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Chapter - 34

# MAGNETIC FIELD EFFECT

## Magnetic Field, B

"Magnetic field is the region or space around a magnet with in which its influence can be felt by other magnetic substances."

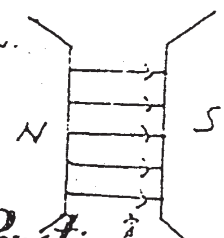
## Magnetic Field Line:

It is the path along which an isolated north pole moves in a magnetic field."

## Uniform Magnetic Field:

The magnetic field is said to be uniform if the magnetic field lines are parallel and equally spaced.

For this purpose the region b/w two flat pole pieces are shown.




## 1- Magnetic Force on a Charged Particle:

It is observed experimentally that a magnetic field exerts a force on a charged particle moving in the field.

Consider a charged particle having charge 'q' projected in a uniform magnetic field of flux density $\vec{B}$ with velocity $\vec{v}$. Then force on the charged particle is given as,

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{— (1)}$$

$$\vec{F} = qvB \sin\theta \, \hat{n}.$$

$\because \vec{F}$ is $\perp$ to the plane of $\vec{V}$ and $\vec{B}$. $\therefore \hat{n}$ is a unit vector $\perp$ to the plane of $\vec{V}$ and $\vec{B}$.

The magnitude of force is given by

$$F = qVB\sin\theta \quad \text{— (2)}$$

From (1) we find that $\vec{F}$ is always $\perp$ to $\vec{V}$ and $\vec{B}$. If we project the particle in different direction, we find that no matter, what the direction of $\vec{V}$, the magnetic force is always $\perp$ to that direction.

From (2) we see that force is zero for $\theta = 0°$ and $\pi$.

So charged particle experiences no force moving parallel or antiparallel to the magnetic field.

From (2) we also note that $F=0$ when $v=0$. i.e magnetic field exerts no force on a charged particle at rest in a magnetic field.

From (2) we see that

$$F \propto v$$
$$F \propto B$$
$$F \propto q$$

$$\therefore F \propto qvB$$

$$F = qvB.$$

We also see that if sign of $q$ is reversed, the direction of $\vec{F}$ is reversed.

From (2) we find that force on the charged particle is max. when $\theta = 90°$.

$\therefore$ Maximum magnitude of force is given by

$$F = qvB.$$

$$\Rightarrow \quad B = \frac{F}{qv}.$$

Where $B$ is called magnetic induction.

It is defined as,

"Force acting on a unit +ve charge moving $\perp$ to a magnetic field with unit velocity."

The SI Unit of $\vec{B}$ is Tesla and is given by

$$1T = \frac{1N}{1C \times 1m/s}$$

Another unit of $\vec{B}$ is Gauss.

$$1\,T = 10^4 \text{ Gauss.}$$

As the magnetic force acts $\perp$ to $\vec{V}$ So this force does [illegible] the magnitude of velocity. Its e.m.f changes the direction [illegible]

So the magnetic force becomes centripetal force. This force [illegible] $\perp$ to the displacement of the particle. So this force can do [illegible] work on the charged particle.

So we find that when a constant force $F = qVB$ acts on the [illegible] this force cannot change the K.E of the particle.

So under the constant magnetic field, the K.E remains constant.

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## Sample Problem-1

**Sample Problem 1** A uniform magnetic field B, with magnitude 1.2 mT, points vertically upward throughout the volume of the room in which you are sitting. A 5.3-MeV proton moves horizontally from south to north through a certain point in the room. What magnetic deflecting force acts on the proton as it passes through this point? The proton mass is $1.67 \times 10^{-27}$ kg.

Sol. $B = 1.2\,mT = 1.2 \times 10^{-3}\,T$

$K.E = 5.3\,MeV = 5.3 \times 10^6 \times 1.6 \times 10^{-19}\,J$, $m = 1.67 \times 10^{-27}\,kg$, $\theta = 90°$

$F = ?$

$$F = qVB \sin\theta$$

$$= 1.6 \times 10^{-19} V \times 1.2 \times 10^{-3} \times \sin 90$$

$$F = 1.92 \times 10^{-22}\, V \quad \text{— (1)}$$

For V ; we have

$$K.E = \frac{1}{2} m v^2.$$

$$V = \sqrt{\frac{2\,K.E}{m}}.$$

$$= \sqrt{\frac{2 \times 5.3 \times 10^6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}}.$$

$$= \sqrt{10.155689 \times 10^{14}} = 3.186 \times 10^7$$

$$V = 3.2 \times 10^7\, m/s.$$

Putting the value of V in (1) we get

$$F = 1.92 \times 10^{-22} \times 3.2 \times 10^7.$$

$$\boxed{F = 6.1 \times 10^{-15}\, N} \quad \text{Ans.}$$

# 2. The Magnetic Force on a Current-Carrying-Conductor:

We know that a magnetic field exerts a force on a charge moving in a magnetic field. As current is the motion of charges. So a current carrying conductor placed in a magnetic field also experiences this force.

Consider a wire of length L carrying current i placed in a uniform magnetic field of flux density $\vec{B}$.

Let A be the area of x-section of the wire. Then its volume is AL.

As current is due to motion of free electron.

Let n be the number of electrons per unit volume then total number of electrons = nAL.

If e is the charge on one electron then total charge flowing in the wire is = q = nALe.

As force on one electron is given by

$$F' = qvB.$$

Here $v = V_d$ the drift velocity and $q = e$.

$$\therefore \quad F' = eV_dB.$$

Suppose electron cover length L in t sec then

$$t = \frac{L}{V_d}.$$

$$\text{Now } i = \frac{q}{t}.$$

$$i = \frac{nALe}{t} = \frac{nALe}{L/V_d} = nAeV_d.$$

$$i = nAeV_d.$$

$$V_d = \frac{i}{nAe}.$$

As force on electron is $F' = eV_dB$.

$\therefore$ Force nAL electron is $F = nALF'$.

$$F = nALeV_dB.$$

$$F = nALe \frac{i}{nAe} B.$$

$$\boxed{F = iLB.}$$

In vector form

$$\vec{F} = i(\vec{L} \times \vec{B}) = iLB\sin\theta\, \hat{n}.$$

The direction of $\vec{F}$ is $\perp$ to the plane of $\vec{L}$ and $\vec{B}$.
If the wire is placed $\perp$ to uniform magnetic field than $\theta = 90°$.

Then $F = iLB$.

If the wire is not straight or the field is not uniform Then we divide the wire into small elements of length $\vec{ds}$. The elections are so small that ~~with~~ the element the wire is straight and field remains uniform for the element.

Then force on each element can be written as,

$$d\vec{F} = i(\vec{ds} \times \vec{B}).$$

The total force on the whole wire is obtained by integrating ove the whole length L.

$$\int d\vec{F} = i\int(\vec{ds} \times \vec{B})$$

$$\vec{F} = i\int(\vec{ds} \times \vec{B}).$$

## Sample Problem-4

**Sample Problem 4** A straight, horizontal segment of copper wire carries a current $i = 28$ A. What are the magnitude and direction of the magnetic field needed to "float" the wire, that is, to balance its weight? Its linear mass density is 46.6 g/m.

Sol. $i = 28A$, $\frac{m}{L} = 46.6\,g/m = 46.6 \times 10^{-3}\,Kg/m.$

$\vec{B} = ?$ To balance the weight of the wire?

For the wire to float

$$mg = iLB.$$

$$B = \frac{mg}{iL}$$

$$B = \frac{m}{L} \times \frac{g}{i}$$

$$= 46.6 \times 10^{-3} \times \frac{9.8}{28} = 16.31 \times 10^{-3}.$$

$$\boxed{B = 16\,mT}$$

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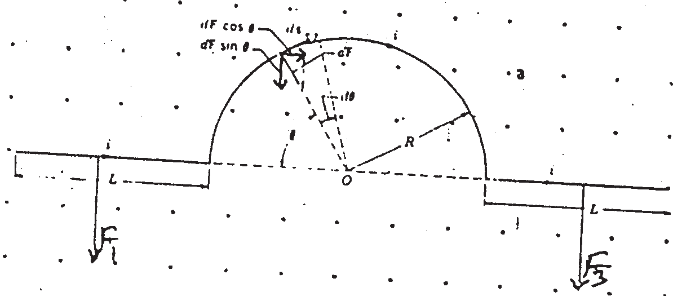
# Sample Problem-5

**Sample Problem 5** Figure 24 shows a wire segment, placed in a uniform magnetic field B that points out of the plane of the figure. If the segment carries a current $i$, what resultant magnetic force F acts on it?




Sol. The magnitude of magnetic orce on each straight portion is

$$F_1 = F_3 = iLB.$$

dF is the force acting on element of length $ds = Rd\theta$.

$$\therefore \; dF = i\,ds\,B = iBRd\theta.$$

It should be noted that only downward component $dF\sin\theta$ is effect. $dF\cos\theta$ is cancelled by the opposite horizontal component due to a symmetrically located element on the opposite side of the arc.

The total force on the central arc portion is

$$F_2 = \int_0^{\pi} dF\sin\theta = \int_0^{\pi} (iBRd\theta)\sin\theta.$$

$$F_2 = iBR\int_0^{\pi} \sin\theta\, d\theta = iBR\left[-\cos\theta\right]_0^{\pi}.$$

$$= iBR\,(-\cos\pi + \cos 0)$$

$$= iBR\,(1+1) = 2iBR.$$

$\therefore$ Resultant force on the entire wire is

$$F = F_1 + F_2 + F_3.$$

$$= iLB + 2iBR + iLB.$$

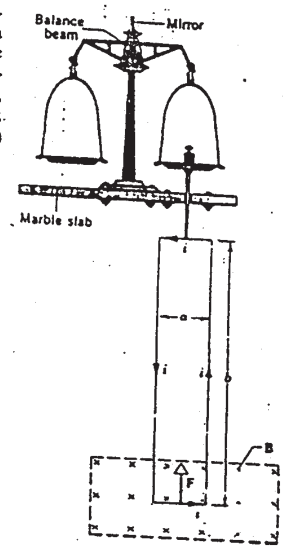
$$= 2iLB + 2iBR.$$

$$\boxed{F = iB(2L + 2R)} \quad \text{Ans.}$$

This is the same force acting on a straight wire of length $(2L+2R)$.

# Sample Problem-6

**Sample Problem 6** A rectangular loop of wire (Fig. 25), consisting of nine turns and having width $a = 0.103$ m and length $b = 0.685$ m is attached to one pan of a balance. A portion of the loop passes through a region in which there is a uniform magnetic field of magnitude $B$ perpendicular to the plane of the loop, as shown in Fig. 25. The apparatus is carefully adjusted so that the weight of the loop is balanced by an equal weight (not shown)




Sol Here width of loop $= a = 0.103\,m$.

length of loop $= b = 0.685\,m$, $\vec{B} = ?$

$i = 0.224$ Amp, $m = 13.7g = 13.7 \times 10^{-3}$ kg.

The forces on two lower portions of long sides cancel.

$\therefore$ Only the force on bottom of loop is

$F = iaB$ for each of nine segments.

For the pan to be in equilibrium.

$$mg = 9F.$$

$$mg = 9iaB.$$

$$B = mg/9ia.$$

$$= \frac{13.7 \times 10^{-3} \times 9.8}{9 \times 0.224 \times 0.103} = 646.57 \times 10^{-3}.$$

$$B = 647 \times 10^{-3}\ T.$$

$$\boxed{B = 0.647\ T}$$

$\vec{B}$ = 0.647T into the plane of page.

## 3- Torque on a Current Loop:

Consider loop of wire carrying current suspended in a uniform magnetic field of flux density $\vec{B}$.

It is found experimentally that magnetic field exerts a torque on the current carrying loop.

Fig. shows a rectangular loop of wire of width 'a' and length 'b'. Then area of the loop is $= ab$.

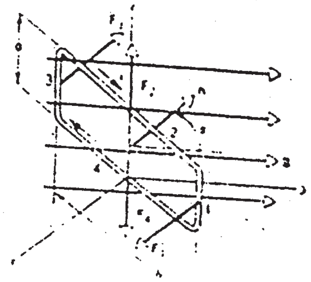
We suppose that the loop is suspended in such a way that it can rotate about any axis.

In the fig. shown the magnetic field is along y-axis and z-axis lies in its plane.

$\hat{n}$ is a unit vector normal to the plane of loop.

Angle between $\hat{n}$ and $\vec{B}$ is $\theta$.

A torque acts on the loop and loop rotates about Z-axis.



The force on each side of loop must be $\perp$ to both $\vec{B}$ and direction of current.

The magnitude of force $F_2$ on side ② of length 'b' is

$$F_2 = ibB \sin(90 - \theta)$$

$$\boxed{F_2 = ibB\cos\theta}$$

The direction of is along +Z-axis.

The force $\vec{F}_4$ on de ④ is of magnitude.

$$F_4 = ibB\sin(90 + \theta) = ibB\cos\theta.$$

The direction of $\vec{F_4}$ is along $-Z$-axis.
So forces $\vec{F_2}$ and $\vec{F_4}$ are equal in magnitude and opposite in direction contribute nothing to the torque because their lines of action pass through Z-axis. So moment arm of both $\vec{F_2}$ and $\vec{F_4}$ is zero.
So torque due to $\vec{F_2}$ and $\vec{F_4}$ is zero.
The forces $\vec{F_1}$ and $\vec{F_3}$ acting on sides ① and ③ have magnitude

$$F_1 = F_3 = F = iaB\sin 90^\circ = iaB.$$

$\vec{F_1}$ and $\vec{F_3}$ both are parallel and antiparallel to x-axis. Net force is zero but net torque is not zero because their lines of action do not pass through Z-axis.
∴ due to torque of $\vec{F_1}$ and $\vec{F_3}$ the loop rotates clockwise.
The moment arms of $\vec{F_1}$ and $\vec{F_3}$ about Z axis are $= \frac{b}{2}\sin\theta$.

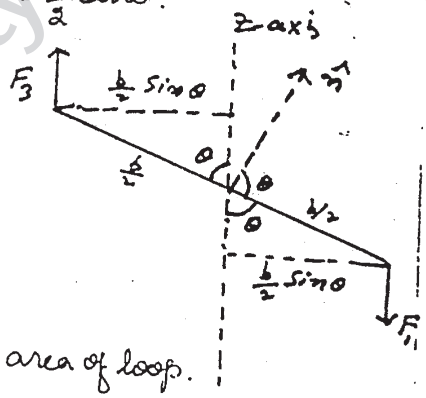
∴ Total torque on the loop is

$$T = F_1 \times \frac{b}{2}\sin\theta + F_3 \times \frac{b}{2}\sin\theta$$

$$= \frac{b}{2}\sin\theta(F_1 + F_3).$$

$$= \frac{b}{2}\sin\theta(iaB + iaB)$$

$$= \frac{b}{2}\sin\theta(2iaB)$$




but $ab = A$ = area of loop.

$$\therefore \boxed{T = iAB\sin\theta}$$

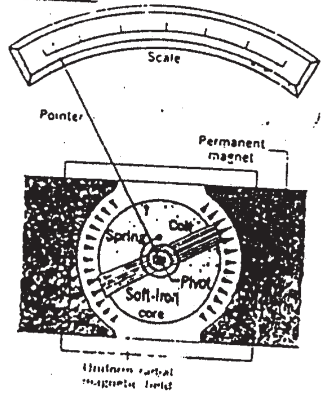
This is the torque for a single turn. For N turns the torque is

$$\boxed{T = NiAB\sin\theta}$$

This is the expression for torque. This equation is valid for loops of any shape whether rectangular or not.

## Sample Problem - 7

...mple Problem 7   Analog voltmeters and ammeters, in which ... reading is displayed by the deflection of a pointer over a scale, ...rk by measuring the torque exerted by a magnetic field on a ...rent loop. Figure 27 shows the rudiments of a *galvanometer*, ...which both analog ammeters and analog voltmeters are ...ed. The coil is 2.1 cm high and 1.2 cm wide; it has 250 turns ...l is mounted so that it can rotate about its axis in a uniform ...al magnetic field with $B = 0.23$ T. A spring provides a coun...orque that balances the magnetic torque, resulting in a steady ...ular deflection $\phi$ corresponding to a given steady current $i$ in ...oil. If a current of 100 μA produces an angular deflection of ...° (= ... rad), what must be the torsional constant κ of the ...ing?



Sol:-

length $= b = 2.1\,cm = 2.1 \times 10^{-2} m$.

Width $= a = 1.2cm = 1.2 \times 10^{-2} m$.

$N = 250$, $B = 0.23T$. $i = 100\,\mu A = 100 \times 10^{-6} A = 10^{-4}$ Ampere.

$\phi = 28^\circ = 0.49\,rad$, $\theta = 90^\circ$.

Torsional constant K of the spring $= K = ?$

According to given condition.

Magnetic torque = Restoring torque

$$NiAB\sin\theta = K\phi.$$

$$K = \frac{NiAB\sin\theta}{\phi}$$

$$= \frac{Niab\sin\theta}{\phi}$$

$$= \frac{250 \times 10^{-4} \times 2.1 \times 10^{-2} \times 1.2 \times 10^{-2} \times 0.23 \times \sin 90^\circ}{0.49}$$

$$= 295.7 \times 10^{-8}$$

$$= 2.957 \times 10^{-6}$$

$$\boxed{K = 3.0\ 10^{-6}\ N\text{-}m/rad.}$$ Ans.

## 4. Magnetic Dipole:

As we know that an electric dipole placed in an electric field of intensity $\vec{E}$ experiences a Torque given by,

$$\vec{T} = \vec{P} \times \vec{E}.$$

where $\vec{P} = q\vec{d}$ is called dipole moment.

The magnitude of this torque is given by

$T = PE\sin\theta$ where $\theta$ is the angle b/w $\vec{P}$ and $\vec{E}$.

Similarly we can obtain an expression for torque acting on a magnetic dipole placed in a magnetic field of flux density $\vec{B}$.

For this purpose, we define a vector $\vec{\mu}$ called magnetic dipole moment having magnitude

$\mu = NIA$. The direction of $\vec{\mu}$ is normal to

" T " " ' " 1st.

A current carrying loop placed in a magnetic field acts as a magnetic dipole.

The torque acting on a current carrying loop in a magnetic field is given by,

$$\tau = NIAB\sin\theta.$$

Putting $\mu = NIA$. we get.

$$\tau = \mu B\sin\theta \quad -(1).$$

In vector form the above expression becomes;

$$\vec{\tau} = \vec{\mu}\times\vec{B} \quad -(2).$$

Equation (2) gives torque on a current carrying loop in a magnetic field of flux density $\vec{B}$. This eq. (2) holds for any shape of loop.

The work done on the magnetic dipole to change its orientation in the magnetic field becomes P.E of magnetic dipole which is given as,

$$U = \text{Work done}$$

$$= \int \tau\, d\theta.$$

From (1) $\tau = \mu B\sin\theta$.

$$\therefore U = \int \mu B\sin\theta\, d\theta. = \mu B\int \sin\theta\, d\theta.$$

$$= \mu B(-\cos\theta)$$

$$= -\mu B\cos\theta$$

$$\boxed{U = -\vec{\mu}.\vec{B}}$$

This is the expression for P.E of magnetic dipole.

The unit of $\mu$ is obtained by dividing the energy unit by unit of magnetic induction.

$\therefore$ Unit of $\mu$ is $\frac{\text{Joule}}{\text{Tesla}} = \frac{J}{T}$ — (A).

There is another unit of $\mu$ from the expression.

$$\mu = NIA.$$

$$\text{Unit} = \text{Ampere}\times m^2 \text{ — (B)}.$$

Both the units of $\mu$ shown by (A) and (B) are identical.

Show that $\frac{J}{T} = Amp\text{-}m^2$.

$$\frac{J}{T} = \frac{Fd}{T} = \frac{ILBd}{T}$$

$$= \frac{Amp \times m^2 \times \cancel{T}}{\cancel{T}}$$

$$\boxed{\therefore \frac{J}{T} = Amp \times m^2.}$$

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## Sample Problem - 8

Sample Problem 8 (a) What is the magnetic dipole moment of the coil of Sample Problem 7, assuming that it carries a current of 85 μA? (b) The magnetic dipole moment of the coil is lined up with an external magnetic field whose strength is 0.85 T. How much work would be done by an external agent to rotate the coil through 180°?

Sol (a) $\mu = ?$ of coil of sample Prob. 7, $I = 85\,\mu A = 85 \times 10^{-6}$ A.

As $\mu = NIA$.

$= 250 \times 85 \times 10^{-6} \times 2.52 \times 10^{-4}$.

$= .53550 \times 10^{-10}$

$$\boxed{\mu = 5.3550 \times 10^{-6} \frac{J}{T}}$$ Ans.

From sample Prob. 7
$N = 25$.
$A = 2.52 \times 10^{-4}\,m^2$.

(b). $B = 0.85T$.

Work done to rotate the coil through $180° = ?$

As work done becomes the increase in P.E.

$\therefore$ Work $= \Delta U$.

$= U_f - U_i$.

$= -\mu B \cos 180° - (-\mu B \cos 0°)$.

$= -\mu B(-1) + \mu B$.

$= \mu B + \mu B = 2\mu B$.

Work $= 2 \times 5.36 \times 10^{-6} \times 0.85$.

$= 2 \times 5.36 \times 0.85 \times 10^{-6}$.

$= 9.1 \times 10^{-6}$ Joule.

$\boxed{\text{Work} = 9.1\,\mu J}$ Ans.

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THE END.