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QUANTUM PHYSICS

1. THERMAL RADIATION

"Radiation emitted by a body due to its temperature is called Thermal Radiation."

All bodies not only emit such radiation but also absorb it from their surroundings. If the temperature of body is higher than its surroundings, its rate of emission is greater than rate of absorption. If the rates of emission and absorption of radiation become equal, the body is said to be in thermal equilibrium with its surrounding.

The spectrum of thermal radiation from a hot solid body is continuous. This spectrum has the following two properties.

(i) The higher the temperature, the more thermal radiation is emitted.

(ii) The higher the temperature, the shorter is the wave length emitted.

Thermal radiation emitted by a hot body depends not only on its temperature but also on the material, shape of body and nature of surface of body e.g a polished surface hot body emits less radiation than a hot body with rough surface.

BLACK BODY RADIATION

"A body which can absorb all the radiation incident on it is called BLACK BODY."

A perfect black body does not exist in nature. a metallic block with a cavity having a narrow hole whose inner walls are lamp blacked is nearly a black body.

The radiation entering the cavity through small hole suffers so many reflections that it is completely absorbed. If a black body is heated, its wall can emit radiation of all frequencies.

PROPERTIES OF BLACK BODY RADIATION

(i) "The radiation coming out from the cavity through small hole is called black body radiation or cavity radiation or full radiation or temperature radiation."

"The radiation is called temperature radiation because it depends only on the temperature of the walls of the cavity and not on its material, shape and size."

(ii) Those radiation exert pressure on the walls of the container. But black body radiations differ from a gas in two aspects.

(a) The gas molecules move with random velocities while black body radiation move the same velocity i.e, velocity of light.

(b) The pressure of gas can be changed by keeping its temp. constant, while the pressure of the radiation can be changed only by changing the temperature.

STEFAN-BOLTZMANN LAW (Fourth power law)

Radiant Intensity I(T):-

"The total radiated power per unit area of the cavity aperture of all wave lengths is called RADIANT INTENSITY."
It is denoted by I(T).

According to Stefan-Boltzmann law
"The radiant intensity is directly proportional to the fourth power of absolute temperature of the black body."

$$I(T) \propto T^4$$

$$I(T) = \sigma T^4$$

Where σ is a universal constant called Stefan-Boltzmann constant. Its value is $\sigma = 5.67 \times 10^{-8}$ watt/m^2-K^4.

The Spectral Radiancy. $R(\lambda)$:

"It is the quantity which shows variation of intensity of black body radiation with wave length for a given temperature."

It is denoted by $R(\lambda)$.

It is a statistical distribution function and is given as,

$$R(\lambda) = Ae^{-c\lambda}$$

The radiant intensity $I(T)$ for any temperature can be obtained by integrating the spectral radiancy over the complete wave length range i.e,

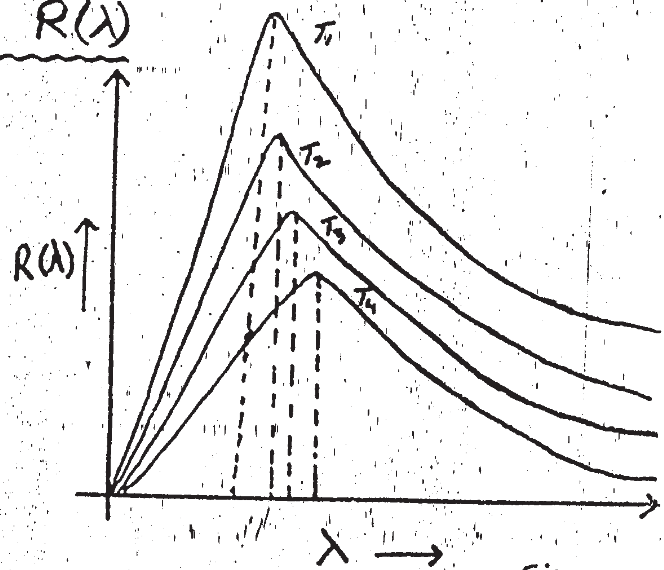
$$I(T) = \int_0^\infty R(\lambda)\,d\lambda \qquad , T = \text{constt.}$$

UNIT:-- The SI unit of spectral radiancy is watt-m^{-2}-m^{-1}. i.e, it is expressed in watt/m^2 per unit wave length.

EXPERIMENTAL CURVES BETWEEN λ AND $R(\lambda)$

The distribution of energy among the black body radiation had been a serious problem for the scientists.

The energy distribution among the black body radiation has been studied experimentally by Lummer and Pringsheim. They plotted curves between $R(\lambda)$ and λ at different temperatures for a black body shown in figure. After a careful study of these curves, some laws and results have been deduced from these curves.



Fig

From these curves we find that,

(i) $R(\lambda)$ increases with increase in λ reaches a maximum value and then starts decreasing.

(ii) The maximum value of $R(\lambda)$ shifts towards smaller wave length as the temperature of black body increases.

THE WIEN'S DISPLACEMENT LAW :-

According to this law,

"The wave length corresponding to maximum spectral radiancy is inversely proportional to the absolute temperature of black body."

i.e,

$$\lambda_{max} \propto \frac{1}{T}$$

$$\lambda_{max} = \text{constant} \times \frac{1}{T}$$

$$T\lambda_{max} = \text{constt.} = 0.0029 \text{ mK}$$

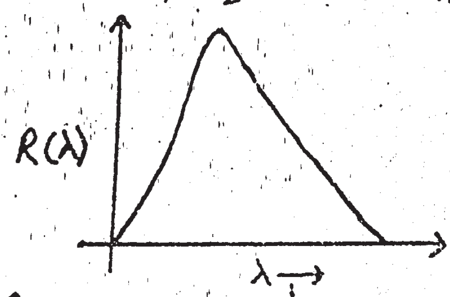
i.e, the product $T\lambda_{max}$ is a universal constant.

i- WIEN'S FORMULA

Wien tried to explain the experimental curves between λ and $R(\lambda)$ theoratically. He gave a formula which is given by,

$$R(\lambda) = \frac{C_1 \lambda^{-5}}{e^{C_2/KT}}$$ where C_1 & C_2 are constants.

Wien's formula holds good for short wave lengths but fails at large wave lengths as shown.

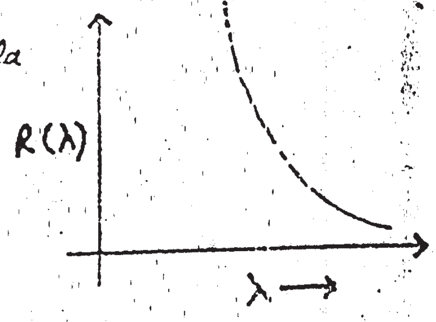


ii- RAYLEIGH-JEANS FORMULA

Rayleigh and Jeans gave a formula according to which

$$R(\lambda) = \frac{2\pi C K T}{\lambda^4}$$

This formula holds good for large wave lengths but fails for short wave lengths as shown.



PLANK'S RADIATION LAW

From the above laws, we find that classical physics fails to explain the peculiar distribution of energy among the black body radiation. Many physicists have tried to give a theoritical explaination of the distribution of energy but succeeded only in explaining parts of the curve. Max Plank however took a bold step and gave a new theory known as Quantum theory to explain this distribution. According to him, every radiation travels in the form of photon which is a packet of energy. The energy of each photon is

$$E = h\nu$$

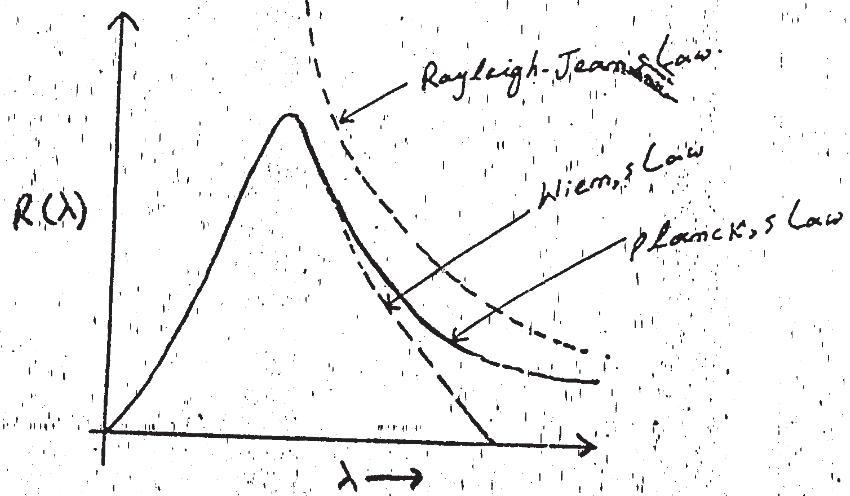
where h is Plank's constant having value $= 6.625 \times 10^{-34}$ J-S

He obtained the following formula on the basis of Quantum theory,

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

This formula is very true and fits quite well with the energy distribution among the black body radiation. This is a complete formula which holds good for all wave lengths. Other formulae can be derived from it as special cases.

Fig shows a comparison of Wien's law, Rayleigh-Jean's law and Plank's law.



SAMPLE PROBLEM 1:

Show that Planck's radiation law approa[ches] the classical Rayleigh - Jeans law at long wavelen[gths].

SOLUTION:

By Planck's radiation law,

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda KT}} - 1}$$

Putting $\frac{hc}{\lambda KT} = x$ we get

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^x - 1} \quad \text{--- } (1)$$

If $\lambda \to \infty$, then $x = \frac{hc}{\lambda KT} \to 0$

and $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{------}$

Then When $x \to 0$

$$e^x = 1 + x$$

$\therefore$ Equation (1) becomes,

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{1 + x - 1}$$

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{x}$$

$$= \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{\frac{hc}{\lambda KT}}$$

$$= \frac{2\pi c^2 h}{\lambda^5} \times \frac{\lambda KT}{hc}$$

$$R(\lambda) = \frac{2\pi c KT}{\lambda^4}$$ Which is Rayleigh - Jean's fo[rmula]

2- THE QUANTIZATION OF ENERGY

The assumptions made by Planck in deriving his radiation law and their consequences and importance of 'h' w[ere] not clear at that time. Planck himself was not clear about.

Plank derived his radiation law by studying the interaction between radiation in the cavity volume and the at[oms] of cavity walls.

He assumed that,

(i) The atoms of cavity behave like tiny linear oscillators.

(ii) These oscillators emit energy into the cavity and also absorb energy from the cavity.

(iii) The emission and absorption of energy by cavity does not take place in a continuous manner.

(iv) The absorption and emission of energy take place as an integral multiple of a certain unit known as quantum.

i.e, $E_n = nE$ where E is the energy of quantum.

(V) A quantum is a packet of energy and this energy is proportional to frequency ν of radiation i.e,

$$E \propto \nu$$

$$E = h\nu$$ where h is a planck's constant.

Thus $E_n = nh\nu$ —— ①

Where n is an integer called Quantum number $n = (1, 2, 3, ----)$

This shows that energy of atomic oscillator is quantized. Eq. ① also shows that oscillator energy levels are uniformly spaced and the space between two consecutive energy levels is $h\nu$.

Let us apply the idea of energy quantization to a large oscillator such as a swinging pendulum. From experience we know that a pendulum can oscillate with any energy and not only with certain discrete set of energies. As friction decreases the amplitude of pendulum. So it seems that energy of pendulum is dissipated continuously and not in jumps or quanta. But it is not true because value of Planck's constant 'h' is very small. So the jumps are very small and can't be detected. Hence we can not dismiss energy quantization.

THE CORRESPONDENCE PRINCIPLE

The classical and Quantum Theories are related by a principle called "correspondence principle."

According to this principle,

"Quantum theory must agree with classical theory in the limit of large quantum numbers."

So according to this principle Quantum theory becomes a limiting case of classical theory.

So in this way the large scale oscillator i.e, swinging pendulum and the atomic oscillators can be related to each other.

In the world of classical systems, the value of Planck's constant 'h' is very small which makes the energy quantum too small to be detected.

So when $h \rightarrow 0$, the quantum formulas correspond to classical formulas.

Similarly we can reduce relativistic formulas to the corresponding classical formulas by taking $c \rightarrow \infty$ or by neglecting $\frac{v}{c}$ where c is the speed of light.

Whether we are in a classical or a quantum situation, we must compare energy quantum $h\nu$ with classical energy measure KT. If $h\nu \ll KT$, then we are in classical situation. Here KT is the average translational K.E of the particle at temperature T. So we approach classical situation when $h\nu$ is v.v. small i.e, $\nu \longrightarrow 0$ or $\lambda \longrightarrow \infty$ or $T \longrightarrow \infty$.

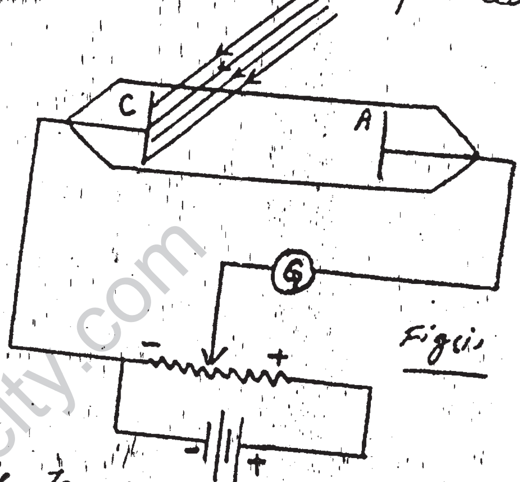
3 PHOTO ELECTRIC EFFECT

"When light of suitable frequency falls a metal surface, the metal emits electrons. These electrons are called photoelectrons and the phenomenon is called photo electric effect."

If these electrons are made to move, the current produced is called photoelectric current.

Circuit Diagram:

Fig(i) shows the circuit diagram to explain the phenomenon of photoelectric effect.

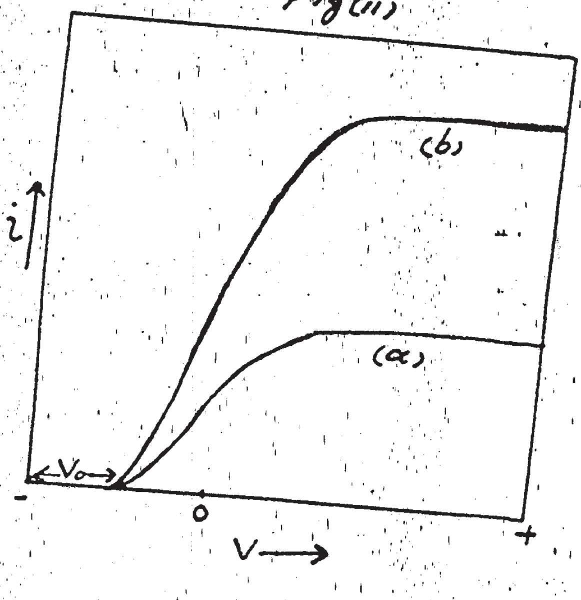


Fig(i)

It consists of a highly evacuated tube containing two electrodes. The tube is called photo tube or photo cell. A light of suitable frequency is made to fall on cathode 'C'. The electrons produced at the cathode 'C' are attracted by the anode 'A' and so a current flows in the galvanometer 'G'. So the galvanometer 'G' shows deflection. When light is switched off, current stops.

Fig(ii) shows the variation of photoelectric current with voltage 'V'. We see from fig(ii) that when V is +ve and large enough, the photo electric current reaches a constant saturation value when all the photoelectrons ejected from the cathode 'C' are collected by the anode 'A'.

Fig(ii)

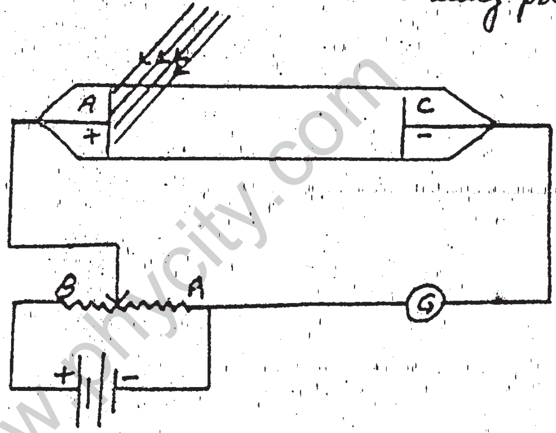


If we reduce V to zero, the photoelectric current does not fall to zero because the photo-electrons are emitted with non zero speed.

MAX. K.E OF PHOTOELECTRONS AND STOPPING POTENTIAL

If the battery connection in the above circuit are reversed, i.e, cathode is made anode and anode is made cathode. Then the electron face a reverse potential. This reverse or retarding potential can be adjusted as shown.

In this case 'C' is made more and more -ve by moving the sliding contact from B to A. So the electrons produced at the anode are repelled by the cathode.



But electrons which have enough energy to overcome the repulsion of cathode reach the cathode and so some current flows. The retarding (reverse) potential is gradually increased and current goes on decreasing.

If the retarding potential is adjusted to such a value that no electron can reach the cathode. This value of retarding potential for which no electron reaches the cathode is called "stopping potential V_0."

This stopping potential V_0 corresponds to the max. energy of photoelectrons.

So max K.E of photoelectrons is given by

$$K_{max} = V_0 e$$

Here V_0 and K_{max} are independent of intensity of incident light.

al as from curve (3) of fig (ii) where by doubling the intensity, the saturation current become doubled but V_0 remains same.

Fig (iii) shows the variation of stopping potential with frequency of incident light. We find that at $\nu = \nu_0$ the value of $V_0 = 0$. But when frequency increases above ν_0, the stopping potential also increases with increase in frequency.

If the frequency of light is below ν_0 called threshold frequency, no photo electrons are emitted, however intense the incident light may be.

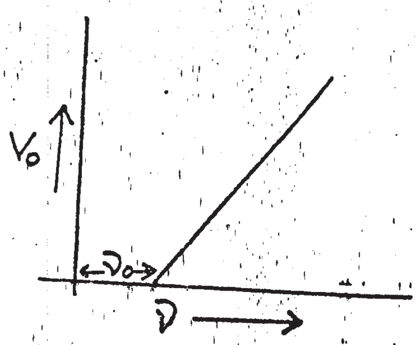


Fig (iii)

Major Features of Photoelectric effect which cannot be explained by classical wave theory

1- THE INTENSITY PROBLEM:

According to wave theory the oscillating electric field vector $\vec{E}$ of light wave increases in amplitude with increase in intensity of light. As force exerted on electron is $F = qE = eE$. So the K.E of electron should increase with increase in intensity of incident light beam.

But fig (ii) shows that $K_{max} = V_0 e$ is independent of light intensity. This fact has been verified over a wide range of intensities of 10^7.

2- THE FREQUENCY PROBLEM:

According to wave theory the photoelectric effect should occur for any frequency of incident light if it is intense enough to supply energy required for electrons to be emitted. But experiment shows that no photo electrons are emitted if frequency is less than threshold frequency ν_0, however intense the light may be.

3- THE TIME DELAY PROBLEM:

According to wave theory a long time is required by the electron to get emitted while actually the electrons are emitted instantaneously. Thus the wave theory cannot account for photoelectric effect. This effect has been explained by Einstein on the basis of photon theory.

4- EINSTEIN'S PHOTON THEORY AND EXPLANATION OF PHOTOELECTRIC EFFECT

The photoelectric effect has been explained by Einstein on the basis of photon theory of radiation.

According to this theory every radiation travels in the form of packets of energy called photon or quantum. The energy E of a photon is proportional to the frequency ν of radiation.

i.e,
$$E \propto \nu$$
$$E = h\nu$$

Where 'h' is the constant of proportionality and is called Planck's constant. Its value is $= 6.625 \times 10^{-34}$ J-S

According to Einstein when a photon of energy $h\nu$ falls on the metal surface, it transfers its entire energy to one electron only. We know that a certain minimum energy is required to remove an electron from an atom. This is called work function of the metal and is denoted by ϕ.

According to Einstein an electron will be emitted if the energy of photon is greater than or equal to ϕ. If the energy of incident photon is less than ϕ, no emission of electrons takes place. That is why when a photon with energy $h\nu > \phi$ falls on a metal, ϕ energy is used for emission and the remaining energy $(h\nu - \phi)$ becomes K.E of electron if it does not lose any energy by internal collisions while coming out of the metal.

Einstein meets the three objections raised against wave theory as follows.

i- ANSWER TO INTENSITY PROBLEM:

Increase in intensity means increase in number of photons. So increase in intensity does not increase the energy of

...vidual photons. So by increasing intensity the energy of photoelectrons does not change, only their number changes. That is why emission of photoelectrons does not occur if frequency of incident light is less than threshold frequency however intense the light may be.

ii- ANSWER TO FREQUENCY PROBLEM:

As we know that a certain minimum energy equal to work function is required by the electron for its emission and this energy depends on frequency.

$$\therefore \quad \phi = h\nu_o$$

So if frequency is less than ν_o, no electron is emitted no matter how intense the light may be.

This explains the significance of threshold frequency.

iii- ANSWER TO TIME DELAY PROBLEM:

According to Einstein an electron will be emitted if energy of photon is greater or equal to ϕ. If the energy of photon is less than ϕ, no emission of electrons takes place and photons are reflected back.

That is why when a photon with energy $h\nu > \phi$ falls on a metal, the electron is emitted instantaneously. This explains the instantaneous nature of photo electric effect.

EINSTEIN'S PHOTO ELECTRIC EFFECT EQUATION

Suppose a photon of energy $h\nu > \phi$ falls on a metal. Then ϕ energy will be used in the emission of electrons while the remaining energy will become the K.E of the electron.

So

$$h\nu = \phi + K.E$$

If ν_o is threshold frequency then $\phi = h\nu_o$

$$\therefore h\nu = h\nu_o + K.E$$

$$h\nu = h\nu_o + \frac{1}{2}mv^2$$

$$\therefore h\nu = h\nu_o + V_o e \qquad \text{But } \frac{1}{2}mv^2 = V_o e$$

$$V_o e = h\nu - h\nu_o \qquad \text{where } V_o \text{ is the stopping pot.}$$

$$V_o e = h(\nu - \nu_o)$$

This is Einstein's equation for photoelectric effect.

In terms of wave length

$$\nu = \frac{c}{\lambda}$$

$$\therefore \quad V_o e = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_o}\right)$$

$$\nu_o = \frac{c}{\lambda_o}$$

$$Ve = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)$$

Where λ_o is the Threshold wave length and is the longest wave length.

This is also Einstein's equation for photo electric effect in Terms of wave length.

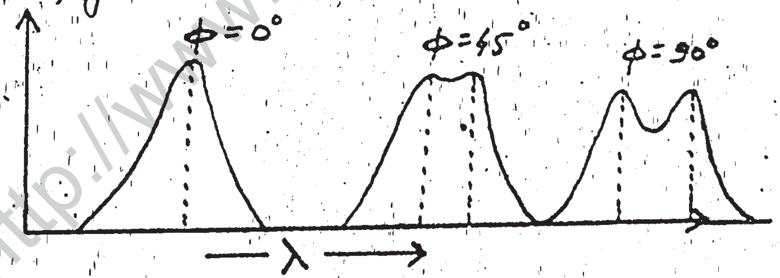
5- THE COMPTON EFFECT

A. H. Compton found that when a radiation (photon) scatters from a stationary particle, its frequency decreases i.e, wave length increases. This phenomenon is known as compton effect.

This effect like photoelectric effect cannot be explained on any wave Theory. Because according to wave theory, the frequency of the scattered radiation should not decrease.

In compton experiments an x-ray beam of particular wave length λ falls on a graphite target T. The scattered beam has been detected for various scattering angles ϕ as shown in the fig(i).

Fig(i)



We see that incident beam consists of single wave length λ but the scattered beam has two wave lengths i.e, λ and λ'. Where $\lambda' = \lambda + \Delta\lambda$. $\Delta\lambda$ is called compton shift. This shift varies with ϕ.

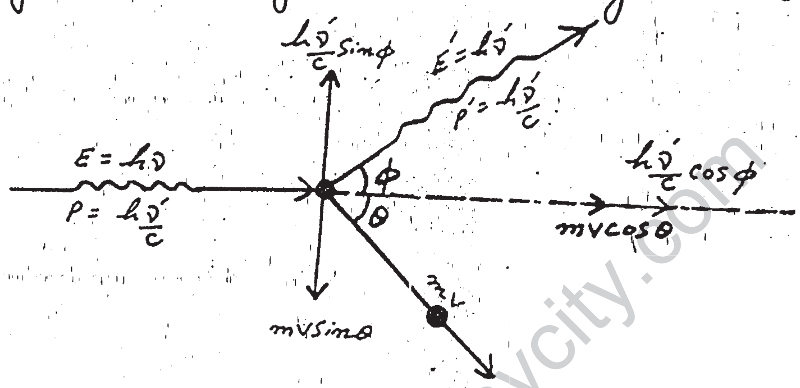
This effect has been explained by A.H. compton on the basis of Quantum theory as follows.

Explaination of compton effect on the basis of Quantum Theory.

Let us consider a single photo-electron collision quantitatively.

Suppose an x-ray photon of energy $h\nu$ strikes a free stationary electron of rest mass m_0. This photon gives some of its energy to the electron which begins to move with a velocity comparable to that of light. The energy of the photon decreases after collision. Decrease in energy $h\nu$ of a photon means decrease in its frequency ν because 'h' is constant.

Suppose photon scatters at angle ϕ with its origional direction and the electron whose mass becomes 'm' moves with velocity 'v' at an angle θ with the origional direction of photon.



Now this is the case of elastic collision between two balls. Both momentum and energy are conserved.

So we can apply law of conservation of energy and law of conservation of momentum to the collision.

Let ν' be the frequency of photon after collision.

$\therefore$ Energy of photon before collision $= h\nu$

Energy of photon after collision $= h\nu'$

Energy of electron before collision $= m_0c^2$

Energy of electron after collision $= mc^2$

$\therefore$ According to the law of conservation of energy.

Total energy before collision = Total energy after collision

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$h\nu - h\nu' + m_0c^2 = mc^2$$

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_0c^2 = mc^2 \qquad \because \nu = \frac{c}{\lambda}$$

$$\nu' = \frac{c}{\lambda'}$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0 c = mc$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0 c = \frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \because m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring both the sides we get,

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0 c\right)^2 = \frac{m_0^2 c^2}{1-\frac{v^2}{c^2}}$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} + m_0^2 c^2 - \frac{2h^2}{\lambda\lambda'} - \frac{2hm_0 c}{\lambda'} + \frac{2hm_0 c}{\lambda} = \frac{m_0^2 c^2}{1-\frac{v^2}{c^2}}$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + \frac{2hm_0 c}{\lambda} - \frac{2hm_0 c}{\lambda'} = \frac{m_0^2 c^2}{1-\frac{v^2}{c^2}} - m_0^2 c^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + 2hm_0 c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \left(\frac{1}{1-\frac{v^2}{c^2}} - 1\right) m_0^2 c^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + 2hm_0 c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \left(\frac{1-1+v^2/c^2}{1-\frac{v^2}{c^2}}\right) m_0^2 c^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + 2hm_0 c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{m_0^2 c^2 (v^2/c^2)}{1-\frac{v^2}{c^2}}$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + 2hm_0 c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{m_0^2}{1-\frac{v^2}{c^2}} \times v^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + 2hm_0 c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = m^2 v^2 \quad \text{——— (1)}$$

Now according to law of conservation of momentum along x-axis.

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + mv\cos\theta \qquad \because \nu = \frac{c}{\lambda}, \quad \nu' = \frac{c}{\lambda'}$$

$$\frac{hc}{\lambda c} = \frac{hc}{\lambda' c}\cos\phi + mv\cos\theta$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + mv\cos\theta.$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\phi = mv\cos\theta \quad \text{——— (2)}$$

According to law of conservation of momentum along Y-axis.

$$0 + 0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta$$

$$\frac{h\nu'}{c}\sin\phi = mv\sin\theta$$

$$\frac{hc}{\lambda' c}\sin\phi = mv\sin\theta \qquad \because \nu' = \frac{c}{\lambda'}$$

$$\therefore \frac{h}{\lambda'}\sin\phi = mv\sin\theta \quad \text{——— (3)}$$

Squaring and adding ② & ③ we get,

$$\left(\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2}\cos^2\varphi - \frac{2h^2}{\lambda\lambda'}\cos\varphi\right) + \left(\frac{h^2}{\lambda'^2}\sin^2\varphi\right) = m^2v^2\cos^2\theta + m^2v^2\sin^2\theta$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2}(\cos^2\varphi + \sin^2\varphi) - \frac{2h^2}{\lambda\lambda'}\cos\varphi = m^2v^2(\cos^2\theta + \sin^2\theta)$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = m^2v^2 \quad \text{———} \quad (4)$$

Subtracting ④ from ① we get,

$$\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'} + 2hm_0c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \cancel{m^2v^2} \quad \text{------} \quad (1)$$

$$\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\varphi = \cancel{m^2v^2} \quad \text{------} \quad (4)$$

$$- \quad - \quad + \qquad\qquad -$$

$$-\frac{2h^2}{\lambda\lambda'} + \frac{2h^2}{\lambda\lambda'}\cos\varphi + 2hm_0c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = 0$$

$$2hm_0c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{2h^2}{\lambda\lambda'} - \frac{2h^2}{\lambda\lambda'}\cos\varphi$$

$$2hm_0c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{2h^2}{\lambda\lambda'}(1-\cos\varphi)$$

$$m_0c(\lambda'-\lambda) = h(1-\cos\varphi)$$

$$\lambda'-\lambda = \frac{h}{m_0c}(1-\cos\varphi)$$

i.e, $$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\varphi)}$$

This is called compton shift in wavelength of scattered photon.

We find that compton shift $\Delta\lambda$ depends only on the scattering angle φ and not on the initial wavelength λ.

From the above equation it is clear that $\Delta\lambda$ changes from $0°$ for $\varphi = 0°$ to $\frac{2h}{m_0c}$ for $\varphi = 180°$.

Q. Prove that wave length of scattered photon varies from λ to $\lambda + \frac{2h}{m_0c}$ as the scattering angle φ changes from $0°$ to $180°$.

SOLUTION:

CASE (i) Put $\varphi = 0°$ in the compton shift we get,

$$\lambda'-\lambda = \frac{h}{m_0c}(1-\cos 0°)$$

$$\lambda'-\lambda = \frac{h}{m_0c}(1-1)$$

$$\lambda' - \lambda = \frac{h}{m_o c}(0)$$

$$\lambda' - \lambda = 0$$

$$\therefore \quad \lambda' = \lambda \quad \text{———} (a)$$

CASE (ii)

Put $\varphi = 180°$ in the compton shift we get,

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos 180°)$$

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - (-1))$$

$$\lambda' - \lambda = \frac{h}{m_o c}(1+1)$$

$$\lambda' - \lambda = \frac{2h}{m_o c}$$

$$\therefore \quad \lambda' = \lambda + \frac{2h}{m_o c} \quad \text{———} (b)$$

From (a) and (b) we find that wave length of scattered photon varies from λ to $\lambda + \frac{2h}{m_o c}$ as φ changes from $0°$ to $180°$.

SAMPLE PROBLEM (8)

Sample Problem 8 X rays with $\lambda = 100$ pm are scattered from a carbon target. The scattered radiation is viewed at 90° to the incident beam. (*a*) What is the Compton shift $\Delta\lambda$? (*b*) What kinetic energy is imparted to the recoiling electron?

SOLUTION:

$\lambda = 100$ Pm $= 100 \times 10^{-12}$ m ; $\varphi = 90°$

$h = 6.625 \times 10^{-34}$ J-S ; $C = 3 \times 10^{8}$ m/s

$m_o = 9.1 \times 10^{-31}$ kg

(a) $\Delta\lambda = ?$

(b) K.E of electron $= ?$

(a) Using compton shift.

$$\Delta\lambda = \frac{h}{m_o c}(1 - \cos\varphi)$$

$$= \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 90°)$$

$$= \frac{6.625}{27.3} \times 10^{-34+31-8}$$

$$= 0.2426 \times 10^{-11}$$

$$= 0.243 \times 10^{-11}$$

$$\Delta\lambda = 2.43 \times 10^{-12} \text{ m}$$

$$\boxed{\Delta\lambda = 2.43 \text{ Pm}}$$

Now $\lambda' = \lambda + \Delta\lambda = 100 \times 10^{-12} + 2.43 \times 10^{-12} = 102.43 \times 10^{-12}$ m.

By the equation,

$$h\nu + m_o c^2 = h\nu' + mc^2$$

$$mc^2 - m_o c^2 = h\nu - h\nu'$$

$$mc^2 - m_o c^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$mc^2 - m_o c^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$mc^2 - m_o c^2 = hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)$$

$$mc^2 - m_o c^2 = \frac{hc\Delta\lambda}{\lambda\lambda'}$$

$\therefore \quad K.E = \frac{hc\Delta\lambda}{\lambda\lambda'}$ But $K.E = mc^2 - m_o c^2$

$$\therefore \quad K.E = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8} \times 2.43 \times 10^{-12}}{100 \times 10^{-12} \times 102.43 \times 10^{-12}}$$

$$K.E = \frac{6.625 \times 3 \times 2.43}{102.43} \times 10^{8-34-12+12+12-2}$$

$$= 0.4715 \times 10^{-16}$$

$$= 0.472 \times 10^{-16}$$

$$= 4.72 \times 10^{-17} \text{ Joule}$$

$$= \frac{4.72 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{4.72}{1.6} \times 10^{-17+19}$$

$$= 2.95 \times 10^{2} \text{ eV}$$

$$\boxed{K.E = 295 \text{ eV}}$$

LINE SPECTRA

Experimental results of photoelectric effect and compton effect support the particle nature of electromagnetic radiation.

There are two types of spectra.

(i) Continuous spectra.

(ii) Line spectra.

The spectrum produced by white light is continuous. But the spectrum produced by vaporization of a salt in a flame is line spectrum. It is the characteristic of the atoms. It consists of definite lines. There are dark spaces between the colours. Line spectrum depends on the nature of the element. The line spectra are produced when electron moves from higher energy level to lower energy le. Dark spaces are due to space between two energy levels.

It is found that if the absorbing or emitting systems are isolated from one another, then spectrum produced is not continuous but discrete. The spectra of visible light are studied by spectrometer with prism or grating. These spectra are called line spectra. The individual lines are called spectral lines. Line spectra are produced due to emission or absorption of radiation by an isolated system e.g. atoms, molecules. By studying the line spectra of atoms, we get information about the energy states in them.