CH#54

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Nuclear Physics

Discovery of Nucleus:

Electron was discovered by J.J Thomson in 1897. Atoms are electrically neutral so they must also contain some +ve charge. In 1911, Ernest Rutherford proposed that +ve charge of the atom was densly concentrated at the centre of the atom; called 'Nucleus'.

Rutherford idea was to probe the forces acting within an atom by firing energetic alph (α) particles through a thin target foil and measuring the extent to which they were deflected as they passed through the foil. Alpha (α) particles which are about 7300 times more massive than an electron and carry a charge of +2e and are spontaneously emitted by many radioactive materials.

Hans Geiger & Ernest Marsden in collaboration with Rutherford performed this experiment. Their experimental arrangement is shown in fig (i). The experiment consists in counting the no. of α-particles deflected through various scattering angles 'θ'.

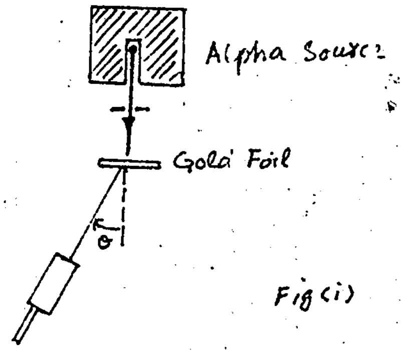


Fig (i)

Fig (ii) shows the results. The verticle scale is logrithmic. We see that most of the α-particles scattered through rather smaller angles, but a very small fraction of them scattered through very large angles approximately 180°.

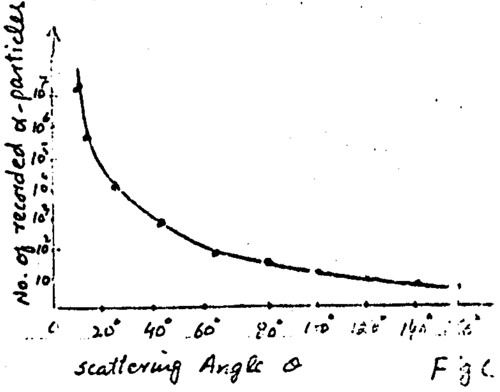


Fig (

This was quite against the J. J. Thomson's Model, according to which the +ve charge of the atom was thought to be spread out through the entire volume of the atom. The electron were thought to be distributed through-out the volume

The maximum deflecting force acting on the alpha particles as it passes through such a sphere of +ve charge proves to be far too small to deflect the α-particle by even as much as one degree.

Rutherford show that to produce such a large deflection there must be a large force, which could be provided if the +ve charge were concentrated tightly at the centre of the atom, instead of being spread through-out its volume. On this model, the incoming α-particles can get very close to the centre of +ve charge without penetrating it, resulting a large deflection.

Fig (iii) shows the paths taken by typical α-particles as they pass through the atoms of target foil. Most are deflected only slightly or not at all, but a few are deflected through large angles.

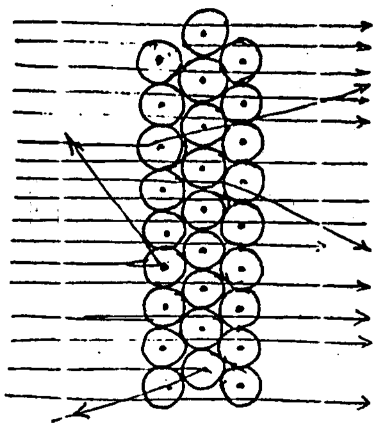


Fig (iii)

From this data Rutherford concluded that the dimension of the nucleus must be smaller than the diameter of an atom by a factor of about 10^4. The atom has most of the space empty.

Some Nuclear Properties

Nuclear Systematics:

Nuclei are made up of proton and neutrons.

Atomic Number:

The no. of protons in a nucleus is called ~~as~~ atomic number. It is denoted by Z.

Neutron Number(N):

"The number of neutrons in the nucleus is called the neutron number N."

The charge on the proton is $q = +e$ while neutron has no charge (i.e $q = 0$). Proton & neutron are very similar particles, having nearly equal masses & experience identical nuclear force inside the nuclei.

Nucleons:

"The protons & neutron are also called nucleons".

Mass Number (A):

"The total no. of nucleons is called the mass no. It is represented by "A". i.e

$$Z + N = A$$

Isotopes:

"The nuclides with same Z but different N & A are called isotopes".

As an example nuclide Bromine (Br) is represented by

$^{81}_{35}Br$ & $^{82}_{35}Br$.

In $^{81}_{35}Br$; No. of protons, $Z = 35$ & No. of Neutrons $= A - Z = 81 - 35 = 46$

In $^{82}_{35}Br$; No. of protons $Z = 35$ & No. of Neutrons $= A - Z = 82 - 35 = 47$

So two nuclides by Bromine with same $Z = 35$ but differen N & A are called isotopes of Bromine.

The Nuclear Force

The nucleus contains proton & neutron. There is coulomb's repulsion among protons. To bind the protons together, there must be a strong attractive force acting b/w neutron & protons. This force is called "Nuclear Force". This force must be strong enough to overcome the coulomb's repulsive force & to bind both protons & neutrons into the tiny nuclear volume.

This strong force has short range, roughly equa to 10^{-15} m. It means that the attractive force between a pair of nucleons drops rapidly to zero for nucleons separation greater than a certain critical value.

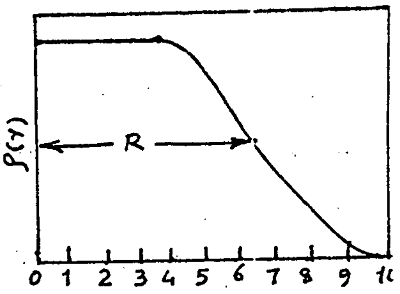
Nuclear Radii:

we have used Bohr's radius $a_0 = 5.29 \times 10^{-11}$ m as a convenient unit for measuring the dimensions of atoms. Nuclei are smaller by a factor of about 10^4 and a convenient unit for measuring distances of this scale is femto meter $= 10^{-15}$ m. This is often called a 'fermi'. So

$$1 \text{ fermi} = 1 \text{ femto meter} = 1\,\text{fm} = 10^{-15}\,\text{m}$$

We can study the size & structure of nuclei by doing scattering experiments, using the incident beam of high energy electrons. The energy of incident electrons must be high enough > 200 Mev so that their De-Broglie wavelength will be smaller enough for them to act as structure sensitive nuclear probes. So these experiments measure the diffraction pattern of scattered particles and so deduce the shape of scattering objects (nucleus).

From these scattering experiments, the nuclear density has been deduced to be of the form as shown in fig.



We see that nucleus does not have a sharply defined surface. It does, however, a characteristic mean radius R. The density $\rho(r)$ has a constant value in the nucleus interior & drops to zero through the fuzzy surface zone.

From these experiments, it has been found that 'R' increases with A approximately as:

$$R = R_0 A^{1/3}$$

where A = mass number, R_o = constant whose value is about 1.2 fm. For ^{63}Cu, for example;

$$R = (1.2\,fm)(63)^{1/3}$$
$$R = 4.3\,fm$$

By comparing the mean radius of a copper ion in the lattice of solid copper is 1.8 Bohr's Radius; about 2×10^4 times larger

Nuclear Masses & Binding Energies:

Atomic masses are measured with great precision using mass spectrometer & nuclear reaction techniques. Such masses are measured in the "unified atomic mass" units (u); which is choosen so that the atomic mass (not nuclear mass) of ^{12}C is exactly 12u. So its reaction with SI mass standards is;

$$1\,a.m.u = 1u = 1.6605\times10^{-27}\,kg$$

From Einsteins mass-energy relation, $E = \Delta m c^2$ the energy equivalent of 1 a.m.u is;

$$E = \Delta m c^2 = 1u\,c^2$$
$$= (1.6605\times10^{-27}\,kg)\times(2.9979\times10^{8}\,m/s)^2\ J$$
$$= \frac{1.6605\times10^{-27}\times(2.9979\times10^8)^2}{1.6\times10^{-19}}\ eV.$$
$$= 931.5\times10^{6}\,eV$$
$$E = 931.5\ MeV$$

$$\boxed{1\,a.m.u = 931.5\ MeV} \Rightarrow$$

If
$\Delta m c^2 = 931.5$
Put $\Delta m = 1u$
$\Rightarrow 1u\,c^2 = 931.5\,Me$
$c^2 = \left(\frac{931.5}{u}\right)$

or

Example:- Consider the deutron, the nucleus of heavy hydrogen atom. It consists of a proton & a neutron bounded together by strong nuclear force. The energy E_B we must add to the deutron to tear it apart into two constituent nucleons is called its "Binding energy".

In fact, the binding energy is the total internal energy of the nucleus due in part to the strong force b/w the nucleons, the coulomb's force b/w the nucleons & K.E of the nucleons relative to the centre of mass of entire nucleus. From the conservation of energy, we have;

$$m_d c^2 + E_B = m_n c^2 + m_p c^2$$

Add $(m_e c^2)$; the energy equivalent of one electron mass to each side of above equation; thus,

$$m_d c^2 + E_B + m_e c^2 = m_n c^2 + m_p c^2 + m_e c^2$$

$$(m_d + m_e) c^2 + E_B = m_n c^2 + (m_p + m_e) c^2$$

$$m(^2H) c^2 + E_B = m_n c^2 + m(^1H) c^2 \text{ ———— } ①$$

where $m(^2H)$ & $m(^1H)$ are the masses of neutral deutron & neutral ordinary H-atom respectively. They are atomic masses, not nuclear masses.

From equ. ①;

$$E_B = m_n c^2 + m(^1H) c^2 - m(^2H) c^2$$

$$= [m_n + m(^1H) - m(^2H)] c^2$$

$$E_B = \Delta m c^2 \text{ ———— } ②$$

where Δm is the "mass difference" & or "mass defect". Hence when a neutron or a proton are combined to form a deutron, there will be a decrease in the mass called the mass defect.

In making calculations, we use atomic rather than nuclear masses.

As $m_n = 1.008665u$, $m(^1H) = 1.007825u$, $m(^2H) = 2.014102u$

Putting the values in equ. ②, we get;

$$E_B = (1.008665u + 1.007825u - 2.014102u) c^2$$

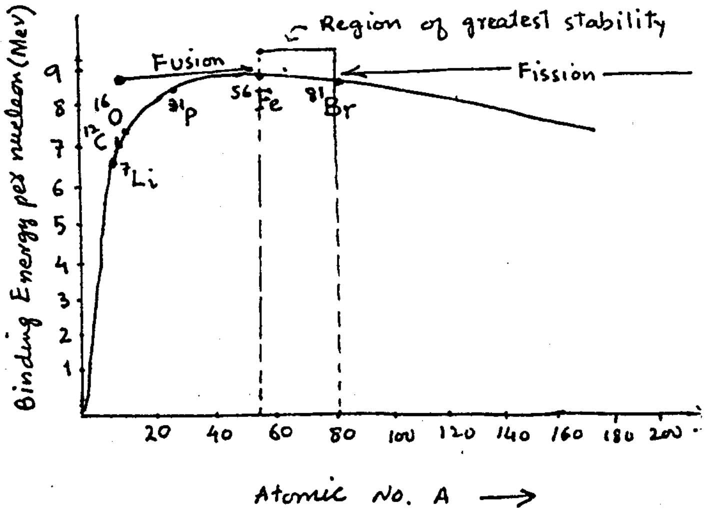
$$= (0.002388 u) c^2$$

$$E_B = 0.002388 \cancel{u} \times \frac{931.5}{\cancel{u}} \text{ Mev} \qquad \because c^2 = \frac{931.5}{u} \text{ Mev}$$

$$\boxed{E_B = 2.224\ \text{MeV}}$$

$$\text{Average Binding energy per nucleon} = \frac{\text{Binding Energy of Nucleons}}{\text{No. of Nucleons}}$$

A graph b/w average binding energy per nucleon & mass no. "A" is shown in fig. below.



The dropping of binding energy curve at both low & high mass no's. tells us that nucleons are more tightly bound when they are assembled into two middle-mass nuclei rather than into a single high mass nucleus. In other words energy can be resol-ved in the nuclear fission of a single massive nucleus into two smaller fragments.

The dropping of binding energy curve both at high and low & mass no. on the other hand, tells us that energy will be released if two nuclei of ~~two~~ small mass number combine to form a single middle-mass nucleus. This process is called "nuclear fusion" the reverse of fission.

Nuclear Spin & Magnetism:

Just like atoms, nuclei also possess an intrinsic angular momentum (nuclear spin) whose maximum value (component along any choosen z-axis is $= J\hbar = J\frac{h}{2\pi}$; where 'J' is a quantum number which may be integral or half integral, called the nuclear spin.

Just as for atoms, angular momentum is associted with 'magnetic moment', nuclear angular momentum is associated with "nuclear magnetic moment."

As the atomic "Bohr's Megneton" is given by;

$$\mu_B = \frac{eh}{4\pi m_e} = 5.79\times10^{-5}\ eV/T$$

is a convenient unit for atomic magnetic moment (& T stands for Tesla)

In Nuclear Physics the corresponding unit of convenience is the "Nuclear Magneton," defined similarly except that the electron-mass 'm_e' is replaced by proton-mass 'm_p'.

$$\mu_N = \frac{eh}{4\pi m_p} = 3.15\times10^{-8}\ eV/T$$

The magnetic moment of the free electron is nearly one "Bohr magneton", However magnetic moment of free proton is not equal to one nuclear magneton but is equal to $+2.7972\,\mu_N$. We can understand the magnetic moment of proton & Neutron only if we know their internal structure. We can analyse the magnetic moments of heavier nuclei in terms of in terms of the magnetic moments of the constituent protons & neutrons.

Radioactivity

Elements with atomic no. $Z \geqslant 82$ are not stable. They emit three types of radiations α, β & γ. These substances are called radioactive substances & the phenomenon is called radioactivity.

With the emission of α & β, the original element which is called parent element is transformed into a new element called daughter element. The transformations go on until a stable substance is obtained.

Soddy & Fajans, in 1913, discovered the following displacement laws governing radioactive transformations

1) Emission of α-particles from an atom reduces its mass-number A by four units and atomic number 'Z' by two units i.e $\,{}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}x + \alpha\ ({}^{4}_{2}He)$

For example;

$$^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

(Radium) (Radon)

Emission of β-particles from an atom, increases the atomic number 'Z' by one unit with mass no. 'A' unchanged

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}x + {}^{0}_{-1}\beta$$

For example;

$$^{232}_{90}Th \longrightarrow {}^{232}_{91}Pa + {}^{0}_{-1}\beta$$

(Thorium) (Protactinium)

When β-particles are named for those electrons emitted from the nucleus.

With the emission of γ-radiations, neither atomic no. Z nor mass no. A changes i.e

$$^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}x + \gamma$$

where X^* shows the excited nucleus.

Half Life

With the emission of α & β particles, the original element called "parent element" transforms into a new element called "daughter element".

"The Time during which half no. of atoms of parent ~~nucle~~ element transform into the atoms of daughter element is called Half-life", denoted by $T_{1/2}$.

The radio-active decay obeys the following laws.

i) The no. of atoms decayed per second (i.e decay rate $R = \frac{dN}{dt}$) is proportional to the original no. of atoms in the radio-active sample.

ii) The complete decay of the sample of a radioactive element takes infinite time.

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = \lambda N$$

where λ is the constant called decay constant

$$\frac{dN}{N} = \lambda\, dt$$

Integrating both sides

$$\int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} \lambda\, dt$$

$$\left| \ln N \right|_{N_0}^{N} = -\lambda \left| t \right|_0^t = \cancel{\lambda}$$

or $$\ln N - \ln N_0 = -\lambda\,(t-0)$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$e^{-\lambda t} = \frac{N}{N_0}$$

$$N_0 e^{-\lambda t} = N$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

where N_0 = No. of atoms in the sample at $t=0$
N = " " " " " " " time t

Put $N = \frac{N_0}{2}$ & $t = T_{1/2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

or $\frac{1}{2} = \frac{1}{e^{\lambda T_{1/2}}}$

$$\Rightarrow \quad 2 = e^{\lambda T_{1/2}}$$

Taking log of both sides;

$$\ln 2 = \ln e^{\lambda T_{1/2}}$$

$$= \lambda T_{1/2} \cdot \ln e$$

$$\ln 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\boxed{T_{1/2} = \frac{0.693}{\lambda}}$$

This gives an expression for half life of radioactive element.

Mean Life of Radioactive Element:

Since possible life of radioactive values from '0' to '∞', Total life of all the atoms, N_0 initially present in the sample is given by;

$$\text{Total life time} = \int_{t=0}^{t=\infty} t\,dN$$

Mean or average life is given by;

$$T^* = \frac{\text{Total life time}}{\text{Total no. of atoms}} = \frac{\int_0^\infty t\,dN}{\int_0^\infty dN}$$

$$T^* = \frac{\int_0^\infty t\,dN}{-N_0} \qquad \left(\because \int_0^\infty dN = |N|_0^\infty = 0 - N_0 = -N_0\right)$$

But $dN = - \quad t$

$$T^* = \frac{1}{N_0} \quad \lambda N t\,dt$$

$$T^* = \frac{1}{N_0} \int_0^\infty \lambda N_0 e^{-\lambda t}\, t\, dt \qquad \because N = N_0 e^{-\lambda t}$$

$$= \frac{\lambda N_0}{N_0} \int_0^\infty t e^{-\lambda t} dt = \lambda \int_0^\infty t e^{-\lambda t} dt$$

Integrating by parts;

$$\therefore T^* = \lambda \left[\left| \frac{t e^{-\lambda t}}{-\lambda} \right|_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \left| -t e^{-\lambda t} \right|_0^\infty + \int_0^\infty e^{-\lambda t} dt$$

$$= 0 + \left| \frac{e^{-\lambda t}}{-\lambda} \right|_0^\infty$$

$$= -\frac{1}{\lambda} \left[e^{-\infty} - e^0 \right]$$

$$= -\frac{1}{\lambda} (0 - 1)$$

$$\boxed{T^* = 1/\lambda}$$ It is the mean or average life of radioactive element.

Note that $T^* > T_{1/2}$

Alpha Decay

In case of α-decay, the parent nucleus is changed into daughter nucleus with charge no. (i.e atomic no.) 'Z' decreases by '2' & mass no. A decreases by 4.

In case of typical α-emitter radio-nuclide ^{238}U it decays spontaneously according to the eqn.

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

Its half life is 4.47×10^9 years. In every such decay, an energy of 4.27 Mev is emitted appearing as kinetic energy shared b/w the α-particle (^4_2He) & recoiling residual nucleus ($^{234}_{90}Th$).

Beta Decay

A nucleus that decays spontaneously by emitting an electron (either +ve or −ve) is said to under-go beta decay. i.e

$$^{32}P \longrightarrow {}^{32}S + \bar{e} + \bar{\nu} \quad (T_{1/2} = 14.3 \text{ days})$$

$$^{64}Cu \longrightarrow {}^{64}Ni + \bar{e} + \nu \quad (T_{1/2} = 12.2 \text{ hours})$$

The symbols ν & $\bar{\nu}$ represent the neutrino & its antiparticle, the 'anti-neutrino'; neutral particles that are emitted from the nucleus along with electron or positron during the decay process.

The electrons & neutrinos are emitted from the nuclei during β-decay. They are both created during the emission process; a neutrino transforming itself into a proton within the nucleus or conversly:

$$n \longrightarrow p + e^- + \bar{\nu} \quad (\beta\text{-decay})$$

$$p \longrightarrow n + e^+ + \nu \quad (\beta^+\text{-decay})$$

Heat and thermodynamics

(PTO)