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ELECTOSTATICS

Electrostatics:

"The branch of Physics which deals with the charges at rest is called electrostatics."

1- Electric Charge:

"It is an agent which endows substances the property of attracting or repelling some other substances."

All bodies in nature are electrically neutral. but when we rub two different bodies with each other, both the bodies are charged. Actually due to rubbing, free electrons are transferred from one body to another and there is no creation of charge. When electrons are transferred from one body to the other then both the bodies become a little bit imbalanced in charge. In a neutral body there is a balance between +ve and -ve charges. If a body gets charged, it means that the body has charge imbalance.

Like charges repel each other while un-like charges attract each other. In electrostatics we consider only those charges that are either at rest with respect to each other or move very slowly.

2- Conductors and Insulators:

"The materials through which charges can easily flow are called conductors." e.g copper (Cu), metals in general, tap water, human body are common examples of conductors.

"The materials which donot allow the flow of charges are called insulators."

e.g Glass, pure water, plastic are common examples of insulators. However perfect conductor or perfect insulator donot exist in nature.

In the solid 'Cu', the outermost electrons don't remain attached to the particular atom but move about in the latice and are called conduction electrons.

In a tipical conductor like copper (Cu), there are about 10^{23} conduction electrons per cm^3. However in insulators there is hardly one electron per cm^3.

In between conductors and insulators there is an other class of materials known as semi-conductors. These are the materials whose resistivity and conductivity lies between those of conductors and insulators. The most common examples of semiconductors are silicon (Si) and germenium (Ge). A tipical semiconductor contains 10^{10} to 10^{12} conduction electrons per cm^3. However the no. of conduction electrons can be changed (per cm^3) by adding acceptor or donor impurity. The addition of impurities to the pure semiconductor called doping.

3- Coulomb Law for point charges

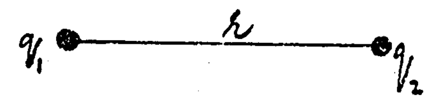
Point Charges: "Two charges are said to be the point charges if distance between them is greater as compare to their sizes"

Coulomb's Law: We know that like charges repel and unlike charges attract each other. Coulomb's law gives the magnitude of the force between two point charges. According to this law;

"The force of attraction or repulsion b/w two point charges is directly proportional to the product of the magnitude of two charges and inversely proportional to the square of the distance b/w them."

Consider two point charges q_1 and q_2 placed in free space at a distance 'r' apart. Then the force b/w q_1 and q_2 is given by

$$F \propto \frac{q_1 q_2}{r^2}$$



$$F = K \frac{q_1 q_2}{r^2} \quad \text{———} (1)$$

Here 'F' gives the magnitude of the force which either charge exerts on the other. 'K' is the constant of proportionality. Its value depends on the medium in which charges are placed and the system of units. For example in system international the value of 'K' for free space is $9\times10^9\ Nm^2/C^2$. For CGS-system for free space $K = 1\ dyne\ cm^2/stat\ coul^2$.

To separate out the constant of free space, the constant K is changed into another constant as;

$$K = \frac{1}{4\pi\varepsilon_0}$$

Where ε_0 is permitivity of free space. Its value is $8.85\times10^{-12}\ C^2/Nm^2$. It is also called the permitivity constant.

so equ. (1) can be written as;

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Unit of Charge:

The SI unit of charge is coulomb. It is defined as;

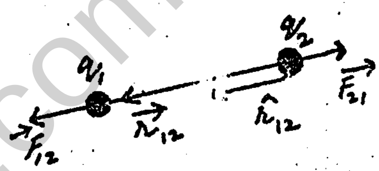
"The amount of charge that flows through any section of wire in one second when steady current of one ampere (1A) is flowing"

The CGS-unit of charge is stat-coulomb. 'Coulomb' (SI-unit) and 'stat-coulomb' (CGS-unit) are related as;

$$1\,C = 3\times10^{9}\ \text{Stat-C}$$

Vector form of "Coulomb's Law"

Consider two point charges q_1 & q_2 lying r_{12} distance apart. then the force exerted by q_2 on q_1 is given by



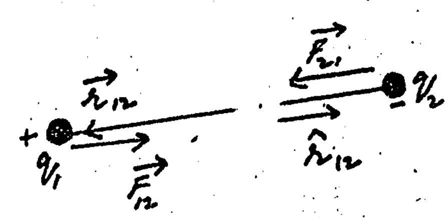
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}^2}\ \hat{r}_{12}$$

where $\hat{r}_{12}$ is a unit vector parallel to $\vec{r}_{12}$ and gives the direction of the force. Here r_{12} gives the magnitude of the vector $\vec{r}_{12}$. $\vec{r}_{12}$ is the position vector of q_1 w.r.t. q_2.

If the charges q_1 and q_2 have the opposite signs. Then the electric force b/w them is attractive. Then $\vec{F}_{12}$ is anti-parallel to $\vec{r}_{12}$..

Similarly force exerted by q_1 on q_2 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{21}^2}\cdot\hat{r}_{21}$$



where $\hat{r}_{21}$ is a unit vector directed from q_1 to q_2.

It must be noted that Coulomb's Law is consistant with newtons third law of motion. i.e. $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude but opposite direction.

The vector form of coulomb's law is useful because it gives the information about the direction of coulomb's-force when attractive or repulsive.

Coulomb's Force on a Single Charge due to distribution of N-point-charges.

Where many charges are present as source charges, then force on any one test charge due to all other charges is equal to the vector sum of all the forces exerted by the source charges on that of the test charge.

EXAMPLE:

The force on charge q_1 due to the charges $q_2, q_3, q_4, \text{----------} q_N$ is given by the principle of superposition as

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \text{--------} + \vec{F}_{1N} \quad \text{—— (A)}$$

$$= K\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + K\frac{q_1 q_2}{r_{13}^2}\hat{r}_{13} + K\frac{q_1 q_2}{r_{14}^2}\hat{r}_{14} \cdots K\frac{q_1 q_2}{r_{1N}^2}\hat{r}_{1N}$$

Where $\vec{F}_{12}$ is the force exerted by q_2 on q_1.

$\vec{F}_{13}$ " " " " " q_3 on q_1.

Equ. (A) is the application of the principle of superposition.

$$\vec{F}_i = \sum_{i \neq j}^{N} K\frac{q_i q_j}{r_{ij}^2}\hat{r}_{ij}$$

Principle of Superposition

"According to this principle force on any charge due to many charges is calculated by measuring the force b/w single pair of charges (a test and a source charge) assuming all other as absent."

e.g. F_{12} is the force exerted by q_2 on q_1, assuming $q_3, q_4, q_5, \text{------} q_N$ as absent.

Siagnificance of Coulomb's Law

This law describes

1) the forces (i.e electrical-forces) that binds the electron of an atom to its nucleus.
2) the electrical forces that binds the atoms together to form a molecule.
3) the electrical forces that binds atoms & molecules together to form solids & liquids.

Conditions (UNDER WHICH COULOMBS LAW HOLDS GOOD)

Coulomb's Law holds good when
1) Charges are point charges.
2) Charges are stationary.

Sample Problem - 1

Sample Problem 1 Figure 6 shows three charged particles, held in place by forces not shown. What electrostatic force, owing to the other two charges, acts on q_1? Take $q_1 = -1.2\ \mu C$, $q_2 = +3.7\ \mu C$, $q_3 = -2.3\ \mu C$, $r_{12} = 15$ cm, $r_{13} = 10$ cm, and $\theta = 32°$.

Sol.

$q_1 = -1.2\ \mu C = -1.2 \times 10^{-6} C$
$q_2 = +3.7\ \mu C = +3.7 \times 10^{-6} C$
$q_3 = -2.3\ \mu C = -2.3 \times 10^{-6} C$
$r_{12} = 15\ cm = 0.15\ m$
$r_{13} = 10\ cm = 0.1\ m$
$\theta = 32°$

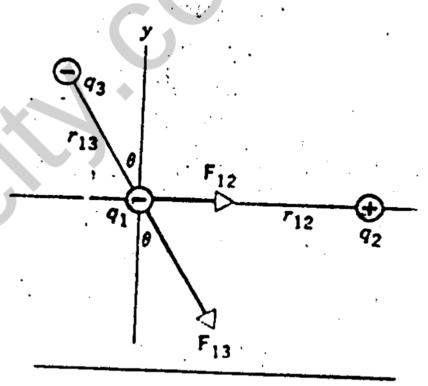


Fig 6

Force exerted by q_2 on q_1 is attractive. Its direction is along x-axis. Its magnitude is

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

$$= 9\times10^{9} \times \frac{1.2\times10^{-6} \times 3.7\times10^{-6}}{0.15\times0.15}$$

$$= \frac{9\times1.2\times3.7}{0.15\times0.15} \times 10^{9-6-6}$$

$$= \frac{39.96}{0.0225} \times 10^{-3}$$

$$= 1776\times10^{-3}$$

$F_{12} = 1.77$

$F_{12} = 1.77\ N$

Force exerted by q_3 on q_1 is repulsive. Its direction is as shown (next page-7). Its magnitude is

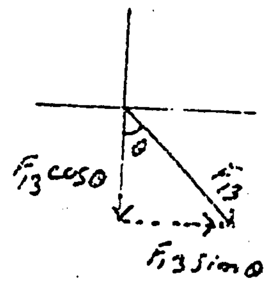
$$F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_{13}^2}$$

$$= 9\times10^9 \times \frac{1.2\times10^{-6} \times 2.3\times10^{-6}}{0.1\times0.1}$$

$$= \frac{9\times1.2\times2.3}{0.1\times0.1} \times 10^{9-6-6}$$

$$= \frac{24.84}{0.01} \times 10^{-3}$$

$$= 2484 \times 10^{-3}$$



$$\boxed{F_{13} = 2.48\ N}$$

Now resolving F_{13} into its rectangular components, we have

$$F_{13x} = F_{13}\sin\theta$$

$$= 2.48 \times \sin 32^\circ$$

$$= 2.48 \times 0.5299$$

$$\boxed{F_{13x} = 1.314\ N}$$

$$F_{13y} = F_{13}\cos\theta$$

$$= 2.48 \times \cos 32^\circ$$

$$= 2.48 \times 0.848$$

$$\boxed{F_{13y} = 2.103\ N}$$

Now the total force along x-axis is

$$F_{1x} = F_{12} + F_{13x}$$

$$= 1.77 + 1.314$$

$$\boxed{F_{1x} = 3.08\ N}$$

Now the total force along y-axis is

$$F_{1y} = F_{13y}$$

$$= -2.103\ N$$

$$\boxed{F_{1y} = -2.10\ N}$$

Magnitude of resultant forces on q_1 is given by

$$F_1 = \sqrt{(F_{1x})^2 + (F_{1y})^2}$$

$$F_1 = \sqrt{(3.08)^2 + (-2.10)^2}$$

$$= \sqrt{9.4864 + 4.41}$$

$$= \sqrt{13.8964}$$

$$= 3.727$$

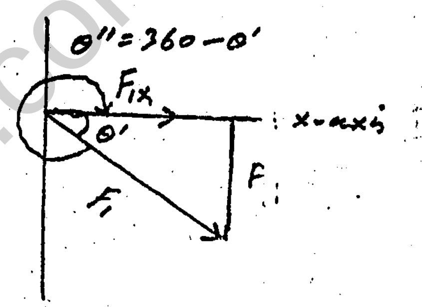
$\boxed{F_1 = 3.73\ N}$ is the magnitude of resultant force on charge q_1

Ans.

Now the direction of F_1 is given by

$$\tan\theta' = \left(\frac{F_{1y}}{F_{1x}}\right)$$

$$= \left(\frac{2.10}{3.08}\right)$$



$$\tan\theta' = 0.68$$

$$\theta' = \tan^{-1} 0.68$$

$\theta' = 34^\circ$ below +ve x-axis

$$\theta'' = 360 - 34$$

$\theta'' = 326^\circ$ with +ve x-axis.

x——————————x

4 - Quantization of Charge

In Franklin's day electric charge was considered as a continuous fluid but we know that fluid consist of atoms and molecules & matter has complete discrete nature. So fluid is not continuous but discrete. Experiment shows that electrical fluid is not continuous but it is made up of multiples of certain basic elementary unit of charge in any charge +q or −q can be written as

$$q = ne \quad \text{where } n = 0, \pm 1, \pm 2, \ldots \qquad (1)$$

and 'e' is the unit of elementary charge.

The value of e is determined experimentally is

$$e = 1.6 \times 10^{-19} C$$

This elementary charge is one of the fundamental constants of nature. When a physical quantity has discrete values, we say that it is quantized.

As we know already that matter, energy and angular momentum are quantized. The quantization of charge is further addition in this list.

Equ. ① shows that it is possible to find a particle having zero, ±10e, +6e & 15e charge but it is impossible to find a particle possessing charge equal to 3.57e, −5.9e or 6.3e etc.

According to new theory, protons and neutrons are made up of more fundamental particle called quarks. It means quarks exist in protons & neutrons but uptill now the science has failed to produce a free quark out of proton or neutron. Perhaps the quarks are bound so strongly in protons or in neutrons that we have insufficient energy to produce free quark. However complete explaination to observe free quark is not yet clear.

There is no theory by which charge of an electron can be calculated nor there is any theory that explains why electronic charge is equal and opposite to that of charge on proton.

Sample Problem - 2

Sample Problem 2· A penny, being electrically neutral, contains equal amounts of positive and negative charge. What is the magnitude of these equal charges?

1.

$m = 3.11\ g$

$M = 63.5\ g/mol.$

$N_A = 6.02 \times 10^{23}$ atom/mol.

$Z = 29$ (for Cu) $e = 1.6 \times 10^{-19}\ C$

... ze; ... be the no. of atoms in penny and 'n' be the ... of moles in penny, then

$$n = \frac{m}{M} \quad \text{——— (1)}$$

As $N = n N_A$ (where N_A is Avogadro no.)

$$n = \frac{N}{N_A} \quad \text{——— (2)}$$

From equ (1) and (2)

$$\frac{N}{N_A} = \frac{m}{M}$$

$$N = \frac{m N_A}{M}$$

$$= \frac{3.11 \times 6.02 \times 10^{23}}{63.5}$$

$$= \frac{5.72 \times 10^{23}}{63.5}$$

$$= 0.2948 \times 10^{23}$$

$$N = 2.95 \times 10^{22} \text{ atoms}$$

Now Ze ... the magnitude of charge in one Cu-atom (either +ve or -ve).

Then magnitude of total +ve or -ve charge in N atoms is given by

$$q = NZe$$

$$= 2.95 \times 10^{22} \times 29 \times 1.6 \times 10^{-19}$$

$$= 2.95 \times 29 \times 1.6 \times 10^{22-19}$$

$$= 136.88 \times 10^{3}$$

Magnitude ... +ve or -ve charge ... Cu-penny $\boxed{q = 1.37 \times 10^{5}\ C}$ Ans.

Sample Problem - 3

Sample Problem 3 In Sample Problem 2 we saw that a copper penny contains both positive and negative charges, each of a magnitude 1.37×10^5 C. Suppose that these charges could be concentrated into two separate bundles, held 100 m apart. What attractive force would act on each bundle?

Sol.

$q_1 = 1.37 \times 10^5 C$

$q_2 = 1.37 \times 10^5 C$

$r = 100 m$

$F = ?$

As $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$= 9 \times 10^9 \times \frac{1.37 \times 10^5 \times 1.37 \times 10^5}{100 \times 100}$

$= \frac{9 \times 1.37 \times 1.37}{10^4} \times 10^{9+5+5}$

$= 9 \times 1.37 \times 1.37 \times 10^{19-4}$

$= 16.89 \times 10^{15}$

$\boxed{F = 1.69 \times 10^{16} N}$ Ans.

Sample Problem - 4

Sample Problem 4 The average distance r between the electron and the proton in the hydrogen atom is 5.3×10^{-11} m. (*a*) What is the magnitude of the average electrostatic force that acts between these two particles? (*b*) What is the magnitude of the average gravitational force that acts between these particles?

Sol.

$q_1 = 1.6 \times 10^{-19} C$

$q_2 = 1.6 \times 10^{-19} C$

$r = 5.3 \times 10^{-11} m$

$m_e = 9.11 \times 10^{-31} kg$

$m_p = 1.67 \times 10^{-27} kg$

$F_e = ?$, $F_g = ?$

As $F_e = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$$= 9\times10^9 \times \frac{1.6\times10^{-19} \times 1.6\times10^{-19}}{5.3\times10^{-11} \times 5.3\times10^{-11}}$$

$$= \frac{9\times1.6\times1.6}{5.3\times5.3} \times 10^{9-19-19+11+11}$$

$$= 0.82\times10^{-7}$$

$$\boxed{F_e = 8.2\times10^{-8} N}$$ Ans.

As $F_g = \frac{G m_e m_p}{r^2}$

$$= \frac{6.67\times10^{-11} \times 9.11\times10^{-31} \times 1.67\times10^{-27}}{5.3\times10^{-11} \times 5.3\times10^{-11}}$$

$$= \frac{6.67\times9.11\times1.67}{5.3\times5.3} \times 10^{-11-31-27+11+11}$$

$$\boxed{F_g = 3.6\times10^{-47} N}$$ Ans.

Sample Problem - 5

Sample Problem 5 The nucleus of an iron atom has a radius of about 4×10^{-15} m and contains 26 protons. What repulsive electrostatic force acts between two protons in such a nucleus if they are separated by a distance of one radius?

Solution:

$q_1 = q_2 = 1.6\times10^{-19}$ C

$r = 4\times10^{-15} m$

$F = ?$

As $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$$= \frac{9\times10^9 \times 1.67\times10^{-19} \times 1.6\times10^{-19}}{4\times10^{-15} \times 4\times10^{-15}}$$

$$= \frac{9\times1.6\times1.6}{16} \times 10^{9-19-19+15+15} = 1.44\times10^1$$

$$\boxed{F = 14.4 N}$$ Ans.

5- Conservation of Charge:

When glass rod is rubbed, with silk, a +ve charge appears on the rod. Measurement shows that an equal and opp--osite -ve charge appears on the silk. This means that rubbing does not create charge, but merely transfers it from one object to another by sligh--tly disturbing the neutrality of the each. So we find that charge is conserved. i.e. it can't be created or destroyed but can be transferred from one body to the other body. So conservation of charge is defined as

"In an isolated system the total amount of charge is always conserved."

Examples of charge conservation;

An interesting example of charge conservation is the phenomenon of pair anihilation. In this process an electron and a positron combine and dis--appear and two γ-ray photons are produced.

The net charge before and after event is zero. So charge is conserved

i.e. $e^- + e^+ \longrightarrow \gamma + \gamma$

The neutral π meson decays into two γ-ray photons as;

$$\pi^\circ \longrightarrow \gamma + \gamma$$

π-meson is chargeless and the two γ-ray photons are also chargeless. So the charge is conserved i.e total charge before and after decay is zero.

In β-decay, the neutron decays into a proton and β-particle i.e electron and another neutral particle neutrino. The total charge before and after decay is zero, hence the ~~to~~ charge is conserved.

i.e $n \longrightarrow p + \beta + \nu$

In $+\beta$ decay and proton decay into neutron and

+β particle i.e positron and another neutral particle antineutrino. So the total charge before and after decay is same, hence charge is conserved

i.e $p \longrightarrow n + {}_{+}\beta + \bar{\nu}$

In pair production, a photon disappears to give a pair of electron and positron. So before and after the pair production, charge is conserved.

i.e $\gamma \longrightarrow e^+ + e^-$

When two nuclei of deuterium are fuse together to conserve charge as

(a) $_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + p$

(b) $_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + n$

On left hand side, net charge is +2e and on R.H.S net charge is also +2e. So on each side of reaction charge is same. Hence charge is conserved. Similarly in second reaction, there is a charge of +2e on either side. Thus the reaction also conserves charge.

It is due to the conservation of charge that we can not see a proton emitted when the 2nd reaction takes place or a neutron emitted when the Ist reaction takes place.

x———x