure along B-C.

(iii) Process 3 occurs at const temp.

(isothermal) process along C-A

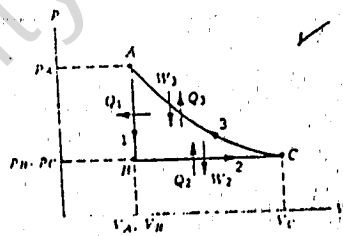
The total work done is +ve

because area under the curve 3 is greater than curve 2.

$$\therefore W > 0 \quad \text{So } Q < 0$$

For any cyclic process in anticlockwise direction $W > 0$ and $Q < 0$.

For any cyclic process in clockwise direction $W < 0$ and $Q > 0$.

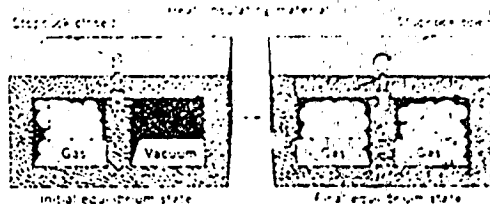


12-A

(iv) **Free Expansion:**

A process in which gas goes from one side of container to the other half initially evacuated side is called free expansion process shown in fig. A.

In this case the gas is initially on one side of the container & when stop cock is opened, the gas enters into the evacuated



half side. No weights are raised so no work is done. The container is insulated, so process is adiabatic.

$$\therefore Q = 0 = W$$

1st law of thermodynamic becomes

$$\Delta E_{int} = 0 \quad (\text{Free Expansion})$$

$$\text{Change in internal energy } \Delta E_{int} = 0 \Rightarrow E_{int} = \text{const.}$$

So during the free expansion the internal energy of the ideal gas remains const. As internal energy only depends upon temp. So temp. must remain constant.

Sample-Problem-4

Sample Problem 4 Let 1.00 kg of liquid water be converted to steam by boiling at standard atmospheric pressure, see Fig. 15. The volume changes from an initial value of $1.00 \times 10^{-3} \text{ m}^3$ as a liquid to 1.671 m^3 as steam. For this process, find (a) the work done on the system, (b) the heat added to the system, and (c) the change in the internal energy of the system.

Solution:

$$m = 1 \text{ kg}$$

$$P = 1 \text{ atm.} = 1.01 \times 10^5 \text{ Pa.}$$

$$V_i = 1.0 \times 10^{-3} \text{ m}^3$$

$$V_f = 1.671 \text{ m}^3$$

(a) Work done on the system = ?

(b) Heat added to the system = ?

(c) Change in internal Energy of the system = ?

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(a) work done is given by

$$W = -P\Delta V$$

$$= -P(V_f - V_i)$$

$$W = -1.01 \times 10^5 (1.671 - 1 \times 10^{-3})$$

$$= -169 \times 10^5 = -169 \times 10^3 \text{ J}$$

$$W = -169 \text{ KJ} \text{ Ans.}$$

(b) $Q = mL$

$$Q = 1 \times 2260 \quad Q = 2260 \text{ KJ} \text{ Ans}$$

$L = 2260 \text{ kJ}$
From

(c)

$$\Delta E_{int} = Q + W$$

$$= 2260 + (-169)$$

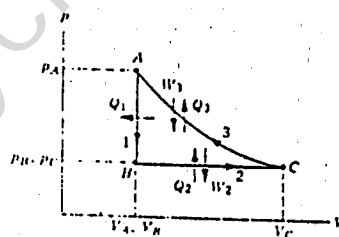
$$= 2260 - 169$$

$$\Delta E_{int} = 2091 \text{ KJ} \text{ Ans.}$$

Sample Problem: 5

Sample Problem 5 The cycle shown in Fig. 13 consists of three processes, starting at point A: a reduction in pressure at constant volume from point A to point B; an increase in volume at constant pressure from point B to point C; an isothermal compression (decrease in volume) from point C back to point A. Let the cycle be carried out on 0.75 mol of a diatomic ideal gas, with $p_A = 3.2 \times 10^3 \text{ Pa}$, $V_A = 0.21 \text{ m}^3$, and $p_B = 1.2 \times 10^3 \text{ Pa}$. For each of the three processes and for the cycle, find Q, W, and ΔE_{int} .

Sol. $R = 8.31 \text{ J/mol K}$, $p_A = 3.2 \times 10^3 \text{ Pa}$,
 $p_B = 1.2 \times 10^3 \text{ Pa}$, $V_A = 0.21 \text{ m}^3$, $C_p = 29.1 \text{ J/mol K}$
 $n = 0.75 \text{ mol}$, $C_v = 20.8 \text{ J/mol K}$
 For each Process $Q = ?$, $W = ?$, $\Delta E_{int} = ?$
 For each cycle $Q = ?$, $W = ?$, $\Delta E_{int} = ?$
 First we calculate T_A , T_B and V_C as follows



Using ideal gas law at pt 'A' we get

$$p_A V_A = nRT_A$$

$$T_A = \frac{p_A V_A}{nR} = \frac{3.2 \times 10^3 \times 0.21}{0.75 \times 8.31} = 108 \text{ K}$$

$$T_A = 108 \text{ K}$$

Using ideal gas law at pt B we get,

$$p_B V_B = nRT_B$$

$$T_B = \frac{p_B V_B}{nR} = \frac{1.2 \times 10^3 \times 0.21}{0.75 \times 8.31} = 40 \text{ K}$$

$$T_B = 40 \text{ K}$$

Using ideal gas law at point 'C' we get

$$p_C V_C = nRT_C$$

$$V_C = \frac{nRT_C}{p_C} = \frac{0.75 \times 8.31 \times 108}{1.2 \times 10^3}$$

$\because T_C = T_A$
 $\& p_C = p_B$

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(i) For process 1 (At const volume) (A-B)

$$Q_1 = nC_v(T_B - T_A)$$

$$= 0.75 \times 20.8 (40 - 108)$$

$$Q_1 = -1060 \text{ J}$$

-ve sign shows that heat is rejected by the system

$W_1 = 0$ because volume is const.

$$\Delta E_{int_1} = Q_1 + W_1$$

$$= -1061 + 0$$

$$\Delta E_{int_1} = -1061 \text{ J}$$

-ve sign shows that internal energy decreases from A-B

(ii) For process 2 at const pressure (B-C)

$$Q_2 = nC_p(T_C - T_B)$$

$$Q_2 = 0.75 \times 29.1 (108 - 40)$$

$$T_C = T_A = 108 \text{ K}$$

$$Q_2 = 1480 \text{ Joule}$$

Now

$$W_2 = -P(V_C - V_B)$$

$$= -1.2 \times 10^3 (0.56 - 0.21)$$

$$\therefore P_B = P_C$$

$$W_2 = -420 \text{ Joule}$$

-ve sign shows that work is done by the gas in moving the piston up

$$\Delta E_{int_2} = Q_2 + W_2$$

$$= 1480 - 420$$

$$\Delta E_{int_2} = 1060 \text{ J}$$

(iii) For process 3 at const temp (C-A)

The work in isothermal process is given by

$$W_3 = -nRT_C \ln \frac{V_A}{V_C}$$

$$= -0.75 \times 8.31 \times 108 \ln \frac{0.21}{0.56}$$

$$= 673.11 \ln 0.375$$

$$= -673.11 \times (-0.980)$$

$$W_3 = 660 \text{ Joule}$$

Now $\Delta E_{int_3} = 0$

\therefore Temperature remains const

Now by first law of thermodynamics

$$\Delta E_{int_3} = Q_3 + W_3$$

$$0 = Q_3 + 660$$

$$Q_3 = -660 \text{ Joule}$$

-ve sign shows that

heat Q_3 is rejected by the gas.

(iv) For the whole cycle.

$$Q = Q_1 + Q_2 + Q_3$$

$$= -1060 + 1484 - 660$$

$$Q = -236 \text{ Joule} \quad \text{Ans}$$

Now $W = W_1 + W_2 + W_3$

$$= 0 - 420 + 660$$

$$W = 240 \text{ Joule} \quad \text{Ans}$$

Now $E_{int} = \Delta E_{int_1} + \Delta E_{int_2} + \Delta E_{int_3}$

$$= -1060 + 1060 + 0$$

$$E_{int} = 0 \quad \text{Answer.}$$

Q. Show that adiabat is steeper than Isotherm:

Ans Adiabatic process is represented by

$$PV^\gamma = \text{const}$$

Taking differentials on both sides we get

$$d(PV^\gamma) = 0$$

$$V^\gamma dP + P\gamma V^{\gamma-1} dV = 0$$

$$V^\gamma dP = -\gamma P V^{\gamma-1} dV$$

$$\frac{dP}{dV} = -\gamma P \frac{V^{\gamma-1}}{V^\gamma}$$

$$= -\gamma P V^{\gamma-1-\gamma} = -\gamma P V^{-1}$$

$$= -\frac{\gamma P}{V}$$

$$\frac{dP}{dV} = -\gamma \left(\frac{P}{V} \right) \quad (1)$$

where $\frac{dP}{dV}$ gives slope of adiabat.

Now isothermal process is given by

$$PV = \text{const}$$

Taking differentials on both sides we get

$$d(PV) = 0$$

$$P dV + V dP = 0$$

$$V dP = -P dV$$

$$\frac{dP}{dV} = -\left(\frac{P}{V} \right) \quad (2)$$

From (1) and (2) we see that adiabat is steeper than isotherm.

The Transfer of Heat:

Heat energy can be transferred from one place to the other by three ways namely

(i) Conduction (ii) Convection (iii) Radiation

We shall study them one by one.

Conduction:

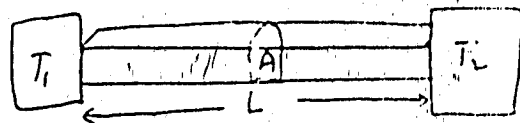
It is the process in which heat energy is transferred from one point to the other due to vibrations and collisions of molecules.

This method of transfer of heat is prominent in solids (metallic solids) and small in liquids and negligible in gases.

If one end of rod is heated, the K.E of the molecules at the hot end is increased so they vibrate with large amplitude. So they collide with neighbouring molecules and give some energy to them. This process goes on and the energy is transferred from molecule to molecule down the rod. Thus it can be said that conduction of heat is due to vibration and collisions of molecules.

Factors on which conduction depend

Consider a rod of length L . The two ends of the rod are at temps T_1 and T_2 respectively and the area of cross-section



of the rod is A . As the two ends of the rod are at two different temperatures, the heat will flow from hot end to cold end through the rod.

The heat which is conducted through the rod depends upon:

(i) Area of cross section 'A' of the rod. Larger the area, greater will be the flow of heat.

$$\text{i.e. } Q \propto A.$$

(ii) The temperature difference :-

The greater the temperature difference b/w the two ends, greater will be the flow of heat.

$$\text{i.e. } Q \propto (T_1 - T_2)$$

iii) The time of conduction :-
 More heat will flow in longer time
 i.e. $Q \propto t$.

iv) The length 'L' of the rod.
 Longer the rod, smaller is the amount of heat across the rod.
 i.e. $Q \propto \frac{1}{L}$

Combining all the above factors, we get

$$Q \propto \frac{A(T_1 - T_2)t}{L}$$

$$\text{or } \frac{Q}{t} \propto \frac{A(T_1 - T_2)}{L}$$

Where $\frac{Q}{t}$ is the rate of flow of heat denoted by H.

$$\therefore H \propto \frac{A(T_1 - T_2)}{L}$$

$$\text{or } H = \frac{KA(T_1 - T_2)}{L}, \text{ where } K \text{ is called thermal conductivity.}$$

It depends upon the nature of material of rod.

If T_1 and T_2 are denoted as T_H and T_L then

$$H = \frac{KA(T_H - T_L)}{L} \quad (1)$$

The substances with the value of 'K' are good thermal conductors and the substances with small value of K are bad thermal conductors.

From eq (1) $H = \frac{A(T_H - T_L)}{L/K}$

$$H = \frac{A(T_H - T_L)}{R}$$

Where $R = \frac{L}{K}$ is called thermal resistance or R-value. So for good thermal conductors the R-value is small and vice versa.

If we have a small rod of length dx and temp. difference dT then rate of flow of heat is given as,

$$H = -KA \frac{dT}{dx}$$

where -ve sign shows that heat flows in the direction of decreasing T. where $\frac{dT}{dx}$ is called Temp. gradient.

Convection:

It is the process in which heat is transferred from one place to another by the actual movement of the molecules.

As the molecules can move only in liquids and gases, (fluids), so fluids are heated by this process. In solids heat can't be transferred by convection because their molecules can't leave their mean positions.

Natural Convection

In natural convection the heated fluid (liquid or gas) rises due to difference in density.

Forced Convection:

In forced convection a blower or pump is used to make the flow of heated fluid to its destination.

Examples:

- (i) At the flame of candle or match, heat energy is transferred upward by convection.
- (ii) All their movements take place by convection. e.g. winds, monsoon, trade winds, sea and land breeze, cyclones are due to convection.

Factors:

Transfer of heat by convection depends on:

- (i) Temperature difference b/w the source and the place to which heat is transferred.
- (ii) Nature of fluid (liquid or gas)
- (iii) Density, viscosity, specific heat and the thermal conductivity of the fluid.
- (iv) Velocity of the fluid.

3. Radiation

"It is the process in which heat is transferred from one place to another place in the form of waves without a material medium."

When we touch a body with our hands, heat reaches the hand by conduction.

If hand is placed above the hot body but not in contact with it, heat reaches the hand by convection.

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If we place our hand at one side of the hot body, it receives heat by radiation and not by conduction and convection. since there is no medium b/w the sun and the earth. Heat reaches the earth from the sun by radiation.

Heat waves are called thermal radiations. Thermal radiation belong to the electromagnetic radiation. Heat waves have longer wave length than light waves. Both thermal radiations and light have similar properties and obey identical laws.

Sample Problem - 6

Sample Problem 6 Consider a compound slab, consisting of two materials having different thicknesses, L_1 and L_2 , and different thermal conductivities, k_1 and k_2 . If the temperatures of the outer surfaces are T_2 and T_1 , find the rate of heat transfer through the compound slab (Fig. 20) in a steady state.

Sol: Let T_x be the temp of boundary b/w two slabs.

Rate of heat transfer through the compound slab in steady state = $H = ?$

$$\text{Now } H_1 = \frac{k_1 A (T_x - T_1)}{L_1} \quad (1)$$

$$H_2 = \frac{k_2 A (T_2 - T_x)}{L_2} \quad (2)$$

For steady state $H_1 = H_2$.

$$\frac{k_1 A (T_x - T_1)}{L_1} = \frac{k_2 A (T_2 - T_x)}{L_2}$$

Now we solve for T_x

$$\frac{k_1 (T_x - T_1)}{L_1} = \frac{k_2 (T_2 - T_x)}{L_2}$$

$$\frac{k_1 T_x - k_1 T_1}{L_1} = \frac{k_2 T_2 - k_2 T_x}{L_2}$$

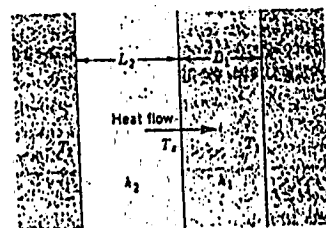
By cross multiplication $k_1 L_2 T_x - k_1 L_2 T_1 = k_2 L_1 T_2 - k_2 L_1 T_x$

$$k_1 L_2 T_x + k_2 L_1 T_x = k_1 L_2 T_1 + k_2 L_1 T_2$$

$$T_x (k_1 L_2 + k_2 L_1) = k_1 L_2 T_1 + k_2 L_1 T_2$$

$$T_x = \frac{k_1 L_2 T_1 + k_2 L_1 T_2}{k_1 L_2 + k_2 L_1}$$

Putting the value of T_x in (1) we get



$$T_2 > T_x > T_1$$

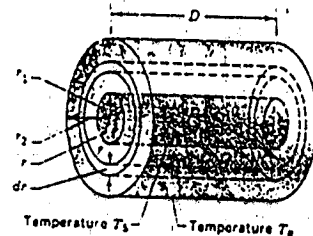
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$$\begin{aligned}
 H_1 &= \frac{K_1 A}{L_1} \left[\frac{K_1 L_2 T_1 + K_2 L_1 T_2}{K_1 L_2 + K_2 L_1} - T_1 \right] \\
 &= \frac{K_1 A}{L_1} \left[\frac{K_1 L_2 T_1 + K_2 L_1 T_2 - K_1 L_2 T_1 - K_2 L_1 T_1}{K_1 L_2 + K_2 L_1} \right] \\
 &= \frac{K_1 A}{L_1} \left[\frac{K_2 L_1 T_2 - K_2 L_1 T_1}{K_1 L_2 + K_2 L_1} \right] \\
 &= \frac{K_1 A}{L_1} \frac{K_2 L_1 (T_2 - T_1)}{K_1 L_2 + K_2 L_1} = \frac{K_1 A K_2 (T_2 - T_1)}{K_1 L_2 + K_2 L_1} \\
 &= \frac{A (T_2 - T_1)}{\frac{K_1 L_2}{K_1 K_2} + \frac{K_2 L_1}{K_1 K_2}} = \frac{A (T_2 - T_1)}{\frac{L_2}{K_2} + \frac{L_1}{K_1}} \\
 &= \frac{A (T_2 - T_1)}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} \quad \therefore \frac{L_1}{K_1} = R_1 \text{ and } \frac{L_2}{K_2} = R_2
 \end{aligned}$$

$$\therefore H_1 = \frac{A (T_2 - T_1)}{R_1 + R_2}$$

∴ For steady state $H_1 = H_2 = H$

$$\therefore H = \frac{A (T_2 - T_1)}{R_1 + R_2} \quad \text{Ans.}$$



Sample Problem 7

Sample Problem 7 A thin cylindrical metal pipe is carrying steam at a temperature of $T_s = 100^\circ\text{C}$. The pipe has a diameter of 5.4 cm and is wrapped with a thickness of 5.2 cm of fiberglass insulation. A length $D = 6.2$ m of the pipe passes through a room in which the temperature is $T_R = 11^\circ\text{C}$. (a) How much heat is lost through the insulation? (b) How much additional insulation must be added to reduce the heat loss by half?

Solution $T_s = 100^\circ\text{C}$, $T_R = 11^\circ\text{C}$

$D =$ length of pipe $= 6.2$ m.

$d =$ diameter of pipe $= 5.4$ cm. Radius of pipe $= r_1 = \frac{d}{2} = \frac{5.4}{2} = 2.7$ cm

Thickness of insulation $= dr = 5.2$ cm [Fiberglass].

Thermal conductivity of fiberglass $= k = 0.048$ W/m-k [From Table].

Radius of shell of insulation $= r_2 = r_1 + dr = 2.7 + 5.2 = 7.9$

(a) How much heat is lost through the insulation.

(b) How much additional insulation must be added to reduce heat loss by half.

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(a) Now $H = -KA \frac{dT}{dr} = -K(\pi r D) \frac{dT}{dr}$

$\therefore H \frac{dr}{r} = -2\pi KD dT$

Integration gives $H \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi KD \int_{T_5}^{T_R} dT$

$H \ln r_2 - \ln r_1 = -2\pi KD (T_R - T_5)$

$H \ln \frac{r_2}{r_1} = -2\pi KD (T_R - T_5)$

$H = \frac{2\pi KD (T_5 - T_R)}{\ln \frac{r_2}{r_1}}$

Putting the values we get

$H = \frac{2 \times 3.14 \times 0.048 \times 6.2 (100 - 11)}{\ln \frac{7.9}{2.7}}$

$H = \frac{6.28 \times 0.048 \times 6.2 \times 89}{1.07}$

$H = 155 \text{ watt}$ Ans.

(b) To reduce H to half the denominator of eq. (1) must be twice i.e.

$\frac{\ln \frac{r_2'}{r_1}}{\ln \frac{r_2}{r_1}} = 2$

$\ln \frac{r_2'}{r_1} = 2 \ln \frac{r_2}{r_1}$

$\ln \frac{r_2'}{r_1} = \ln \left(\frac{r_2}{r_1} \right)^2$

$\frac{r_2'}{r_1} = \left(\frac{r_2}{r_1} \right)^2$

$r_2' = \frac{r_2^2}{r_1} = \frac{7.9^2}{2.7}$

$r_2' = \frac{7.9^2}{2.7}$

$r_2' = 23 \text{ cm}$ Ans

So to reduce heat loss to half, we should increase the thickness of insulation four times.

THE END