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Chapter - 36

Faradays Law of Electromagnetic Induction

Magnetic Flux:-

"The number of magnetic lines of force passing normally through certain area is called magnetic flux."

It is denoted by ϕ_B. It is a scalar quantity. It is measured by the product of area and component of $\vec{B}$ normal to the area. If we have surface element of vector are $\vec{dA}$ placed in uniform magnetic field having magnetic field strength $\vec{B}$. Then the magnetic through $\vec{dA}$ is given by

$$d\phi_B = \vec{B}.\vec{dA}.$$

The magnetic flux thru the whole surface is obtained by integrating the above expression over the entire surface through which we want to calculate the flux.

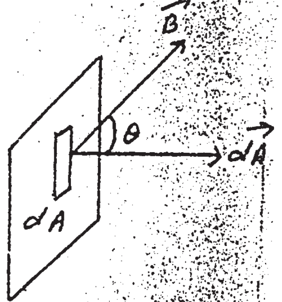
$$\therefore \int d\phi_B = \int \vec{B}.\vec{dA}$$

$$\phi_B = \int \vec{B}.\vec{dA}$$

$$\phi_B = \int B\,dA\cos\theta$$

Where θ is the angle b/w magnetic field $\vec{B}$ and normal to the plane of area. Now $\int dA = A$, the area of the whole surface. If $\vec{B}$ is uniform then above relation becomes

$$\phi_B = BA\cos\theta.$$



Unit of Magnetic Flux:

The S.I unit of magnetic flux is Weber (Wb).

It is given as

$$\phi_B = BA.$$

$$\boxed{1\,Wb = 1\,Tesla \times 1\,m^2}$$

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Faraday's Law

Statement: According to this law

"The induced emf in a circuit is equal to the negative of the rate at which the magnetic flux through the circuit is changing with time." OR.

"The magnitude of induced e.m.f in a circuit is directly propotional to the rate of change of magnetic flux."

Explaination:

We know that when current flows through a conductor, a magnetic field is produced around it. The question arises whether current can be produced by a magnetic field? Faraday showed that when a magnet is moved towards or away from a coil having galvanometer joined between its ends, the galvanometer shows deflection. The current which flows in the coil is called induced current and the e.m.f set up across the coil is called induced emf. This phenomenon is called electromagnetic induction. Same is the result when the magnet is kept stationary and the coil is moved towards or away from the magnet.

According to Faraday whenever the magnetic flux passing through a coil changes, an emf is induced across it which lasts till the flux remains changing. The Faraday's law gives the magnitude of induced emf.

For a coil having single turn, this law is expressed mathematically as,

$$e \propto \frac{d\phi}{dt}$$

$$e = -\text{constt} \times \frac{d\phi}{dt}.$$

In S.I system the constant is unity.

$$\therefore e = -\frac{d\phi}{dt}.$$

Where -ve sign occurs due to direction of induced emf which is given by a law known as Lenz's law.

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If a coil consisting of N turns this law can be written as

$$e = -N\frac{d\phi}{dt}.$$

Where ϕ is the flux linked with one turn of the coil.
It should be noted that flux can be changed in many ways.

e.g (i) By moving a magnet relative to coil.
(ii) By changing the current in the nearby circuit.
(iii) By changing the size or shape of loop.
(iv) By moving the loop in a non-uniform magnetic field.

Causes of Induced e.m.f

The cause of induced emf in a loop is the passage of variable magnetic flux through the loop.
Due to relative motion b/w magnet and the loop, the flux changes and induced emf is produced.
Due to variable current, the flux passing through the loop changes and an emf is produced in it.
When a constant magnetic flux passes through a loop no emf is produced.

So from the above discussion we arrive at the result that induced emf is produced due to the change of flux.

Sample Problem 1

Sample Problem 1 The long solenoid S of Fig. 4 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter d is 3.2 cm. At its center we place a 130-turn close-packed coil C of diameter $d_C = 2.1$ cm. The current in the solenoid is increased from zero to 1.5 A at a steady rate over a period of 0.16 s. What is the absolute value (that is, the magnitude without regard for sign) of the induced emf that appears in the central coil while the current in the solenoid is being changed?

Solution: $n = 220$ turns/cm $= 220 \times 100$ turns/m. $i = 1.5$ Amp
$d = 3.2$ cm $= 3.2 \times 10^{-2}$ m, $N = 130$ turns, $\theta = 0$
Diameter of central coil $= d_c = 2.1$ cm $= 2.1 \times 10^{-2}$ m.
Radius of central coil $= r_c = \frac{d_c}{2} = 1.05 \times 10^{-2}$ m.
Magnitude of induced emf in the central coil $= E = ?$

$$\phi_B = BA \quad \text{--- (1)}$$

Now for a Solenoid $B = \mu_0 n i$.

$$= 4\pi \times 10^{-7} \times 220 \times 100 \times 1.5$$

$$= 4.144 \times 10^{-2}$$

$$\boxed{B = 4.15 \times 10^{-2}\ T}$$

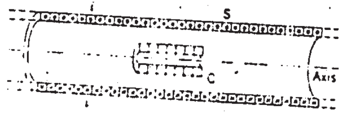

Fig.

Putting B in (1) we get

$$\phi_B = 4.15 \times 10^{-2} \times A \quad \text{--- (2)}$$

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Now area of central coil is given by

$$A = \pi r^2.$$
$$= \pi \times (1.05 \times 10^{-2})^2.$$
$$= 3.14 \times 1.05 \times 1.05 \times 10^{-4}.$$
$$= 3.46 \times 10 \, m^2.$$

$\therefore$ Equation (2) becomes

$$\phi_B = 4.15 \times 10^{-2} \times 3.46 \times 10^{-4}.$$
$$= 4.15 \times 3.46 \times 10^{-6} = 14.359 \times 10^{-6}$$
$$= 14.4 \times 10^{-6} \, wb.$$
$$\phi_B = 14.4 \, \mu wb$$

$\therefore$ The magnitude of induced emf in the central coil is given by

$$E = N \frac{\Delta \phi}{\Delta t}.$$

$$E = \frac{130 \times 14.4 \times 10^{-6}}{0.16} = 11700 \times 10^{-6}.$$
$$= 1.17 \times 10^{-2}$$
$$= 1.2 \times 10^{-2} \text{ volt.} = 12 \times 10^{-3} \text{ volts.}$$

$$\boxed{E = 12 \, mV} \quad \text{Ans.}$$

3. Lenz's Law:

This law gives the direction of induced emf.

According to this law

"The direction of induced emf is always s.t it opposes its own cause."

Explaination:

To explain this law we consider a bar magnet pushed towards a stationary conducting loop as shown in fig (A)

As the magnet is pushed towards the loop towards right, the magnetic flux through the loop increases. To oppose this

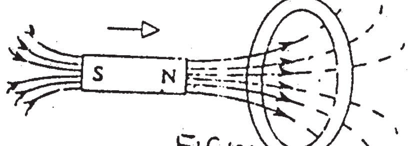

FIG (A)

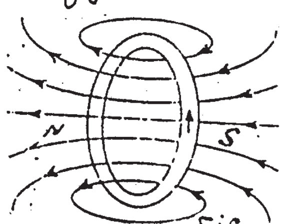

FIG B

increase in flux to the right

the induced current produces a flux to the left as shown in fig (B)

It should be noted that the magnetic field lines associated with

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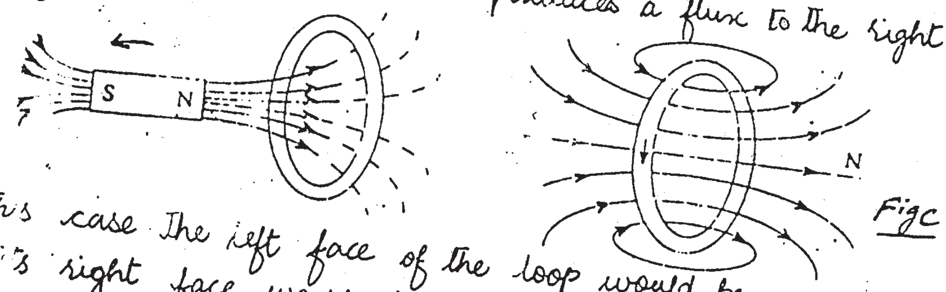
the induced current oppose the motion of magnet. So the left fac of the loop acts as a north pole and the right face as a s uth pole.

On the other hand the magnet is pulled away from the loop, the fl x through the loop will decrease. To compensate this decrease in f x the induced current produces a flux to the right as shown in f c



Figc

In this case the left face of the loop would be a south pole and its right face would be north pole.

L 's Law & Conservation of Energy:-

Lenz's law is the result of law of conservation of energy.
When we pull or push the magnet, its motion is automatically opposed.
So the agent that moves the magnet towards or away from the loop always experiences a resistive force.
So when we move the magnet, we have to do work against this resistive force which arises due to intersection of original magnetic field and the magnetic field due to the induced current. If we move the magnet more rapidly, the rate of work becomes greater.
So this work which is done in moving the magnet becomes the source of induced emf. So this work done is converted into electrical energy. So electromagnetic induction is exactly in accordance with the law of conservation of energy.
Thus Lenz's law is a particular form of law of conservation of energy.
However, it must be noted that Lenz's law is applied only to a close loop. e.g if we cut the loop and then move the magnet, no induced current is set up. There is still an emf in the loop like a battery connected to an open circuit, no current is set up.

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Eddy Currents

"The induced currents produced due to change of magnetic flux through a large piece of conducting material are called Eddy current."

In some cases eddy currents produce undesirable effects. e.g They increase the internal energy and hence temperature of material increases. For this reason the material subjected to changing magnetic fields are made in the form of small layers insulated from each other.

So instead of one large loop, the eddy currents flow in smaller loops and the length path of eddy currents is increased. In this way resistance increases and so power loss $\frac{V^2}{R}$ is reduced.

So increase in internal energy is smaller. On the other hand the eddy current heating can be used to develop induction furnace. In an induction furnace a material is heated by rapidly changing magnetic field.

Induction furnaces are used in those cases where direct thermal contact with the material to be heated is not possible.

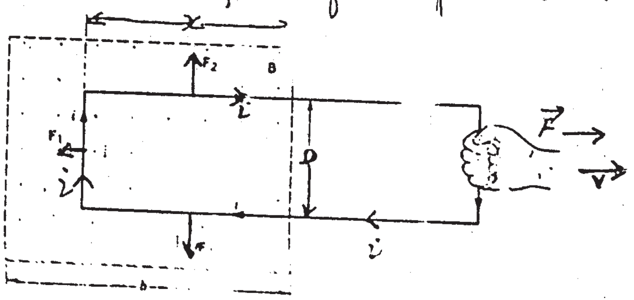
Eddy currents are real currents and their effect is the same as that of real currents.

Motional E.m.f

"The emf induced in a loop by moving it in a magnetic field is called Motional emf."

Consider a rectangular loop of wire of width 'D' having one end in a uniform magnetic field $\vec{B}$ ⊥ to the plane of the loop as shown.

We pull the loop towards right by applying a force F by our hand. Suppose the loop is pulled with constt speed 'V'.



We want to calculate mechanical power $P = FV$ expended by hand

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in pulling the loop.

The flux ϕ_B enclosed by the loop is given by

$$\phi_B = BDx.$$

Where Dx is the area of the part of loop within magnetic field. By Faraday's law of electromagnetic induction, we have

$$E = -\frac{d\phi_B}{dt}.$$

$$= -\frac{d}{dt} BDx. = -BD\frac{dx}{dt}$$

$$E = BD\left(-\frac{dx}{dt}\right).$$

Putting $-\frac{dx}{dt} = V =$ speed at which the loop is pulled out of the magnetic field and x is decreasing.

$$\therefore E = BDV \quad \text{—} (1)$$

Due to the emf an induced current is produced in the loop and is given by

$$i = E/R$$

$$i = \frac{BDV}{R} \quad \text{—} (2)$$

Where 'R' is the resistance of the loop.

This induced current gives rise to magnetic forces F_1, F_2 and F_3 acting on three sides of the loop.

Therefore these can be written by the expression

$$\vec{F} = i(\vec{L}\times\vec{B})$$

Because $\vec{F}_1$ and $\vec{F}_3$ being equal and opposite cancel away.

So only the force $\vec{F}_1$ which is opposite to the direction of motion of loop has magnitude

$$F_1 = iDB\sin 90^\circ$$

$$F_1 = iDB.$$

Putting the value of i from (2) we get

$$F_1 = \frac{BDV}{R}(DB).$$

$$F_1 = \frac{B^2D^2V}{R} \quad \text{—} (3).$$

As the loop is being pulled with uniform speed then $\vec{F}_1$ and $\vec{F}$ should be equal and opposite.

$$\therefore F_1 = .$$

$$\therefore F = \frac{B^2D^2V}{R}.$$

$\therefore$ the power expended by hand is

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$$= \frac{B^2D^2V}{R}(V)$$

$$\boxed{P = \frac{B^2D^2V^2}{R}} \quad — (4).$$

Now the power dissipated due to Joule heating is given by

$$P = i^2R$$

Putting the value of i from (2) we get

$$P = \left(\frac{BDV}{R}\right)^2 R.$$

$$= \frac{B^2D^2V^2}{R^2}\cdot R$$

$$P = \frac{B^2D^2N^2}{R} \quad — (5)$$

From (4) and (5) we find that they are identical.
So rate of doing work by external agent is dissipated as Joule heating of the loop.

Sample Problem - 3

Sample Problem 3 A copper rod of length R rotates at angular frequency ω in a uniform magnetic field **B** as shown in Fig. 11. Find the emf $\mathcal{E}$ developed between the two ends of the rod. (We might measure this emf by placing a conducting rail along the dashed circle in the figure and connecting a voltmeter between the rail and point O.)

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Solution. Length of rod = R, Angular frequency = ω, Emf = E = ?
If a wire of length dr is moved $\perp$ to $\vec{B}$ then emf produced is given by,

$$dE = VBdr$$

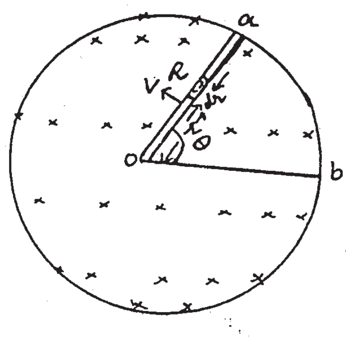
$$\therefore \quad dE = r\omega Bdr. \qquad \because V = r\omega$$

The emf induced by moving the entire rod of length R is given by.

$$\int dE = \int_0^R r\omega Bdr = \omega B\int_0^R rdr$$

$$= \omega B \left| \frac{1}{2}r^2 \right|_0^R$$

$$\boxed{E = \frac{1}{2}B\omega R^2} \quad \text{Ans}$$



ALTERNATE METHOD.

Now flux enclosed by sector aob is

$$\phi_B = BA$$

$$= B \times (\text{Area of sector } aob).$$

But area of sector $aob = \frac{1}{2}R^2\theta$.

$$\therefore \phi_B = B\times\frac{1}{2}R^2\theta.$$

Diff. w.r.t 't' we get

$$\frac{d\phi_B}{dt} = \frac{1}{2}BR^2\frac{d\theta}{dt}$$

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But $\frac{d\theta}{dt} = \omega$

$\therefore \frac{d\phi_B}{dt} = \frac{1}{2} BR^2\omega$

But $\frac{d\phi_B}{dt} = E.$

$\boxed{E = \frac{1}{2} B\omega R^2}$ Ans.

Rough $\rightarrow$

By Geometry

$\frac{\text{Area of sector aob}}{\text{area of circle}} = \frac{\theta}{2\pi}$

Area of sector aob $= \frac{\theta}{2\pi} \times \pi r^2$

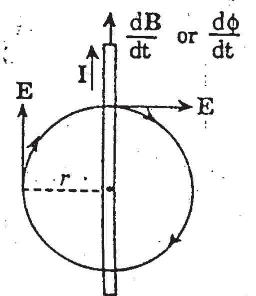
$= \frac{1}{2} R^2\theta.$

Induced Electric Field

We know that straight wire carrying current is surrounded by a magnetic field. This magnetic field is represented by magnetic lines of force which are found to be concentric circles having their centre on the wire.

Similarly a changing magnetic field is surrounded by an electric field called induced electric field. This induced electric field is represented by electric lines of force which are found to be concentric circles.

Let us find expression for induced electric field. Consider a loop of conducting wire placed in a uniform magnetic field. This magnetic field may be applied by an external electromagnet. By varying the current in the electromagnet, we can change the strength of the magnetic field. When $\vec{B}$ is changed, the magnetic flux through the loop changes, so by Faraday's law and Lenz's law we can find the magnitude and direction of induced emf and induced current in the loop.



The induced electric field occurs due to changing magnetic field. This induced electric field is as real as the electric field set up by static charges. Moreover the induced electric field remains present in the presence a[illegible]ll as in the absence of conducting loop. So electric field will be pr[illegible] even if we remove the loop of wire completely.

So now consider a circular path of radius 'R' instead of conducting loop. Suppose the magnetic field changes at the rate $\frac{dB}{dt}$. The [illegible] which is changing at the rate

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around the circular path and an induced electric field is produced around the circular path.

Now work done on a unit +ve charge in moving it once around the circular path is called electromotive force.

$$\therefore \ e = \oint \vec{E} \cdot \vec{dl} = \oint E\,dl \cos 0^\circ = E \oint dl$$

$$e = E \times 2\pi r \quad \text{—(1)}$$

$$e = \frac{d\phi}{dt} \quad \text{—(2)} \ \text{(Numerically)}$$

From (1) and (2).

$$E \times 2\pi r = \frac{d\phi}{dt}.$$

$$\boxed{E = \frac{1}{2\pi r} \frac{d\phi}{dt}}.$$

This is the required expression for induced electric field.

Sample Problem - 4

Sample Problem 4 In Fig. 12b, assume that $R = 8.5$ cm and that $dB/dt = 0.13$ T/s. (*a*) What is the magnitude of the electric field E for $r = 5.2$ cm? (*b*) What is the magnitude of the induced electric field for $r = 12.5$ cm?

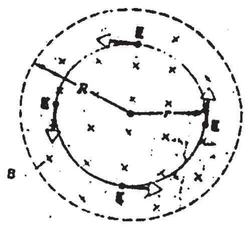

Fig 12 b

Solution. $R = 8.5\text{cm} = 8.5 \times 10^{-2}\text{m}$. $\frac{dB}{dt} = 0.13\,T/s$

(a) $E = ?$ for $r = 5.2$ cm $= 5.2 \times 10^{-2}$ m

(b) $E = ?$ for $r = 12.5$ cm $= 12.5 \times 10^{-2}$ m.

(a) As we know that

$$E \times 2\pi r = -\frac{d\phi_B}{dt} \quad \text{—(1)}.$$

Now $\phi_B = B \times A$.

$$\phi_B = B \times \pi r^2.$$

$\therefore$ Eq. (1) becomes

$$E \times 2\pi r = -\frac{d}{dt}(B\pi r^2)$$

$$E \times 2\pi r = \pi r^2 \left(-\frac{dB}{dt}\right)$$

$$2E = r\left(-\frac{dB}{dt}\right).$$

Taking magnitude only

$$E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$= \frac{1}{2} \times 5.2 \times 10^{-2} \times 0.13$$

$$= 0.34 \times 10^{-2} = .0034\,V/m$$

$$E = 3.4 \times 10^{-3}\,V/m = \boxed{3.4\,mV/m} \ \text{Ans.}$$

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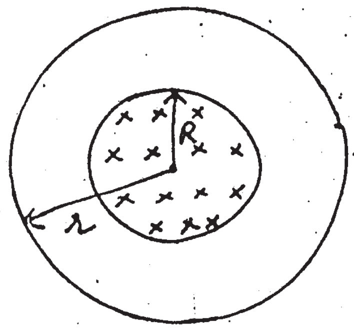
11.

(b) $r > R$.

So the entire flux passes through circular path.

$$\therefore \phi_B = B \times \pi R^2$$

$\therefore$ Eq. (1) becomes



$$E \times 2\pi r = -\frac{d}{dt}(B\pi R^2)$$

$$E = \frac{R^2}{2r} \cdot \frac{dB}{dt}.$$

Putting values of $r, R, \frac{dB}{dt}$ we get,

$$E = \frac{(8.5 \times 10^{-2})^2}{2 \times (12.5 \times 10^{-2})} \times 0.13$$

$$= \frac{8.5 \times 8.5 \times 0.13}{25} \times 10^{-2}$$

$$= 0.3757 \times 10^{-2} = .00375.$$

$$= .0038 = 3.8 \times 10^{-3}\,N/m$$

$$\boxed{E = 3.8\ mv/m}$$ Ans

Then end.

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